Cosmology 1

2021/2022
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Third intermediate test
Topic: Early universe.
Deadline: June 10, 13:00.

We have computed the number of e-folds that inflation must provide to solve the horizon problem, assuming an inflationary De Sitter phase. Now assume that inflation is driven by an inflaton that has an equation of state parameter

$$-1 < w < -\frac{1}{3}$$

thus giving rise to a power-law inflation. If the number of e-folds is defined as

$$\mathrm{e}^{\mathcal{N}_e} = \frac{d_{Hi}}{d_{Hf}}$$

where d_{Hi} and d_{Hf} are the initial and final values of the comoving Hubble horizon, the computation of the number of e-folds will not change with respect to the De Sitter case, but the duration of inflation will change as a function of w, and the energy density will vary in the inflation phase.

(1) Compute the duration of inflation as a function of w, and compare it with that of the De Sitter model.

(2) Compute the evolution of energy density during inflation, and use it to argue what values of w should be preferred in this context.

(3) Later, in front of an aperitivo, you chat with a friend that knows physics but does not study cosmology (think to an engineering student). He/she asks you what inflation is, and both of you are nerd enough to take the question seriously. Try to answer in simple words, but using all terms in the most correct way.

Solution

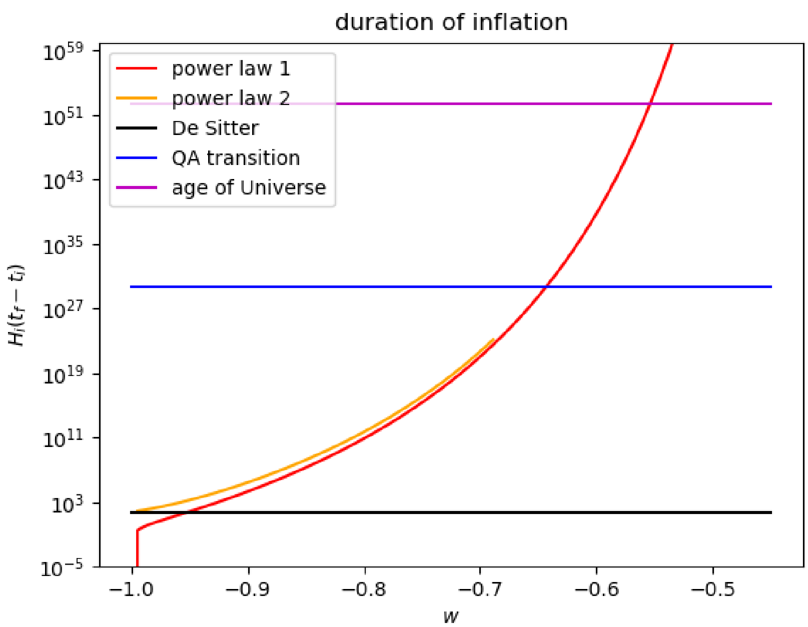
Inflation should last long enough to let the Hubble horizon decrease by a factor of $e^{\mathcal{N}_e}$, where $\mathcal{N}_e \geq 60$. We will stick to the lower limit of this parameter. Considering that $a(t) \propto t^{\alpha}$ where $\alpha = 2/3(1+w)$, it is easy to show that the Hubble horizon scales like

$$d_H \propto t^{\frac{1+3w}{3(1+w)}}$$

I will express the duration as $H_i(t_f - t_i)$, where $t_i = 10^{-35}$ s, $H_i = 1/2t_i$ is the initial Hubble parameter computed assuming radiation dominated era at the inflation start, and t_f the time at which the Hubble horizon has shrinked as required, so that inflation can end. It is easy to work out:

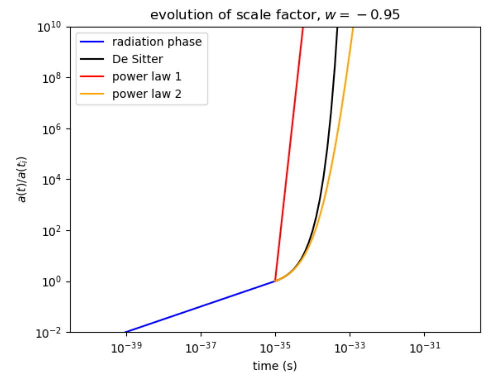
$$H_i(t_f - t_i) = \frac{1}{2}\left(e^{-\mathcal{N}_e \frac{3(1+w)}{1+3w}} - 1\right)$$

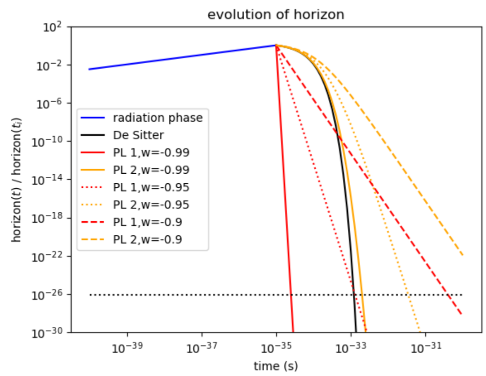
Of course for De Sitter expansion we have $H_i(t_f - t_i) = \mathcal{N}_e$.



This figure shows how the duration of inflation (red line) dramatically increases if w becomes larger than -1; we report in the plot, as a reference, the increase for the De Sitter model and the age of the Universe today and at the Quark-Hadron transition (where all the events leading to nucleosynthesis begin). This already casts doubts on the validity of a power-law model if $w > -0.7$.

However, for $w \to -1$ the duration of inflation tends to zero, instead of tending to the De Sitter value. The reason for this behaviour is illustrated in these two figures below.





We show here the evolution of the scale factor and of the Hubble horizon, both normalized to their value at the beginning of inflation, across the start of inflation. The red lines give our naive expectation, only for $w = -0.95$ for the scale factor and for three values of w for the horizon. The scale factor is continuous at t_i but its first derivative is not, while this is not true for the De Sitter expansion (black lines). But we do not expect a discontinuity in the Hubble parameter. A way to fix this problem is to follow the procedure outlined in the proposed problem of Lecture 21, or in the third test of 2019: assume that the formal big bang during the power-law expansion is not at $t = 0$ but at a time t_0, and fix its value by requiring that the Hubble parameter is continuous at t_i. Then, with some algebra:

$$a(t) = a_i \left(\frac{t - t_0}{t_i - t_0} \right)^{\frac{2}{3(1+w)}}$$

with

$$t_0 = \frac{w - 1/3}{w + 1} t_i$$

and

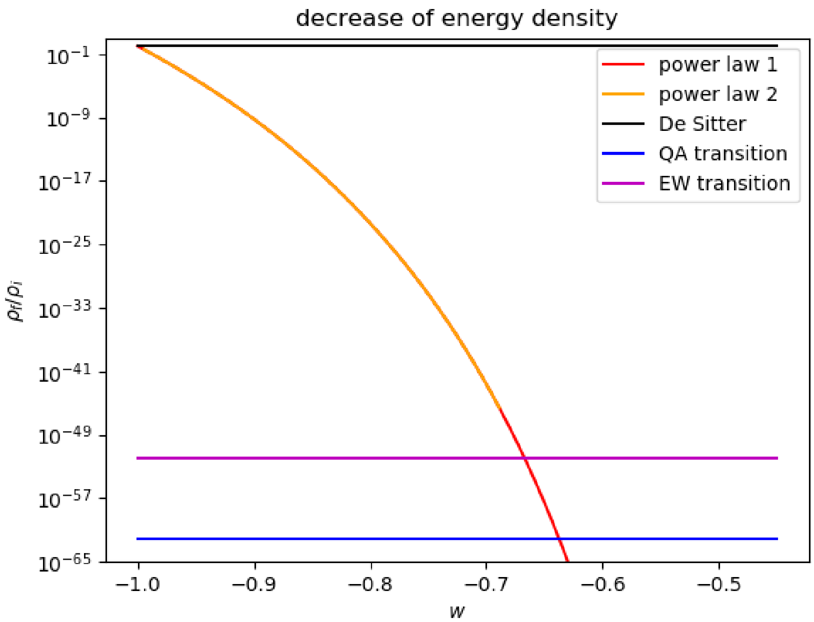
$$\frac{d_{Hi}}{d_{Hf}} = \left(\frac{t_i - t_0}{t_f - t_0} \right)^{\frac{1+3w}{3(1+w)}}$$

The condition $\log(d_{Hi}/d_H f) = \mathcal{N}_e$ must be solved numerically. This model (power law 2) is given by the orange lines above: the duration of inflation this time converges to the De Sitter value for $w \to -1$, the scale factor evolution is smooth in its first derivative and the horizon evolution converges to the De Sitter one.

Energy density decreases during these power-law inflations, as

$$\rho_f = \rho_i \left(\frac{t_f}{t_i} \right)^{-2}$$

In this case taking into account the continuity of Hubble parameter does not give a large difference. We show below the predicted decrease in energy density as a function of w.



For w not very similar to -1 this decrease can be substantial; we report as a reference the ratio of energy densities between 10^{15} GeV and the Electro-Weak and Quark-Hadron transition (here energies must be interpreted as temperatures, and $\rho \propto T^4$). Inflation is a process that must end, and at the end of inflation we need to recouple the inflaton with all other fields, something that presumably needs very high energies. We do not want this to happen at too low energies, for instance we do not want to compromise Big Band nucleosynthesis. Again, values above $w > -0.7$ are clearly disfavoured.

As for the third question, remember that, as Albert Einstein used to say, “If you can’t explain it to an engineer, you don’t understand it yourself”!

(Disclaimer: this is a joke, come on!)