$$x_G = \frac{\ell}{2}\cos\theta + r$$

$$y_G = \frac{3}{2}\ell \,\text{sen}\theta$$

$$x_A = r$$

$$y_A = 2\ell \,\text{sen}\theta \qquad \dot{y}_A = 2\ell\cos\theta\,\dot\theta$$

$$\delta y_A = r\,\delta\varphi \rightarrow \dot\varphi = \frac{\dot{y}_A}{r}$$

$$T = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\dot\varphi^2 + \frac{1}{2}\left(\frac{1}{12}m\ell^2\right)\dot\theta^2 + \frac{1}{2}\left(\frac{1}{12}\,2m\,4\ell^2\right)\dot\theta^2$$

$$+ \frac{1}{2}m\left(2\ell\dot\theta\cos\theta\right)^2 + \frac{1}{2}m\left[\frac{1}{4}\ell^2\dot\theta^2\,\text{sen}^2\theta + \frac{9}{4}\ell^2\dot\theta^2\cos^2\theta\right]$$

$$= T = \frac{m}{2}\left(\frac{3\ell^2}{4}\right)\dot\theta^2 + 2m\ell^2\dot\theta^2\cos^2\theta + \frac{1}{8}m\ell^2\dot\theta^2 + m\ell^2\dot\theta^2\cos^2\theta$$

$$+ \frac{1}{2}\left(\frac{1}{2}mr^2\right)\frac{4\ell^2}{r^2}\cos^2\theta\,\dot\theta^2$$

$$\searrow m\ell^2\cos^2\theta\,\dot\theta^2$$

$$= \frac{m}{2}\ell^2\dot\theta^2\left(1 + 8\cos^2\theta\right)$$

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$$V = mg\,\frac{3}{2}\ell\,\text{sen}\theta + mg\,2\ell\,\text{sen}\theta + \frac{1}{2}k\left(2\ell - 2\ell\,\text{sen}\theta\right)^2$$

$$= \frac{7}{2}mg\ell\,\text{sen}\theta + 2k\ell^2\left(1 + \text{sen}^2\theta - 2\,\text{sen}\theta\right)$$

$$V' = \left(\frac{7}{2}mg\ell - 4k\ell^2 + 4k\ell^2\,\text{sen}\theta\right)\cos\theta$$

$$V\left(\frac{\pi}{2}\right) = \frac{7}{2}mg\ell$$

$$V\left(\frac{3}{2}\pi\right) = -\frac{7}{2}mg\ell + 8k\ell^2$$

$$V' = 0: \qquad \theta = \frac{\pi}{2}, \frac{3}{2}\pi \qquad e \quad \theta^*$$

$$V'\left(\tfrac{\pi}{2}\right) = -\left(\tfrac{7m\ell}{2}\right) < 0$$

$$\theta^* \text{ k.c.} \qquad \sin\theta^* = \frac{4k\ell^2 - \frac{7}{2}mg\ell}{4k\ell^2} = 1 - \frac{7mg}{8k\ell}$$

$$\theta^* \text{ c'è quando} \qquad -1 \leq 1 - \frac{7mg}{8k\ell} \leq 1,$$

$$\text{cioè} \qquad 0 \leq \frac{7mg}{8k\ell} \leq 2$$

$$\overbrace{1 - \sin^2\theta}$$

$$V'' = -\left(\tfrac{7}{2}mg\ell - 4k\ell^2 + 4k\ell^2 \sin\theta\right)\sin\theta + 4k\ell^2 \cos^2\theta$$

$$-4k\ell^2\left(\frac{7mg}{8k\ell} - 1\right)\sin\theta + 4k\ell^2 - 8k\ell^2 \sin^2\theta$$

$$V''\left(\tfrac{\pi}{2}\right) = -4k\ell^2\left(\frac{7mg}{8k\ell} - 1\right) - 4k\ell^2 = -\frac{7mg\ell}{2} < 0$$

$$V''\left(\tfrac{3}{2}\pi\right) = +4k\ell^2\left(\frac{7mg}{8k\ell} - 1\right) - 4k\ell^2 = 4k\ell^2\left(\frac{7mg}{8k\ell} - 2\right)$$

$$\text{STAB} \quad \text{h} \quad \frac{7mg}{8k\ell} \geq 2$$

$$V''(\theta^*) = 4k\ell^2\left(\frac{7mg}{8k\ell} - 1\right)^2 + 4k\ell^2 - 8k\ell^2\left(\frac{7mg}{8k\ell} - 1\right)^2$$

$$e \ \pi - \theta^*$$

$$= 4k\ell^2\left(1 - \left(\frac{7mg}{8k\ell} - 1\right)^2\right) \geq 0 \qquad \text{STAB}$$

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