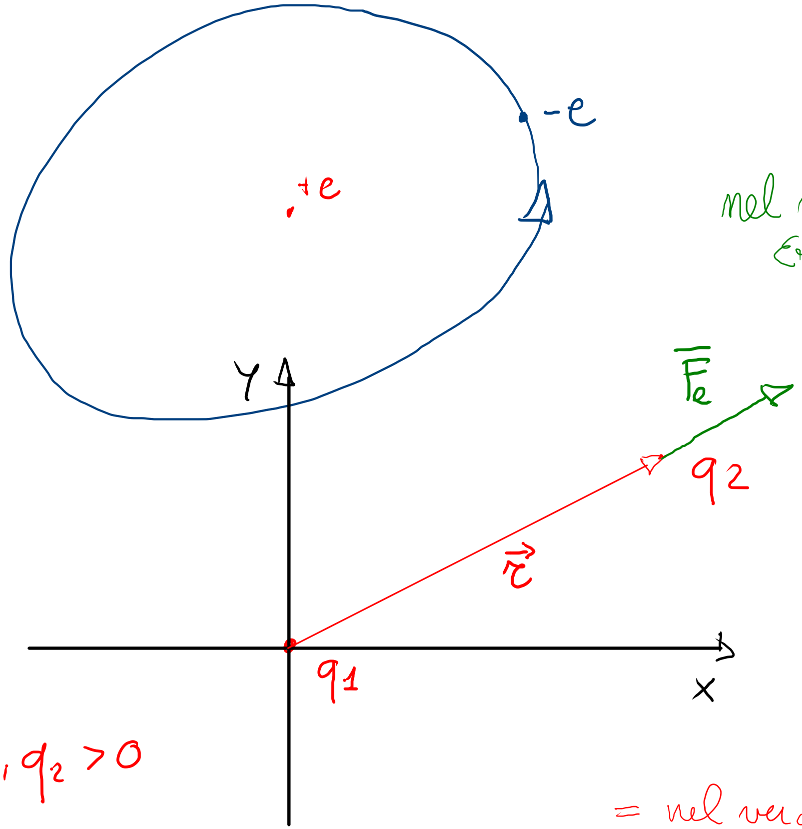
FORZE E CORRENTI ELETTRICHE

Carica elementare $e = 1{,}602 \cdot 10^{-19}$ C (Coulomb)



$q_1, q_2 > 0$

nel vuoto $\varepsilon_r = 1$

$$\vec{F}_e = k \frac{q_1 q_2}{r^2} \hat{r} \qquad k = 9 \cdot 10^9 \frac{Nm^2}{C^2}$$

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2} \hat{r}$$

$$r = |\vec{r}|$$

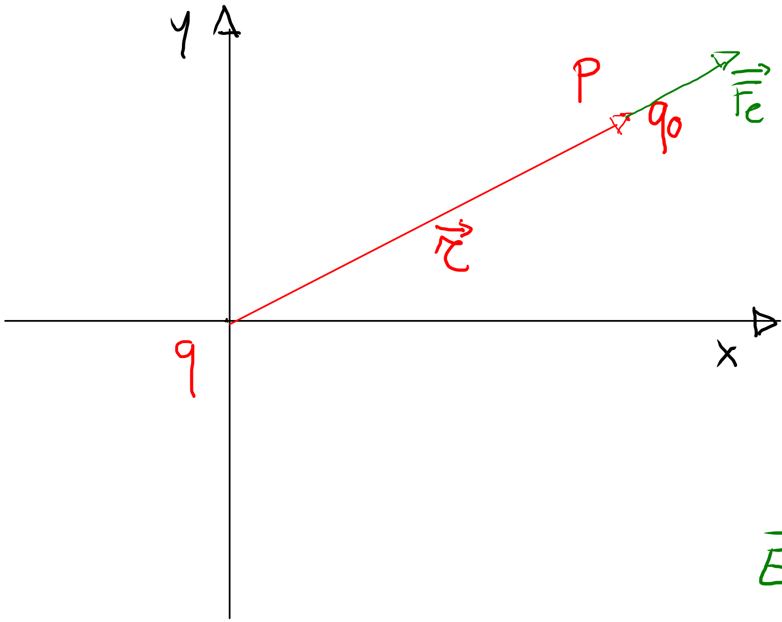
$$\hat{r} = \frac{\vec{r}}{r}$$

$$\varepsilon_0 = 8{,}854 \cdot 10^{-12} \frac{C^2}{Nm^2}$$

costante dielettrica del vuoto

$\varepsilon_r \geq 1$ = nel vuoto

costante dielettrica relativa



$$\vec{E}(P) = \,?$$

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q q_0}{r^2}\hat{r}$$

$$\vec{E}(P) = \frac{\vec{F}_e}{q_0}$$

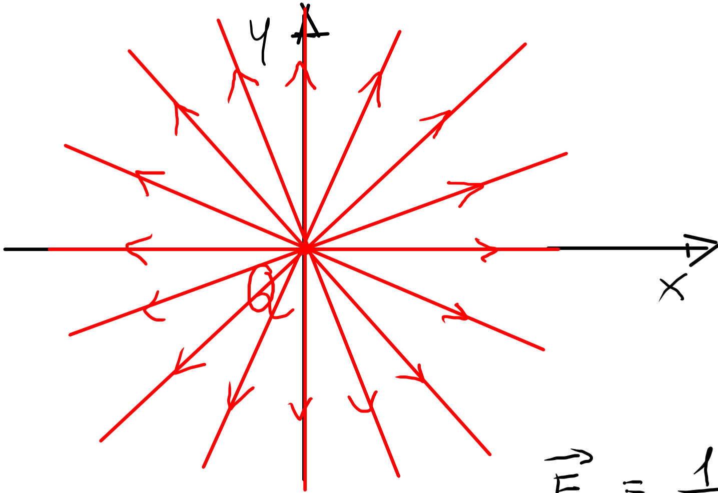
$$\vec{E}(\vec{r}) = \vec{E}(P) = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r^2}\hat{r}$$

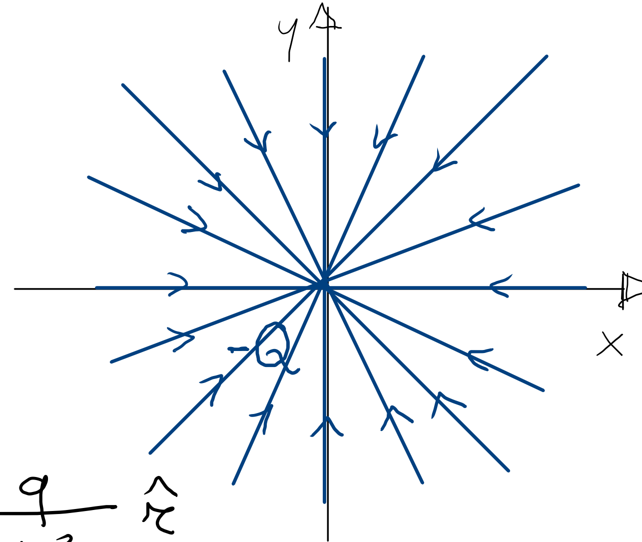
$$[\vec{E}] = \frac{N}{C}$$

$$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r^2}\hat{r}}$$

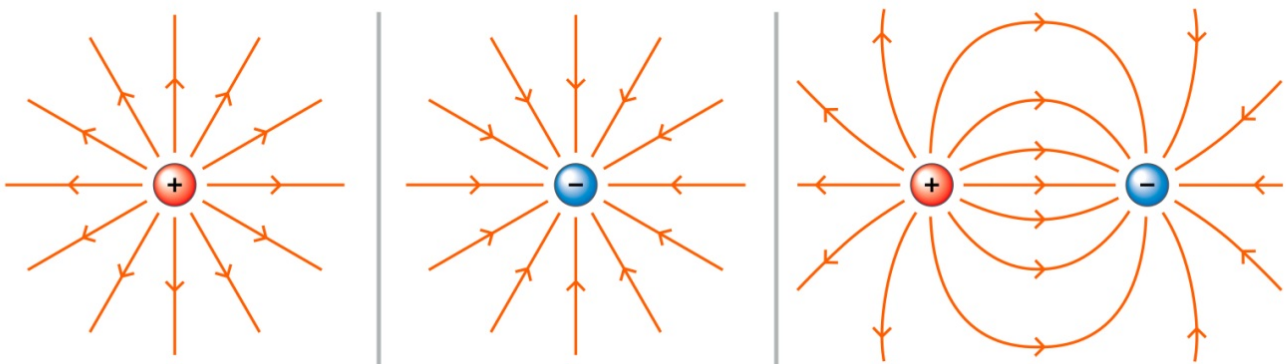
se metto q_0 in $\vec{r}$, ella sarà soggetta a:

$$\vec{F}_e = q_0\vec{E}(\vec{r})$$





$$\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r^2}\hat{r}$$



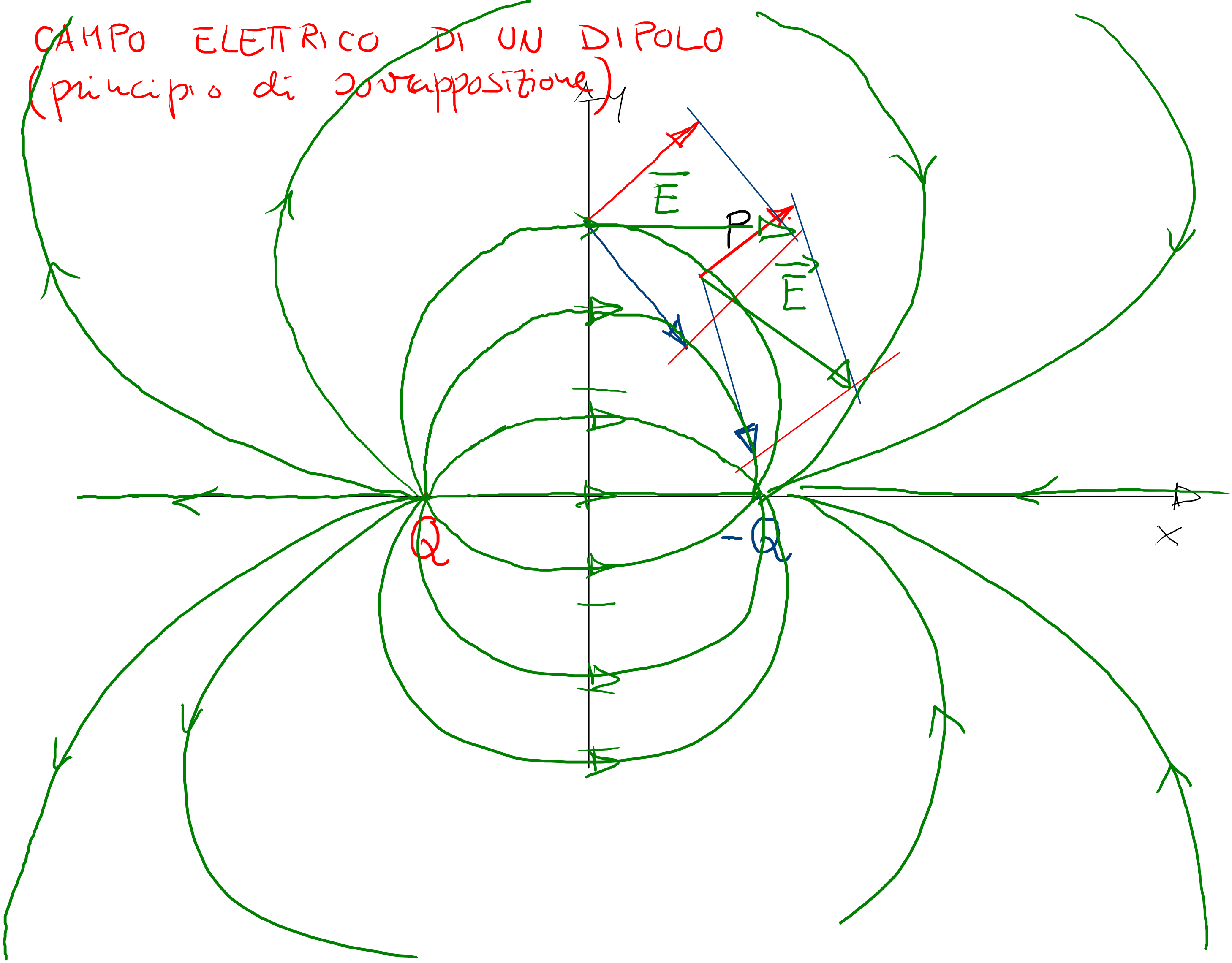
a Linee di forza del campo elettrico creato da una carica positiva.

b Linee di forza del campo creato da una carica negativa.

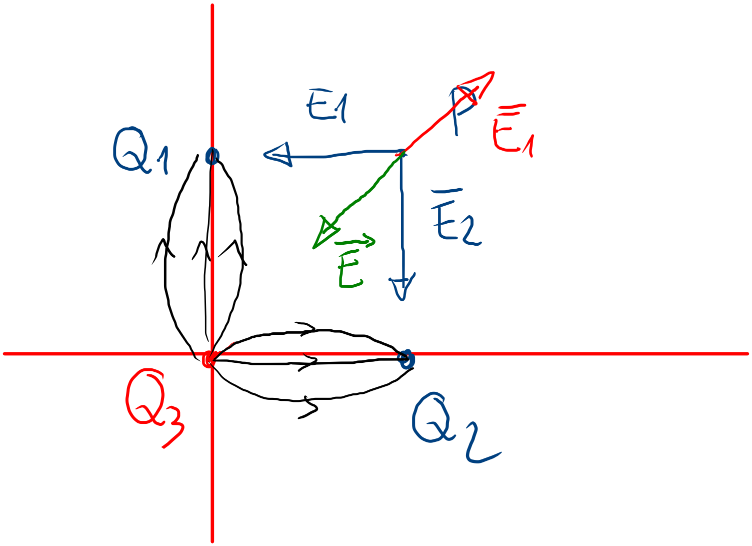
c Linee di forza del campo creato da due cariche uguali e opposte (dipolo elettrico).

CAMPO ELETTRICO DI UN DIPOLO

(principio di sovrapposizione)

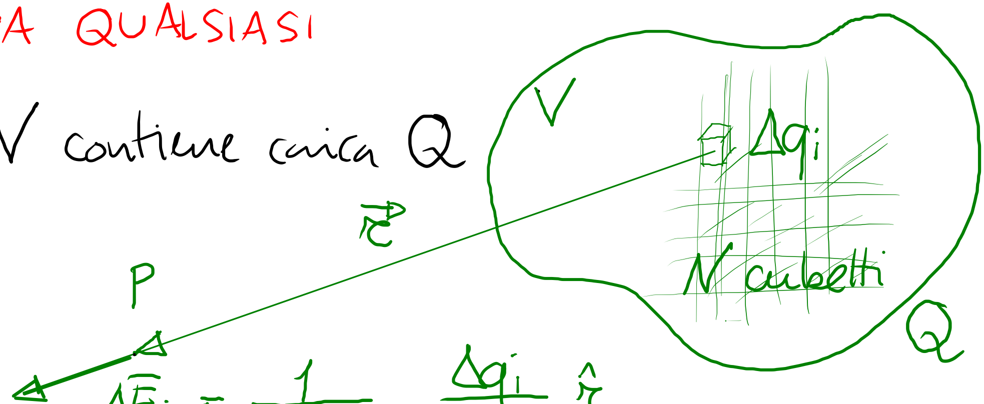


CALCOLO DEL CAMPO ELETTRICO GENERATO DA UN INSIEME DI CARICHE PUNTIFORMI



CALCOLO DEL CAMPO ELETTRICO GENERATO DA UNA DISTRIBUZIONE DI CARICA QUALSIASI

Volume V contiene carica Q



$$\sum_{i=1}^{N} \Delta q_i = Q$$

Q carica totale

$$\Delta E_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{\Delta q_i}{r^2} \hat{r}$$

$$\vec{E} \simeq \sum_{i=1}^{N} \Delta\vec{E}_i = \sum_{i=1}^{N} \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{\Delta q_i}{r_i^2} \hat{r}_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \sum_{i=1}^{N} \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

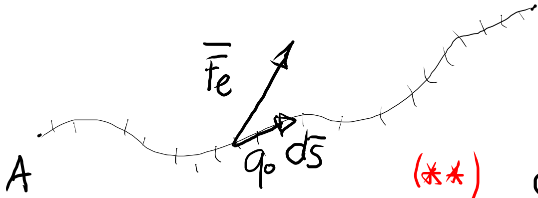
$$\vec{E} = \lim_{\Delta q_i \to 0} \sum_{i=1}^{N} \Delta\vec{E}_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \int_V \frac{dq}{r^2} \hat{r}$$

$\rho = \frac{Q}{V}$ è costante, $dq = \rho\, dV$ $\qquad \vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \int_V \frac{\rho\, dV}{r^2} \hat{r}$

$\sigma = \frac{Q}{S}$ " , $dq = \sigma\, dA$ $\qquad \vec{E} = \dots \int_A \frac{\sigma\, dA}{r^2} \hat{r}$

$\lambda = \frac{Q}{\ell}$ " , $dq = \lambda\, d\ell$ $\qquad \vec{E} = \dots \int_L \frac{\lambda\, d\ell}{r^2} \hat{r}$

ENERGIA POTENZIALE ELETTRICA (U)



$$\mathcal{L} = -\Delta U = U_A - U_B \qquad (*)$$

$$(**) \quad \mathcal{L} = \int_A^B \vec{F}_e \cdot d\vec{s} = \int_A^B q_0 \vec{E} \cdot d\vec{s} = q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

$(*) = (**)$

$$\Delta U = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

Energia potenziale elettrica

POTENZIALE ELETTRICO (V)

$$\boxed{\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot d\vec{s}}$$

$$[V] = \frac{J}{C} = 1\,V \qquad [E] = \frac{N \cdot m}{C \cdot m} = \frac{V}{m} = \frac{N}{C}$$

POTENZIALE ELETTRICO (V) IN UN PUNTO P

$$\boxed{\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot d\vec{s}}$$

$B \rightarrow P$

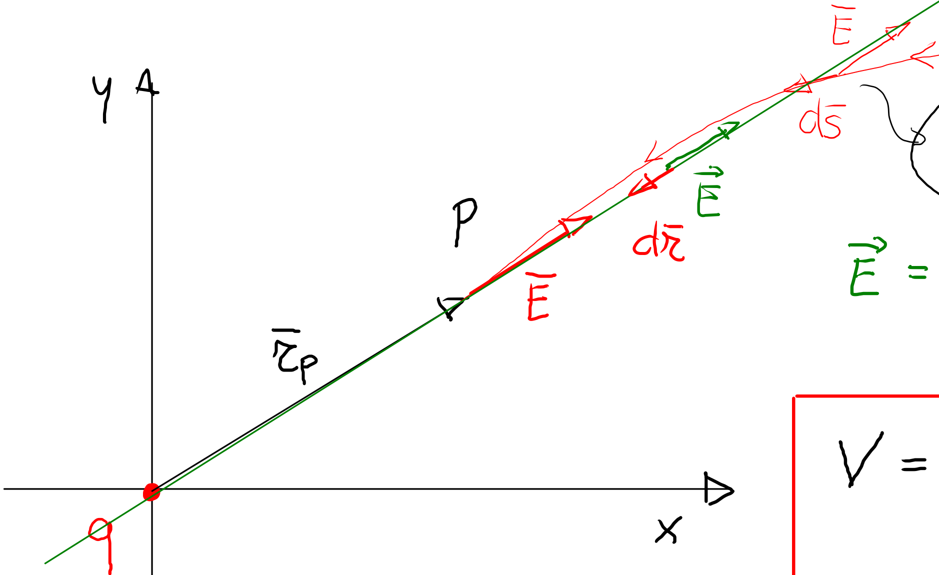
$A \rightarrow \infty$ (punto all'infinito)

$V_A \rightarrow V_\infty = 0$

$$V_P = V_P - \cancelto{0}{V_\infty} = -\int_\infty^P \vec{E} \cdot d\vec{s} \qquad V_P = \int_P^\infty \vec{E} \cdot d\vec{s}$$

$$\boxed{V_P = -\int_\infty^P \vec{E} \cdot d\vec{s}}$$

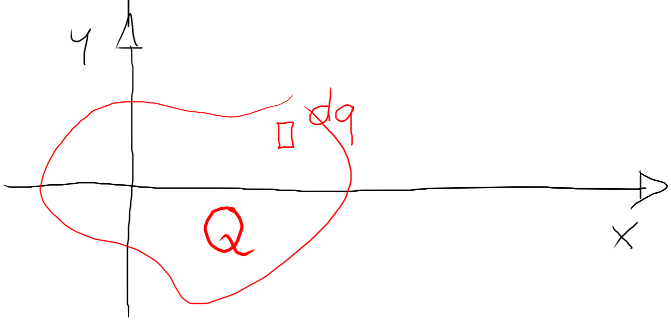
ESEMPIO: CARICA PUNTIFORME POSTA IN O



$$dV = -\vec{E}\cdot d\bar{s} \quad \longrightarrow \; >>>>$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r^2}\hat{r}$$

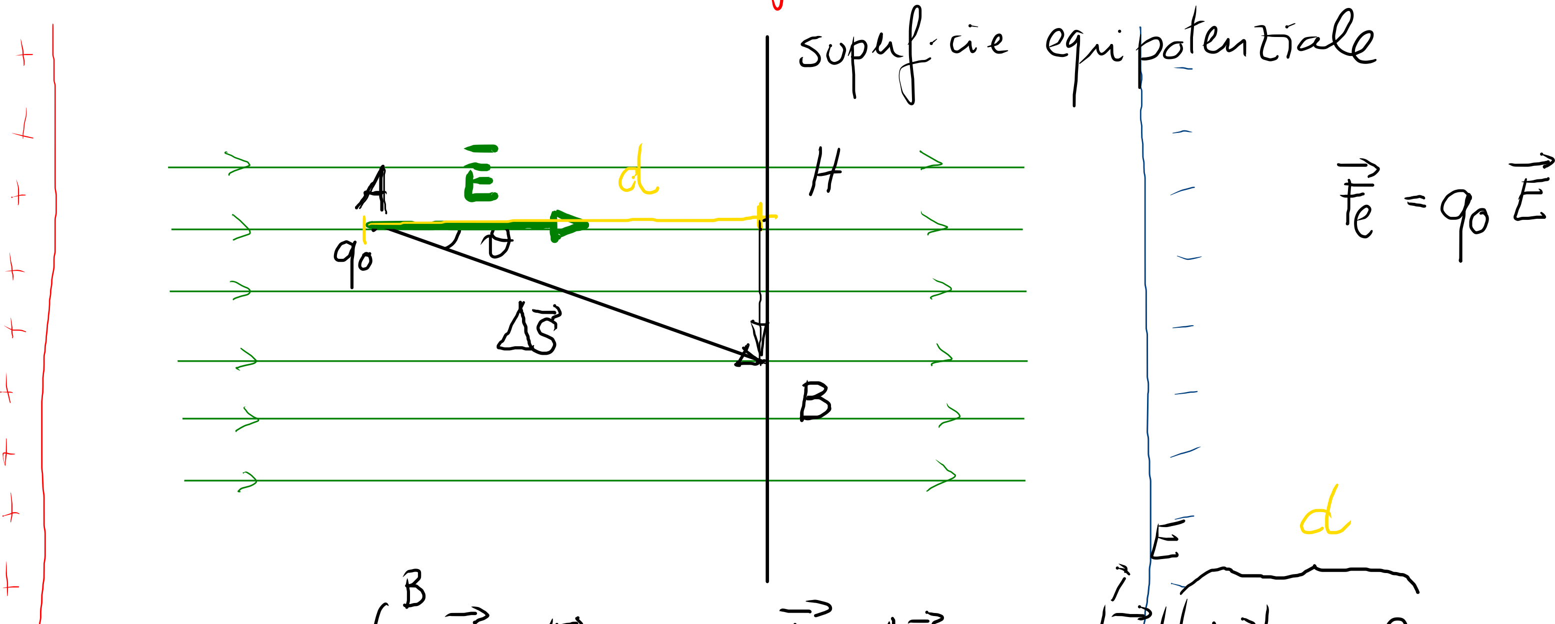
$$\boxed{V = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r}}$$



$$dV = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{dq}{r}$$

$$V = \int_Q \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{dq}{r}$$

ESEMPIO: ΔV in $\vec{E}$ uniforme



$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{s} = -\vec{E}\cdot\Delta\vec{s} = -|\vec{E}||\Delta\vec{s}|\cdot\cos\vartheta$$

$$\Delta V = -Ed = V_B - V_A \longrightarrow V_A - V_B = Ed$$

$$\Delta U = q_0\Delta V = -q_0 E d$$

$$\mathcal{L} = -\Delta U = q_0 E d \longrightarrow \mathcal{L} = q_0(V_A - V_B)$$

POTENZIALE & CAMPO ELETTRICO (* approfondimento)

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$dV = -\vec{E} \cdot d\vec{s} \xrightarrow{1D} d\vec{s} = dx\,\hat{i} \quad \dots \quad d\vec{s} = dy\,\hat{j} \quad \dots$$

$$dV = -E_x\,dx$$

$$E_x = -\frac{dV}{dx} \quad \dots \quad E_y = -\frac{dV}{dy} \quad \dots$$

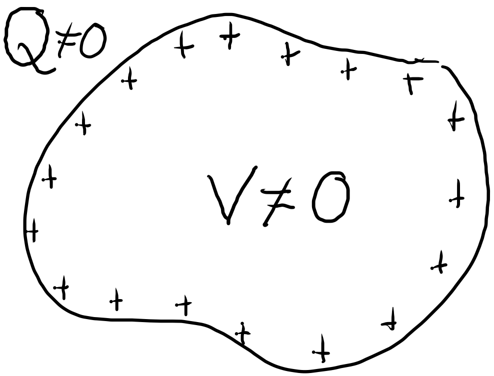
$$\vec{E} = -\frac{dV}{dx}\hat{i} - \frac{dV}{dy}\hat{j} - \frac{dV}{dz}\hat{k}$$

$$\boxed{\vec{E} = -\vec{\nabla} V}$$

CONDUTTORI (metallici)

$Q = 0 \qquad V = 0$ (conduttore neutro)

$Q \neq 0 \qquad V \neq 0$ ed è lo stesso in tutti i punti del conduttore



se V è costante

$\Rightarrow \quad \bar{\nabla} V = 0$

$\bar{E} = 0$

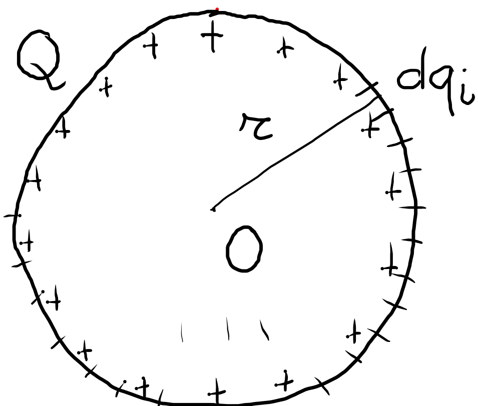
$$\boxed{C = \frac{Q}{V}}$$ é costante

Capacità del conduttore

$Q = C V$

ESEMPIO: C di una sfera metallica

$$C = \frac{Q}{V}$$



Data Q voglio trovare V

V è lo stesso in ogni punto

$\Rightarrow$ lo calcolo in O

$$\sum_i dq_i = Q$$

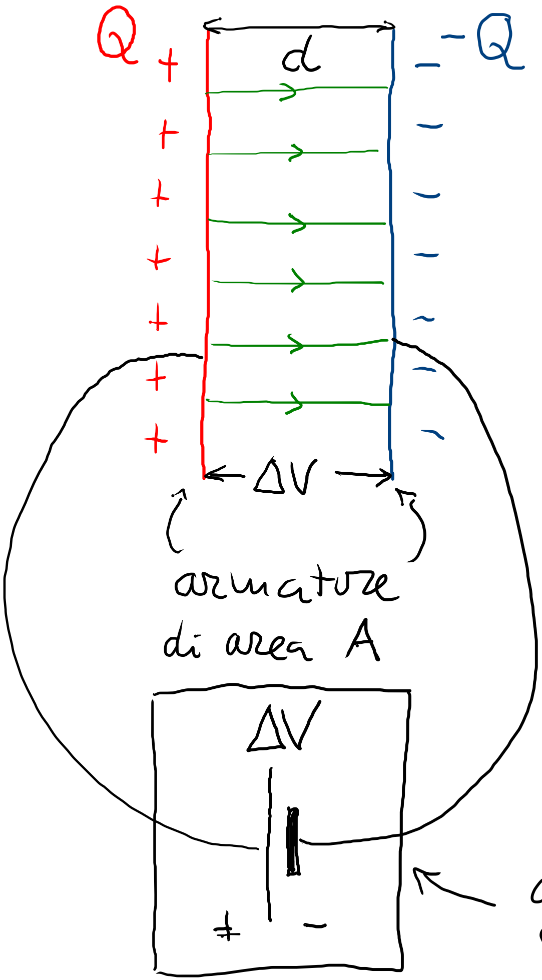
Contributo al potenziale in O da parte di dq_i

$$dV_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{dq_i}{r}$$

$$V = \sum_i dV_i = \sum_i \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{dq_i}{r} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{1}{r}\underbrace{\sum_i dq_i}_{Q}$$

$$V = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{r} \qquad C = \frac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r\, r$$

CONDENSATORE A FACCE PIANE & PARALLELE



generatore di tensione ideale

$$\vec{E} \qquad E = \frac{1}{\varepsilon_0 \varepsilon_r} \frac{Q}{A}$$

(dal teorema di Gauss)

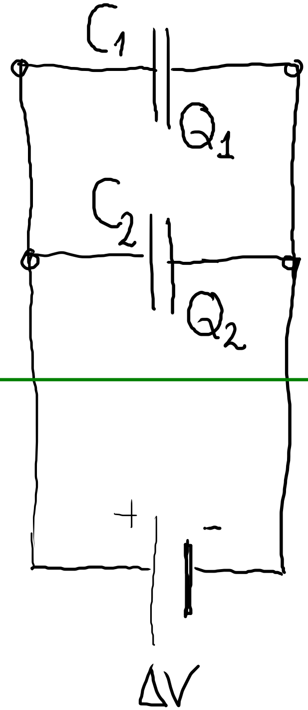
$$\Delta V = E d$$

$$C = \frac{Q}{\Delta V} = \frac{A \varepsilon_0 \varepsilon_r E}{\Delta V} = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

$$Q = C \Delta V$$

$$[C] = \frac{1\,C}{1\,V} = 1 \text{ Farad} = 1\,F$$

CONDENSATORI IN PARALLELO



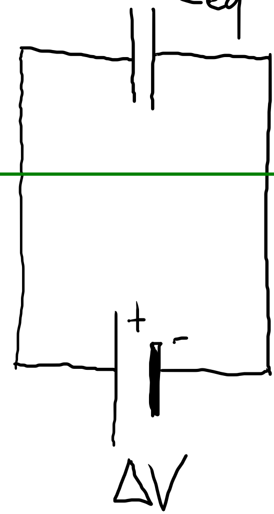
• $\Delta V_1 = \Delta V_2 = \Delta V$

$Q_1 = C_1 \Delta V$

$Q_2 = C_2 \Delta V$

•• $Q = Q_1 + Q_2$

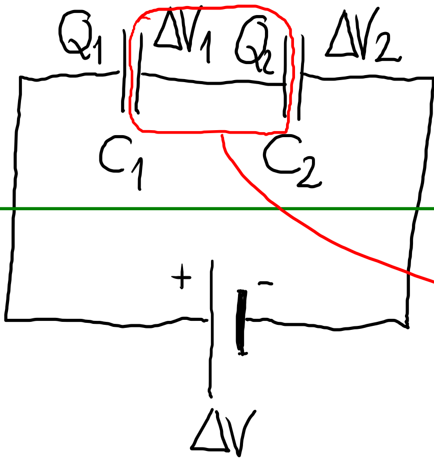
$C_{eq} \equiv \frac{Q}{\Delta V}$



$$C_{eq} = \frac{Q}{\Delta V} = \frac{Q_1 + Q_2}{\Delta V} = \frac{Q_1}{\Delta V} + \frac{Q_2}{\Delta V} \Rightarrow C_{eq} = C_1 + C_2$$

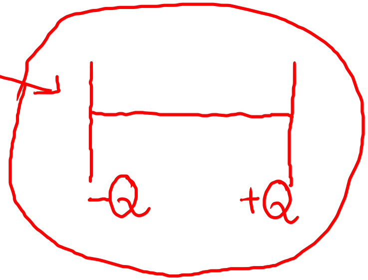
per N condensatori in parallelo $C_{eq} = C_1 + C_2 + \dots + C_N$

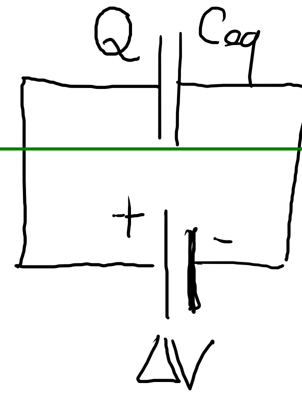
CONDENSATORI IN SERIE



- $\Delta V_1 + \Delta V_2 = \Delta V$

•• $Q_1 = Q_2 = Q$





$$C_{eq} = \frac{Q}{\Delta V}$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \overset{\bullet\bullet}{=} \frac{Q}{C_1} + \frac{Q}{C_2}$$

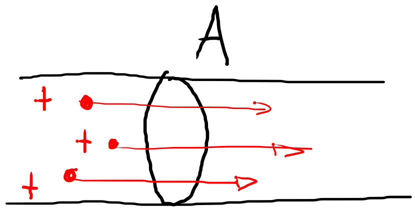
$$\frac{\cancel{Q}}{C_{eq}} = \frac{\cancel{Q}}{C_1} + \frac{\cancel{Q}}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

per N condensatori in serie

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

CORRENTE ELETTRICA (continua)



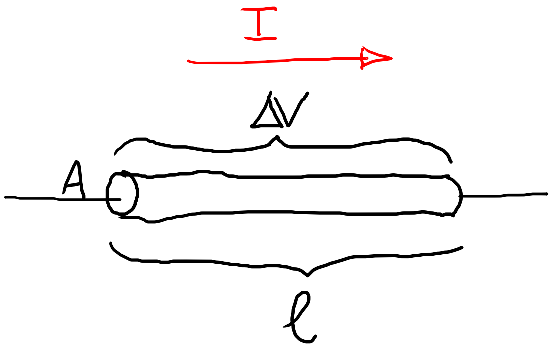
ΔQ attraversa A in Δt

$I_m = \dfrac{\Delta Q}{\Delta t}$ intensità corrente media

$I = \lim_{\Delta t \to 0} \dfrac{\Delta Q}{\Delta t} = \dfrac{dQ}{dt}$ " " istantanea

$[I] = \text{Ampere}$ $1A = \dfrac{1C}{1s}$

LEGGI DI OHM



R resistenza del conduttore

1) $$\boxed{\Delta V = R\, I}$$

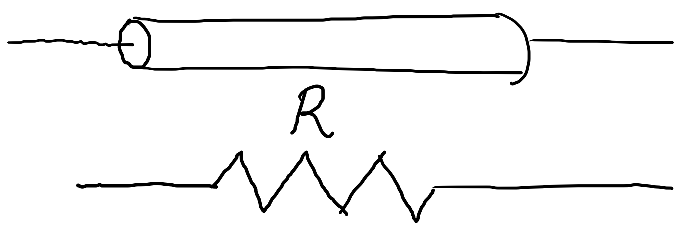
$$[R] = \frac{V}{A} = \Omega \quad \text{Ohm}$$

2) $$R = \rho \, \frac{\ell}{A}$$

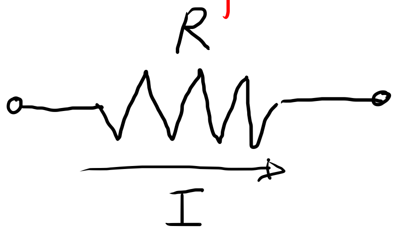
$$[\rho] = \Omega \cdot m$$

ρ: resistività del materiale

Simbolicamente :



Potenza trasferita alla resistenza

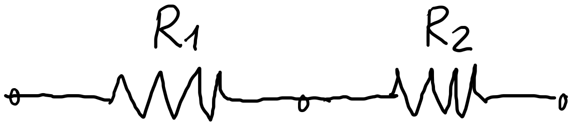


ΔV

$$P = \frac{\Delta U}{\Delta t} = \frac{q\,\Delta V}{\Delta t} = I\,\Delta V = R\,I^2 = \frac{\Delta V^2}{R}$$

I legge Ω

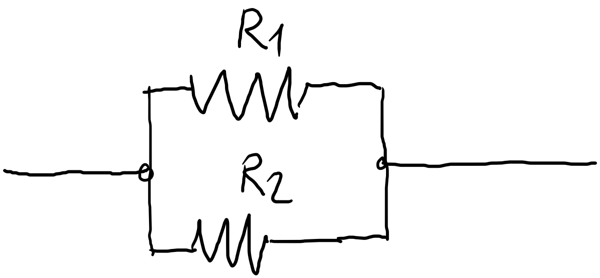
RESISTENZE IN SERIE



$$R_{eq} = R_1 + R_2$$

$$\left(= R_1 + R_2 + \dots + R_N\right)$$

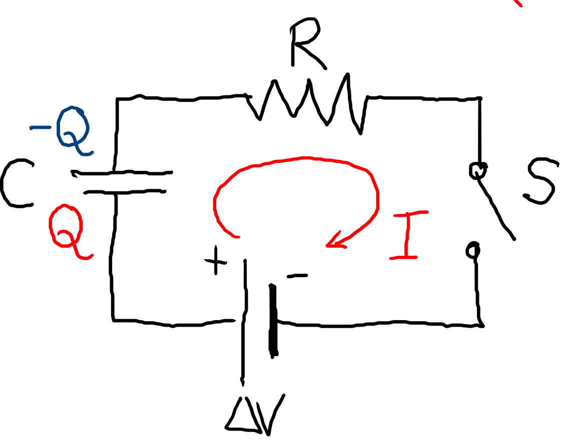
RESISTENZE IN PARALLELO



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\left(= \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}\right)$$

CIRCUITO RC (carica)

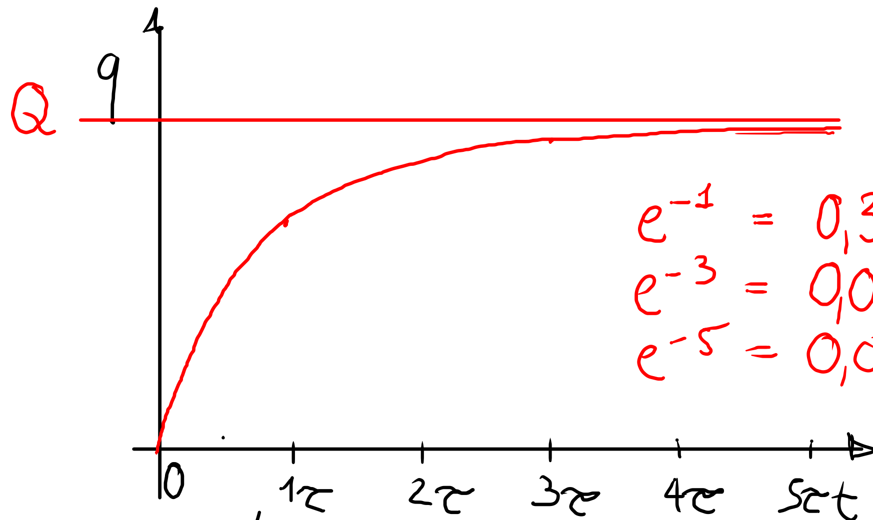


$t=0$ chiudo S

$$Q = C\,\Delta V$$

$$q(t) = Q\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\tau = RC$$

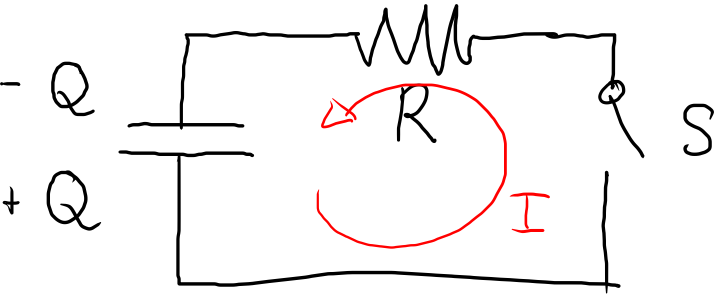


$$e^{-1} = 0{,}37$$
$$e^{-3} = 0{,}05$$
$$e^{-5} = 0{,}007$$

$$I = \frac{dq}{dt} = \frac{d}{dt}\left[Q\left(1 - e^{-\frac{t}{\tau}}\right)\right] = Q\left[-e^{-\frac{t}{\tau}}\left(-\frac{1}{\tau}\right)\right]$$

$$I = \frac{Q}{\tau}e^{-\frac{t}{\tau}} = \frac{Q}{RC}e^{-\frac{t}{\tau}} = I_0\,e^{-\frac{t}{\tau}}$$

CIRCUITO RC (scarica)



$$q(t) = Q\, e^{-\frac{t}{\tau}}$$

$$I(t) = -I_0\, e^{-\frac{t}{\tau}}$$

corrente circola in senso opposto rispetto alla fase di carica