Vectors and Axes

Vectors are physical quantities that have a magnitude and a direction and that are defined only with respect to a given coordinate system.

All geological orientations have a direction in space with respect to a given coordinate system. However, it makes no difference on which end of the line you put the arrow for many calculations. Thus, we make an informal distinction between vectors, which are lines with a direction (i.e., an arrow at one end of the line) and axes, which are lines with no directional significance.

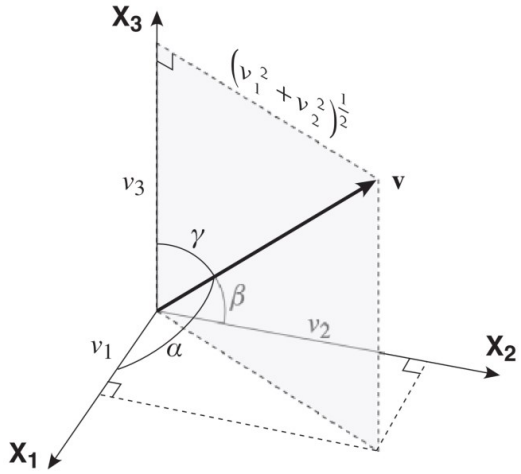
When we use a lower hemisphere stereographic projection, we are automatically treating all lines as axes, as we assume all lines point downwards.

BASIC VECTOR NOTATION

$$\vec{v} = \mathbf{v} = [v_1\ v_2\ v_3]$$

Vectors in 3dimensional space can be described by **three scalar components**.

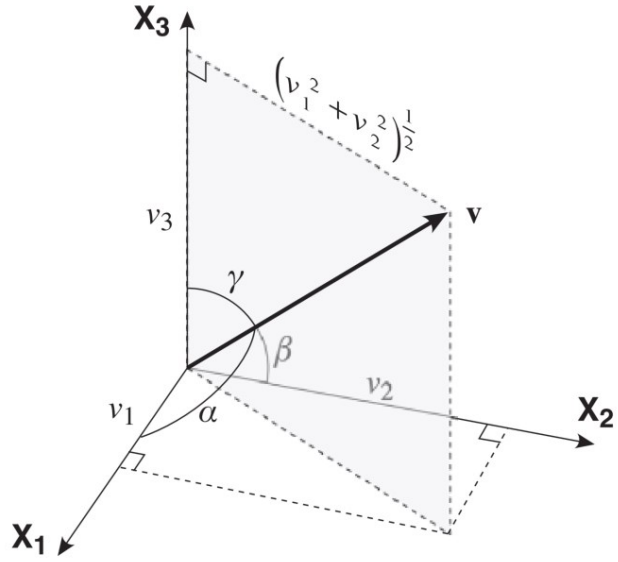
In a Cartesian coordinate system, they give the magnitude of the vector in the direction of the three axes.



BASIC VECTOR NOTATION

$$v = |\mathbf{v}| = \sqrt[2]{v_1^2\ v_2^2\ v_3^2}$$

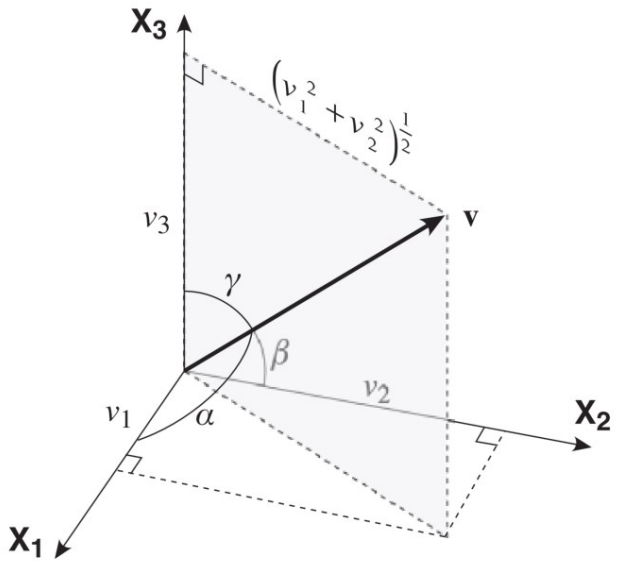
The **magnitude of a vector** is, graphically, just the length of the arrow.



DIRECTION COSINES

A unit vector is just a vector with a magnitude of 1.

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left[\frac{v_1}{|\mathbf{v}|} \; \frac{v_2}{|\mathbf{v}|} \; \frac{v_3}{|\mathbf{v}|}\right]$$



If we pay attention to the figure on the left, we see that the cosine of the angle that a vector makes with a particular axis is just equal to the **component of the vector along that axis divided by the magnitude of the vector**.

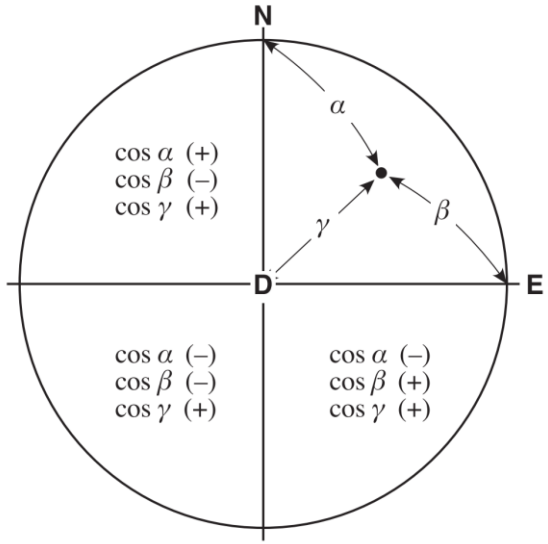
$$\cos\alpha = \frac{v_1}{|\mathbf{v}|} \qquad \cos\beta = \frac{v_2}{|\mathbf{v}|} \qquad \cos\gamma = \frac{v_3}{|\mathbf{v}|}$$

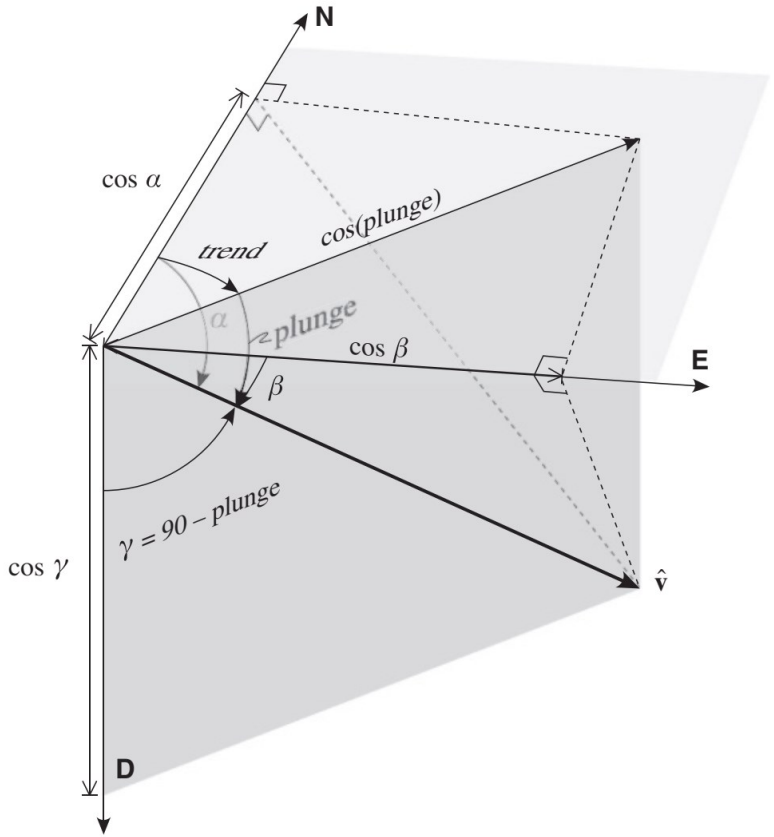
$$\hat{\mathbf{v}} = [\cos\alpha \cos\beta \cos\gamma]$$

These scalars are known as **direction cosines**!

SPHERICAL COORDINATES TO N-E-D CARTESIAN COORDINATES

Any orientation can be expressed as a unit vector, whose components are the direction cosines.





North $\cos\alpha = \cos(trend)\cos(plunge)$

East $\cos\beta = \sin(trend)\cos(plunge)$

Down $\cos\gamma = \sin(plunge)$

MATLAB FUNCTION

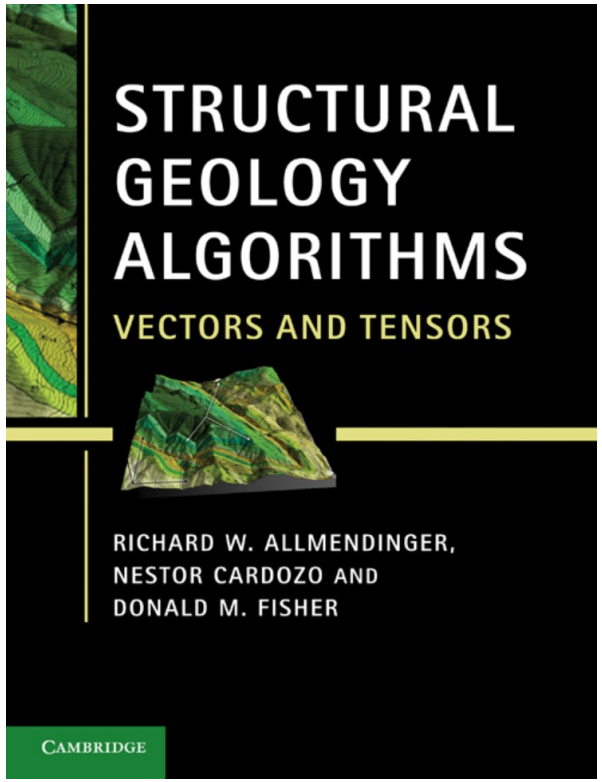
```
% dati in input
trd = 127
plg = 2

trd = deg2rad(trd);
plg = deg2rad(plg);

cos_alpha = (cos(plg)*cos(trd));
cos_beta = (cos(plg)*sin(trd));
cos_gamma = (sin(plg));

x = [cos_alpha cos_beta cos_gamma]

norm (x)                    %verifica che sia uguale a uno
```



For further reading