FORMULARIO METODI

Analisi complessa:

Determinazione principale logaritmo: $Log(\omega) = \log(|\omega|) + iArg(\omega)$

Identità: $\sin(x + iy) = \sin(x)\cosh(y) + i\cos(x)\sinh(y)$

Identità: $\cos(x + iy) = \cos(x)\cosh(y) - i\sin(x)\sinh(y)$

Residuo polo ordine n: $Res(f, z_0) = \dfrac{1}{(n-1)!}\lim_{z\to z_0}\dfrac{d^{n-1}}{dz^{n-1}}((z - z_0)^n f(z))$

Laplace:

Trasformate notevoli e proprietà:

$f(t) = \sin(at) \implies \mathcal{L}\{f(t)\}(s) = \dfrac{a}{s^2 + a^2}$

$f(t) = \sinh(at) \implies \mathcal{L}\{f(t)\}(s) = \dfrac{a}{s^2 - a^2}$

$f(t) = \cos(at) \implies \mathcal{L}\{f(t)\}(s) = \dfrac{s}{s^2 + a^2}$

$f(t) = \cosh(at) \implies \mathcal{L}\{f(t)\}(s) = \dfrac{s}{s^2 - a^2}$

$\mathcal{L}\{t^k u(t)\} = \dfrac{k!}{s^{k+1}} = \dfrac{\Gamma(k+1)}{s^{k+1}}$

$g(t) = f(t-a)\cdot u(t-a) \implies G(s) = F(s)e^{-sa}$

$g(t) = f(t)e^{\gamma t} \implies G(s) = F(s-\gamma)$

$g(t) = f(t\omega) \implies G(s) = \dfrac{1}{\omega}F\left(\dfrac{s}{\omega}\right)$

Derivata trasformata: $F^k(s) = (-1)^k\mathcal{L}\{t^k f(t)\}(s)$

Trasformata segnali periodici: $F^*(s) = F(s)(1 - e^{-sT})$

Convoluzione: $(f * g)(x) = \displaystyle\int_{\mathcal{R}^N} f(y)g(x-y)dy$

Fourier:

Coefficienti serie Fourier: $a_n = \dfrac{2}{T}\displaystyle\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cos(n\omega t)dt$ $\quad b_n = \dfrac{2}{T}\displaystyle\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sin(n\omega t)dt$ $\quad c_n = \dfrac{1}{T}\displaystyle\int_0^T x(t)e^{-in\omega t}dt$

Trasformate notevoli e proprietà:

$f(t) = e^{-a|t|} \implies \mathcal{F}\{f(t)\}(\omega) = \dfrac{2a}{a^2 + \omega^2}$

$f(t) = \dfrac{1}{a^2 + t^2} \implies \mathcal{F}\{f(t)\}(\omega) = \dfrac{\pi}{a}e^{-a|\omega|}$

$f(t) = e^{-at^2} \implies \mathcal{F}\{f(t)\}(\omega) = e^{-\frac{\omega^2}{4a}}\sqrt{\dfrac{\pi}{a}}$

$f(x) = \chi_{[a,b]}(x) \implies \mathcal{F}\{f(x)\}(\omega) = \dfrac{e^{-i\omega a} - e^{-i\omega b}}{i\omega}$

$\hat{p}_{2d}(\omega) = 2d\,sinc\left(\dfrac{d\omega}{\pi}\right)$

$\hat{\Lambda}_a = a^2 sinc\left(\dfrac{\omega a}{2\pi}\right)$

$g(x) = f(x - x_0) \implies \hat{g}(\omega) = e^{-i\omega x_o}\hat{f}(\omega)$

$g(x) = e^{i\omega_o x}f(x) \implies \hat{g}(\omega) = \hat{f}(\omega - \omega_o)$

$\mathcal{F}\{f^k\}(\omega) = (i\omega)^k\mathcal{F}\{f\}(\omega)$

$g(x) = \overline{f(x)} \implies \hat{g} = \overline{\hat{f}(-\omega)}$

$g(x) = f(\alpha x) \implies \hat{g}(\omega) = \dfrac{1}{|\alpha|}\hat{f}\left(\dfrac{\omega}{\alpha}\right)$

$g(x) = x\cdot f(x) \implies \dfrac{d\hat{f}}{d\omega}(\omega) = -i\hat{g}(\omega)$

Plancherel: $\|\hat{f}\|_{L^2}^2 = 2\pi\|f\|_{L^2}^2$