from
An Introduction to Computer Simulation Methods Third Edition (revised)
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or Edition II of the previous book (By Gould and Tobochnik only)

Project 17.28. Zero temperature dynamics of the Ising model We have seen that various kinetic growth models (Section 14.3) and reaction-diffusion models (Section 12.4) lead to interesting and nontrivial behavior. Similar behavior can be seen in the zero temperature dynamics of the Ising model. Consider the one-dimensional Ising model with $J > 0$ and periodic boundary conditions. The initial orientation of the spins is chosen at random. We update the configurations by choosing a spin at random and computing the change in energy ΔE. If $\Delta E < 0$, then flip the spin; else if $\Delta E = 0$, flip the spin with 50% probability. The spin is not flipped if $\Delta E > 0$. This type of Monte Carlo update is known as Glauber dynamics. How does this algorithm differ from the Metropolis algorithm at $T = 0$?

The quantity of interest is $f(t)$, the fraction of spins that flip for the first time at time t. As usual, the time is measured in terms of Monte Carlo steps per spin. Published results (Derrida, Bray, and Godrèche) for $N = 10^5$ indicate that $f(t)$

$$f(t) \sim t^{-\theta} \tag{17.68}$$

for $t \approx 3$ to $t \approx 10,000$ with $\theta \approx 0.37$. Verify this result and extend your results to the one-dimensional q-state Potts model. In the latter model each site is initially given a random integer between 1 and q. A site is chosen at random and set equal to either of its two neighbors with equal probability. The value of the exponent θ is not understood at present, but might be related to analogous behavior in reaction-diffusion models.

B. Derrida, A. J. Bray, and C. Godrèche, "Non-trivial exponents in the zero temperature dynamics of the 1D Ising and Potts models," *J. Phys.* A **27**, L357 (1994).