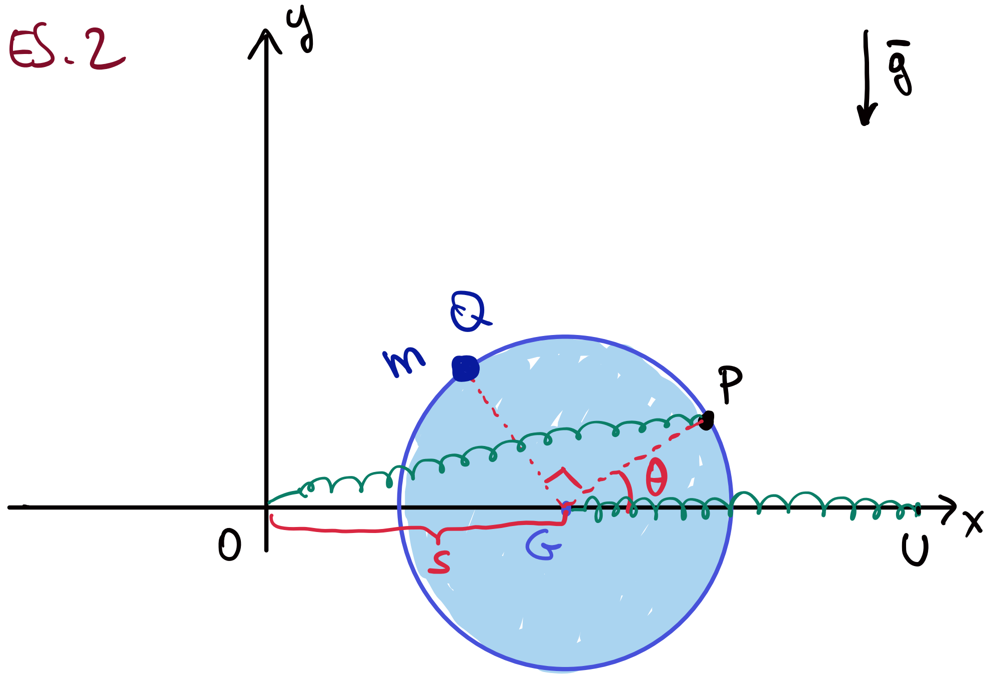
ES. 2



$x_Q = s - R\,\mathrm{sen}\,\theta$

$y_Q = R\cos\theta$

$x_G = s$

$y_G = 0$

$\dfrac{\partial \bar r_Q}{\partial s} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\dfrac{\partial \bar r_Q}{\partial \theta} = \begin{pmatrix} -R\cos\theta \\ -R\,\mathrm{sen}\,\theta \end{pmatrix}$

Sen = se $\theta = 0, \pi$

$U = (\ell, 0)$ Disco omogeneo di raggio R e massa M

Entrambe molle con cost. elastica K

1. $$T = \frac{1}{2}m\left[(\dot s - R\dot\theta\cos\theta)^2 + R^2\dot\theta^2\,\mathrm{sen}^2\theta\right] + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot\theta^2 + \frac{1}{2}M\dot s^2$$

$$= \frac{1}{2}m\left(\dot s^2 - 2\dot s\dot\theta R\cos\theta + R^2\dot\theta^2\right) + \frac{1}{4}MR^2\dot\theta^2 + \frac{1}{2}M\dot s^2$$

$$= \frac{1}{2}\left\{(m+M)\dot s^2 - 2m\dot s\dot\theta R\cos\theta + \left(m+\frac{M}{2}\right)R^2\dot\theta^2\right\}$$

Matrice cinetica $$Q = \begin{pmatrix} m+M & -mR\cos\theta \\ -mR\cos\theta & \left(m+\frac{M}{2}\right)R^2 \end{pmatrix} \leftarrow 2.$$

FAC. → def. pos. : $\det Q = R^2\left\{(m+M)\left(m+\frac{M}{2}\right) - m^2\cos^2\theta\right\} > 0$

$$V = mgR\cos\theta + \frac{1}{2}k\Big(\underbrace{s^2 + 2Rs\cos\theta + R^2}_{(s+R\cos\theta)^2 + (R\,\mathrm{sen}\,\theta)^2} + s^2 + \ell^2 - 2\ell s\Big)$$

Trascuro termini costanti

$$L = \frac{1}{2}(m+M)\dot s^2 - m\dot s\dot\theta R\cos\theta + \left(m+\frac{M}{2}\right)R^2\dot\theta^2 - mgR\cos\theta - ks^2 + k\ell s$$

$$-(mg + ks)R\cos\theta$$

3. $\frac{d}{dt}\frac{\partial L}{\partial \dot s} = \frac{d}{dt}\left[(m+M)\dot s - m\dot\theta R\cos\theta\right] = (m+M)\ddot s - m\ddot\theta R\cos\theta + m\dot\theta^2 \operatorname{sen}\theta$

$\frac{\partial L}{\partial s} = -2ks + kl - kR\cos\theta$

$\hookrightarrow \frac{(m+M)}{m}\ddot s - \ddot\theta R\cos\theta = -\dot\theta^2\operatorname{sen}\theta - \frac{2ks}{m} + \frac{kl}{m} - \frac{kR}{m}\cos\theta$

$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta} = \frac{d}{dt}\left[-m\dot s R\cos\theta + (m+2M)R^2\dot\theta\right] = -m\ddot s R\cos\theta +$
$+ \cancel{m\dot s\dot\theta} R\operatorname{sen}\theta + (m+2M)R^2\ddot\theta$

$\frac{\partial L}{\partial \theta} = \cancel{m\dot s\dot\theta} R\operatorname{sen}\theta + mgR\operatorname{sen}\theta + (mg+ks)R\operatorname{sen}\theta$

$\hookrightarrow -\ddot s R\cos\theta + \frac{m+2M}{m}R^2\ddot\theta = 2gR\operatorname{sen}\theta + \frac{ks}{m}R\operatorname{sen}\theta$

4. Se $k=0 \Rightarrow s$ coord. ciclica $\Rightarrow \frac{\partial L}{\partial \dot s}$ è cost. del moto

(comp. quantità di moto lungo asse x).

(Se $g=0 \rightarrow$ sist. inv. per rotaz. attorno asse z
$\rightarrow M_z$ cost. del moto.)

5. $V = (mg + Ks)R\cos\theta + Ks^2 - Kls + \text{cost.}$

$$\frac{\partial V}{\partial s} = KR\cos\theta + 2Ks - Kl \rightsquigarrow s = \frac{l - R\cos\theta}{2} \quad \circ \quad \cos\theta = \frac{l-2s}{R}$$

$$\frac{\partial V}{\partial \theta} = -(mg + Ks)R\,\mathrm{sen}\,\theta$$

$$\theta = 0 \longrightarrow s = \frac{l-R}{2}$$

$$\theta = \pi \longrightarrow s = \frac{l+R}{2}$$

$\exists$ se $l + \frac{2mg}{K} < R$

$$s = -\frac{mg}{K} \longrightarrow \cos\theta = \frac{l + \frac{2mg}{K}}{R} = \frac{Kl + 2mg}{KR}$$

$$\partial^2 V = \begin{pmatrix} 2K & -KR\,\mathrm{sen}\,\theta \\ -KR\,\mathrm{sen}\,\theta & -(mg + Ks)R\cos\theta \end{pmatrix}$$

$$\partial^2 V\left(0, \frac{l-R}{2}\right) = \begin{pmatrix} 2k & 0 \\ 0 & -\left(mg + \frac{Kl}{2} - \frac{KR}{2}\right)R \end{pmatrix}$$ stab. se $mg + \frac{Kl}{2} < \frac{KR}{2}$

$$\partial^2 V\left(\pi, \frac{l+R}{2}\right) = \begin{pmatrix} 2k & 0 \\ 0 & \left(mg + \frac{Kl}{2} + \frac{KR}{2}\right)R \end{pmatrix}$$ def. pos STAB

$$\partial^2 V\left(\theta^*, -\frac{mg}{K}\right) = \begin{pmatrix} 2K & -KR\,\mathrm{sen}\,\theta^* \\ -KR\,\mathrm{sen}\,\theta^* & 0 \end{pmatrix}$$ det < 0 INSTAB

6.

$$A = a\left(\pi, \frac{l+R}{2}\right) = \left.\begin{pmatrix} m+M & -mR\cos\theta \\ -mR\cos\theta & \left(m + \frac{M}{2}\right)R^2 \end{pmatrix}\right|_{\left(\pi, \frac{l+R}{2}\right)} = \begin{pmatrix} m+M & mR \\ mR & \left(m + \frac{M}{2}\right)R^2 \end{pmatrix}$$

$$\det\left(B - \lambda A\right) = \det\begin{pmatrix} 2k - \lambda(m+M) & -\lambda m R \\ -\lambda m R & \left(mg + \frac{k\ell}{2} + \frac{kR}{2}\right)R - \lambda\left(m + \frac{M}{2}\right)R^2 \end{pmatrix}$$

$$R \cdot \Big\{ \lambda^2 \left((m+M)\left(m + \frac{M}{2}\right) - m^2 \right) R$$

$$- \lambda \left((m+M)\left(mg + \frac{k\ell}{2} + \frac{kR}{2}\right) + 2kR\left(m + \frac{M}{2}\right) \right) + 2k\left(mg + \frac{k\ell}{2} + \frac{kR}{2}\right) \Big\}$$

$$\lambda_{1/2} = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4ac}}{2} \quad \leftarrow \quad \lambda^2 + b\lambda + c = 0$$

$$L_{lin} = \frac{1}{2}\left(\delta\dot{s}, \delta\dot{\theta}\right) A \begin{pmatrix} \delta\dot{s} \\ \delta\dot{\theta} \end{pmatrix} - \frac{1}{2}\left(\delta s, \delta\theta\right) B \begin{pmatrix} \delta s \\ \delta\theta \end{pmatrix}$$

$$\delta s = s - \left(\frac{\ell + R}{2}\right) \qquad \delta\theta = \theta - \pi$$

7.

$m=0 \quad \ell = 7R$

$\det(B-\lambda A)=0$

$$A=\begin{pmatrix} M & 0 \\ 0 & \frac{MR^2}{2} \end{pmatrix} \qquad B=\begin{pmatrix} 2k & 0 \\ 0 & 4kR^2 \end{pmatrix}$$

$$\lambda_1 = \frac{2k}{M} \qquad \lambda_2 = \frac{8k}{M}$$

Scrivere i modi normali di oscillazione

$$\delta s = s - \left(\frac{\ell+R}{2}\right) \qquad \delta\theta = \theta - \pi$$

$$\delta s(t) = A_s \cos\left(\sqrt{\lambda_1}\, t + \varphi_s\right) \qquad \delta\theta(t) = A_\theta \cos\left(\sqrt{\lambda_2}\, t + \varphi_\theta\right)$$

8.

Se $M=0$, (s,θ) non è buon sist. di coord. Falliscono in $\theta = 0, \pi$ dove $\frac{\partial \vec{r}_Q}{\partial s}$ e $\frac{\partial \vec{r}_Q}{\partial \theta}$.

ES. 1

4. $\{w_i, w_j\} = \sum_{kl} \frac{\partial w_i}{\partial \tilde{x}_k} E_{kl} \frac{\partial w_j}{\partial \tilde{x}_l} = \sum_{kl} J_{ik} E_{kl} J_{jl} = (JEJ^T)_{ij} \overset{\uparrow}{=} E_{ij}$ //

$x_i = w_i(\tilde{x}, t)$

J simplettica

6. $\delta_G H = 0 \iff \{G, H\} = 0 \iff G(p,q)$ cost. del moto

7. $\{M_3, H\} = \{\{M_1, M_2\}, H\} \overset{}{=}$ id. di Jacobi

$= -\{\{M_2, H\}, M_1\} - \{\{H, M_1\}, M_2\} = 0$

$\sum_{i=1}^{3} \{M_i^2, H\} = \sum_{i=1}^{3} 2M_i \{M_i, H\} = 0$

8. Coord. polari r, θ e mom. coniugati p_r, p_θ

p_θ è comp. di mom. ang. lungo asse z ($p_\theta = M_z$)

$\{p_\theta, \theta\} = -1$

$\Downarrow$

$\delta\theta = \epsilon\{\theta, p_\theta\} = \epsilon \Rightarrow \theta \mapsto \theta + \epsilon$, cioè p_θ genera shift costanti in θ, che non sono altro che le rotaz. sul piano x, y.