Study both maps (tent and sine), including the Lyapunov exponent; determine similarities and differences

Example 7.5.1

Consider the so-called *tent map*

$$f(x) = \begin{cases} rx & 0 \le x \le 1/2 \\ r - rx & 1/2 \le x \le 1 \end{cases}$$

(for $0 \le r \le 2$ and $0 \le x \le 1$).

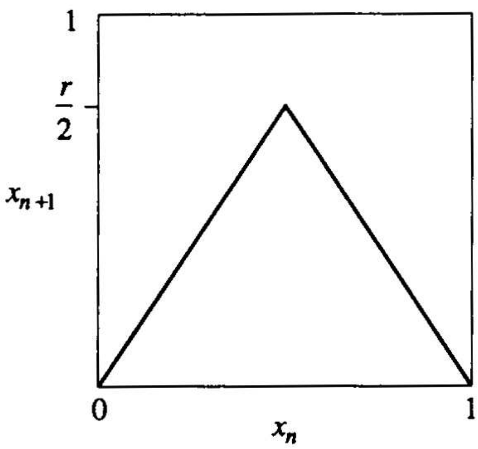


Fig. 7.5.1

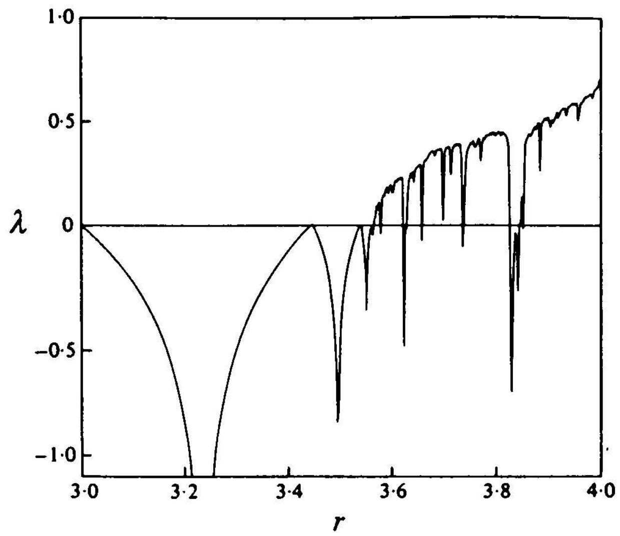
This looks similar to the logistic map, but is much easier to analyse!

Since $f'(x) = \pm r$ for all x, we find

$$
\begin{aligned}
\lambda &= \lim_{n\to\infty} \left(\frac{1}{n} \sum_{i=0}^{n-1} \ln |f'(x_i)| \right) \\
&= \lim_{n\to\infty} \left(\frac{\ln r}{n} \sum_{i=0}^{n-1} 1 \right) \\
&= \ln r
\end{aligned}
$$

This suggests that the tent map has chaotic solutions for all $r > 1$, since $\lambda = \ln r > 0$.

In general one needs a computer to calculate λ!



e.g. λ for the Logistic Map

7.6 Universality

Consider the *sine map* $x_{n+1} = r \sin \pi x_n$ for $0 \le r \le 1$ and $0 \le x \le 1$.

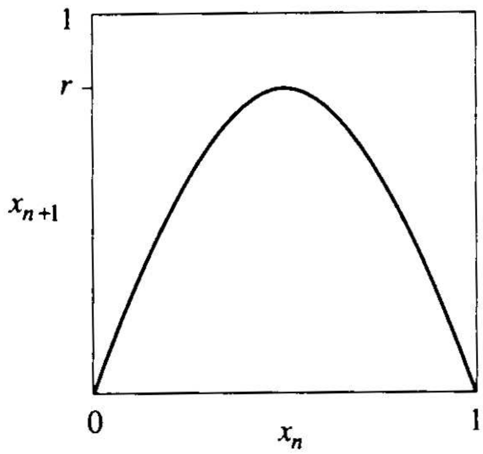


Fig. 7.6.1

It has qualitatively the same shape as the logistic map - such maps are called *unimodal*.

We now compare the orbit diagrams for the sine map and the logistic map...

the resemblance is quite amazing...

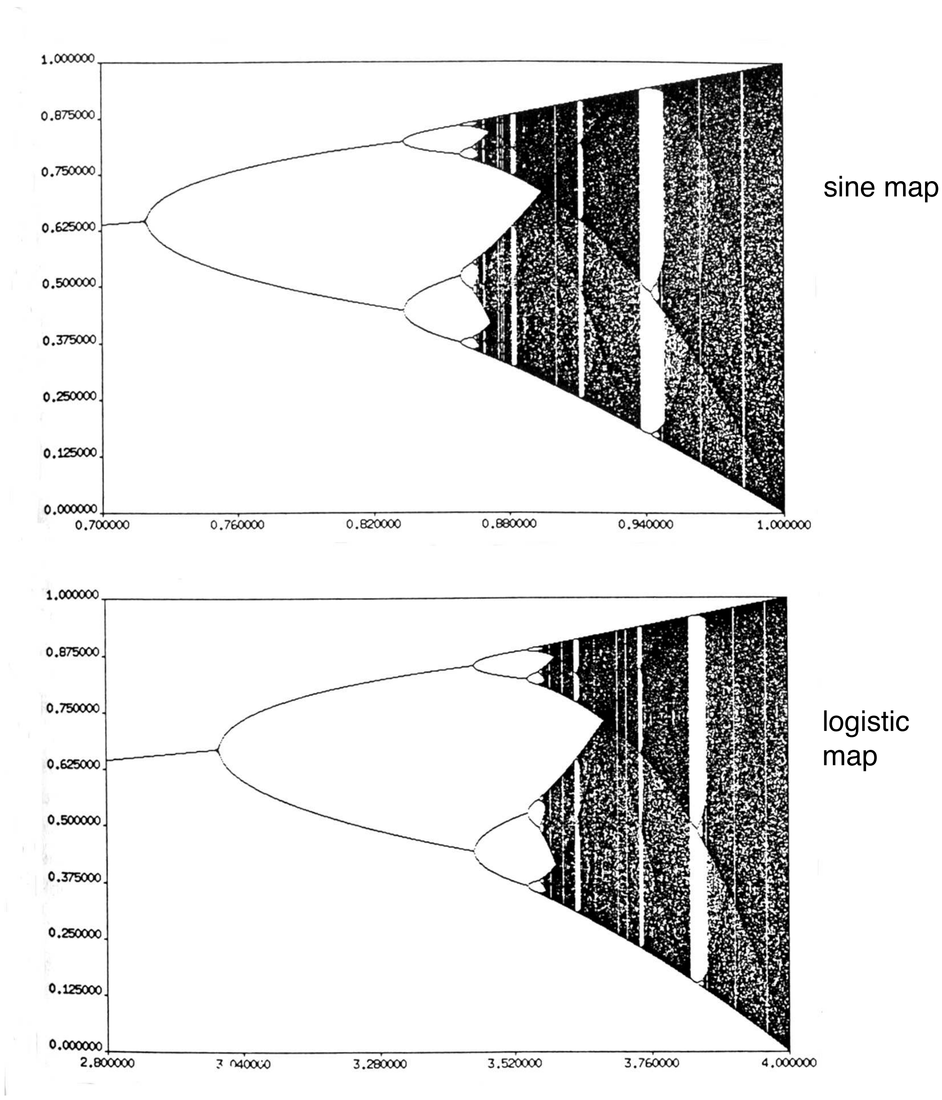


Fig. 7.6.2

The *qualitative* dynamics of the two maps are identical! Metropolis (1973) proved that all unimodal maps have periodic attractors (i.e. stable periodic solutions) occurring in the same sequence. This implies that the algebraic form of the map $f(x)$ is irrelevant - only its overall shape matters!