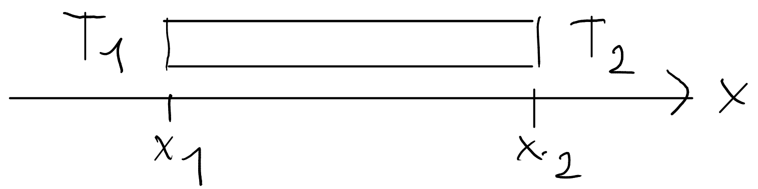
Conduzione termica in regime stazionario

Eq. del calore : $\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_v} \frac{\partial^2 T}{\partial x^2}$ Regime stazionario : $\frac{\partial}{\partial t} = 0 \Rightarrow \frac{\partial T}{\partial t} = 0$, $T(x)$



$\frac{\partial^2 T}{\partial x^2} = 0$ $\frac{d^2 T}{dx^2} = 0$ (stazionario)

$\sum \vec{F} = m\vec{a}$

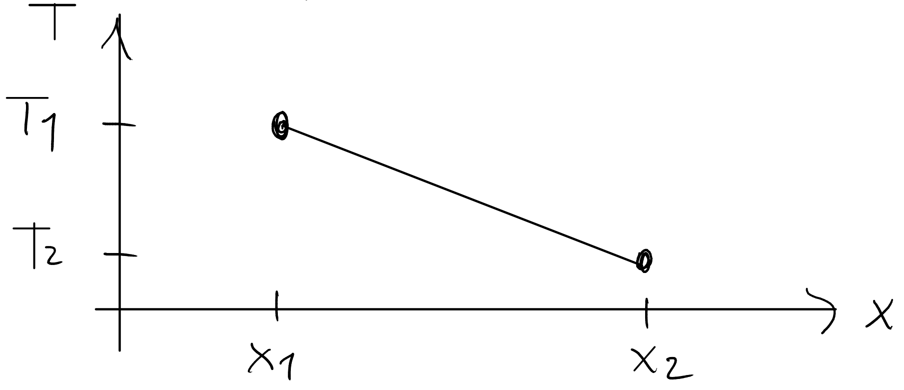
$\vec{0} = \vec{a} = \frac{d^2 \vec{r}}{dt^2}$

$\frac{dT}{dx} = A$

↑ cost

$\int_{x_1}^{x_2} \frac{dT}{dx} dx = A \int_{x_1}^{x_2} dx$

$\int_{T_1}^{T_2} dT = A(x_2 - x_1) \Rightarrow \underline{T_2 = T_1 + A(x_2 - x_1)}$

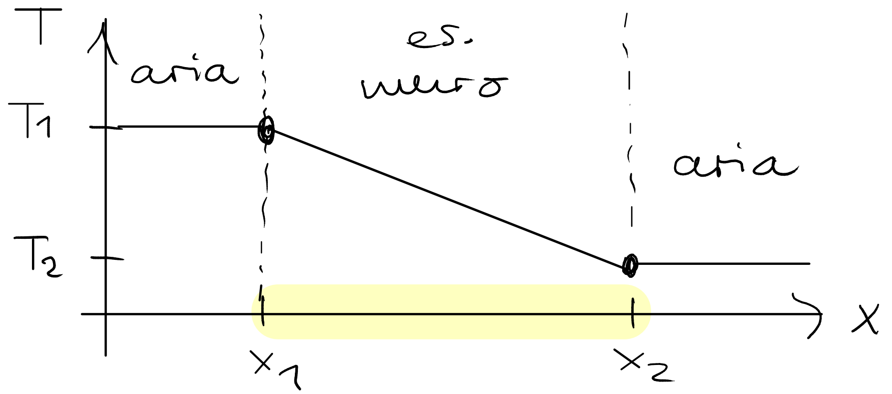


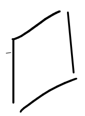
$T_2 = T_1 + A(x_2 - x_1) = T_1 + AL$ $L \equiv x_2 - x_1$

Legge di Fourier : $J_t = -\lambda \frac{dT}{dx} \Rightarrow \frac{dT}{dx} = -\frac{J_t}{\lambda} = cost$

Regime stazionario : $J_t = cost \Rightarrow I_t = cost$

$T_2 - T_1 = -\frac{J_t}{\lambda} L$





S Σ_s sezione materiale ($\to A$)

$$J_t S = I_t$$

$$T_1 - T_2 = \frac{I_t}{S\lambda} L = \frac{L}{\lambda S} I_t$$

$$\Delta T = \underbrace{\frac{L}{\lambda S}}_{\text{resistenza termica}} I_t \qquad \Delta T \sim I_t$$

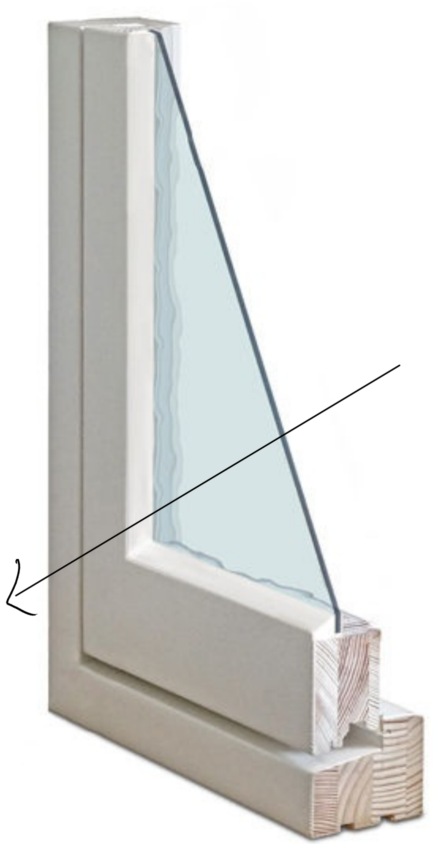
$R_t \ (\equiv R) \qquad SI: \ \frac{K}{W} = K W^{-1}$

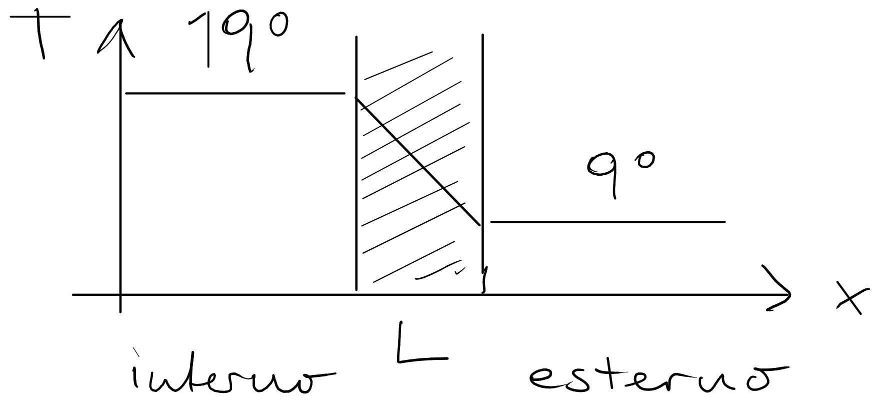
$$\Delta T = R I_t \qquad \Sigma \vec{F} = m \vec{a} \qquad \Delta V = R I$$

$$I_t = \frac{1}{R} \Delta T$$

Es.: Vetro semplice

$S = 5\,m^2 \quad L = 5\,mm \quad \lambda = 1\,\frac{W}{m\cdot K}$ Corrente termica $I_t = ?$





$R = \frac{L}{S\lambda} \qquad \Delta T = R I_t$

$R = \frac{\cancel{5}\times 10^{-3}\,m}{\cancel{5}\,m^2 \times 1\,\frac{W}{mK}} = 10^{-3}\,\frac{K}{W}$

$I_t = \frac{\Delta T}{R} = \frac{10\,K}{10^{-3}\,K}\cdot W = 10\,KW$

Costo elettricità:

0,25 €/KW.h → energia

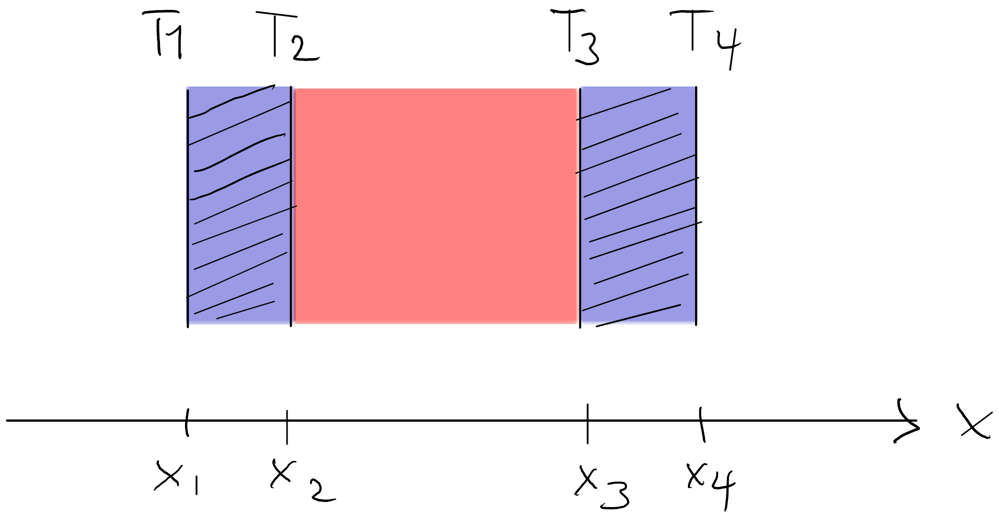
1 giorno, 1 mese → € ? 60 €/giorno 1900 €/mese

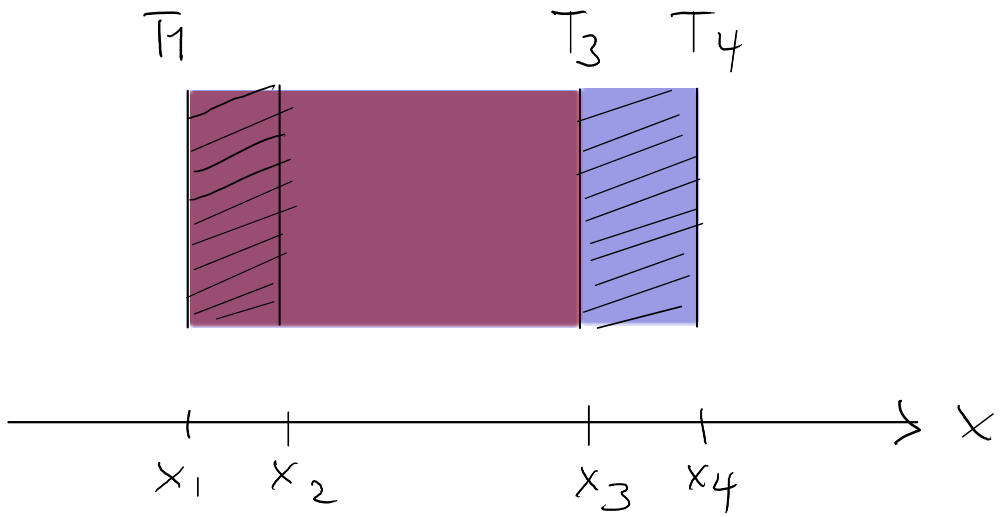
$Q = I_t\,\Delta t = 10\,KW \cdot 24h = 240\,KW\cdot h$

costo $= 240\,kWh \cdot 0.25\,\frac{€}{kWh} = 60\,€$ (1 giorno)

costo $= 1800\,€$ (1 mese)

Resistenze termiche in serie





Regime stazionario : $\boxed{I_t = \text{cost}}$

$$\begin{cases} T_1 - T_2 = R_{12} I_t \\ T_2 - T_3 = R_{23} I_t \end{cases}$$

$$\left.\begin{array}{l} T_1 - T_3 = (R_{12} + R_{23}) I_t \\ \underbrace{T_1 - T_3}_{\Delta T} = R_{13} \, I_t \end{array}\right\} \Rightarrow R_{13} = R_{12} + R_{23}$$

Resistenze termiche in serie si sommano !

$$\left.\begin{array}{l} T_1 - T_3 = R_{13} I_t \\ T_3 - T_4 = R_{34} I_t \end{array}\right\} \Rightarrow R_{14} = R_{12} + R_{23} + R_{34}$$

$$R_{tot} = \sum_{i=1}^{M} R_i$$

Es.: doppio vetro

$A = 5\,m^2 \quad L = 5\,mm \quad \lambda = 1\,\frac{W}{m\cdot K} \quad \lambda_{aria} = 0.026\,\frac{W}{m\cdot K} \quad x_3 - x_2 = 10\,mm$

Corrente termica: $I_t = ?$

Associazione in serie

$$R_{12} = R_{34} = 10^{-3}\,\frac{K}{W} \qquad (\text{vetro})$$

$$R_{23} = 0.08\,\frac{K}{W} \qquad (\text{aria})$$

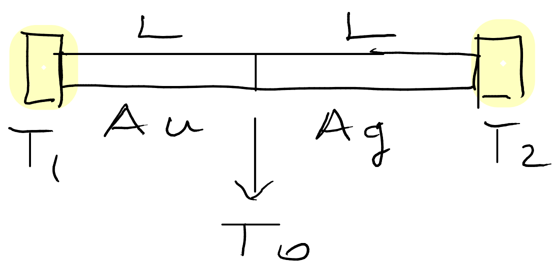
$$= \frac{L}{\lambda_{aria} S} = \frac{2}{0.026} R_{12} \approx 80\, R_{12}$$

$$R_{tot} = R_{12} + R_{34} + R_{23} = 2\times10^{-3}\,\frac{K}{W} + 8\times10^{-2}\,\frac{K}{W}$$

$$= 8{,}2\times10^{-2}\,\frac{K}{W}$$

$$I_t = \frac{\Delta T}{R_{tot}} = \frac{10\,K}{8{,}2\times10^{-2}\,K}\,W \approx 120\,W \approx \frac{1}{83}\, I_t^{singolo}$$

ES: barre di Au e Ag in serie



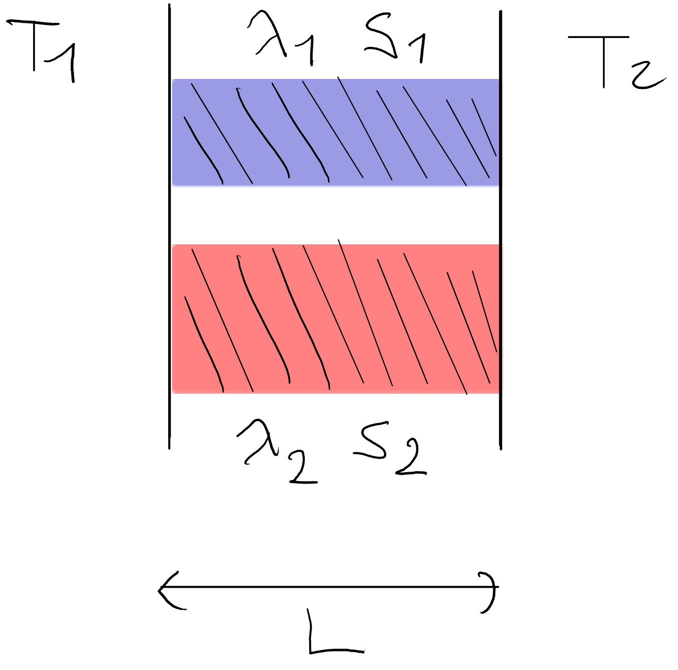
$T_1 = 80°$

$T_2 = 30°$

regime stazionario

$T_0 = ?$

Resistenze termiche in parallelo



$\Delta T = T_1 - T_2$

$$I_t = I_{t1} + I_{t2} \qquad [\, I = I_1 + I_2 \,]$$

$$\begin{cases} I_t = \dfrac{\Delta T}{R_1} + \dfrac{\Delta T}{R_2} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)\Delta T \\ I_t = \dfrac{\Delta T}{R_{tot}} \end{cases}$$

$$\boxed{\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}} \qquad \boxed{R_{tot} = \frac{R_1 R_2}{R_1 + R_2}}$$

Resistenze in parallelo : sommiamo i reciproci delle resistenze

$$\frac{1}{R_{tot}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

$R_1 = R_2 = R$

serie : $R_{tot} = R_1 + R_2 = 2R$ parallelo : $\dfrac{1}{R_{tot}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{2}{R} \Rightarrow R_{tot} = \dfrac{R}{2}$