

Fluid Dynamics

Lecture II: Statics

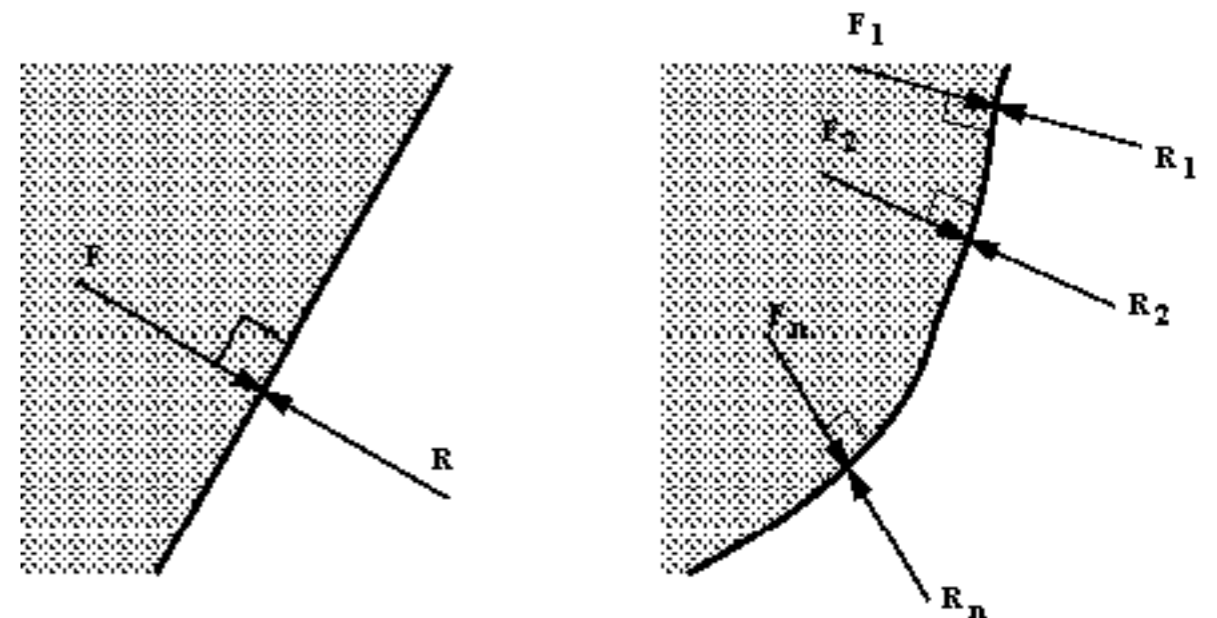


Summary of previous lecture

- What is a fluid? definitions
- Properties (mass, weight, density)
- Ideal fluids vs Real fluids
- Viscosity is a very important fluid property
- Newton's law of viscosity: τ is proportional to fluid μ and the velocity gradient
- Newtonian fluid vs Non-Newtonian fluid
- liquid (gases) have high (low) viscosity

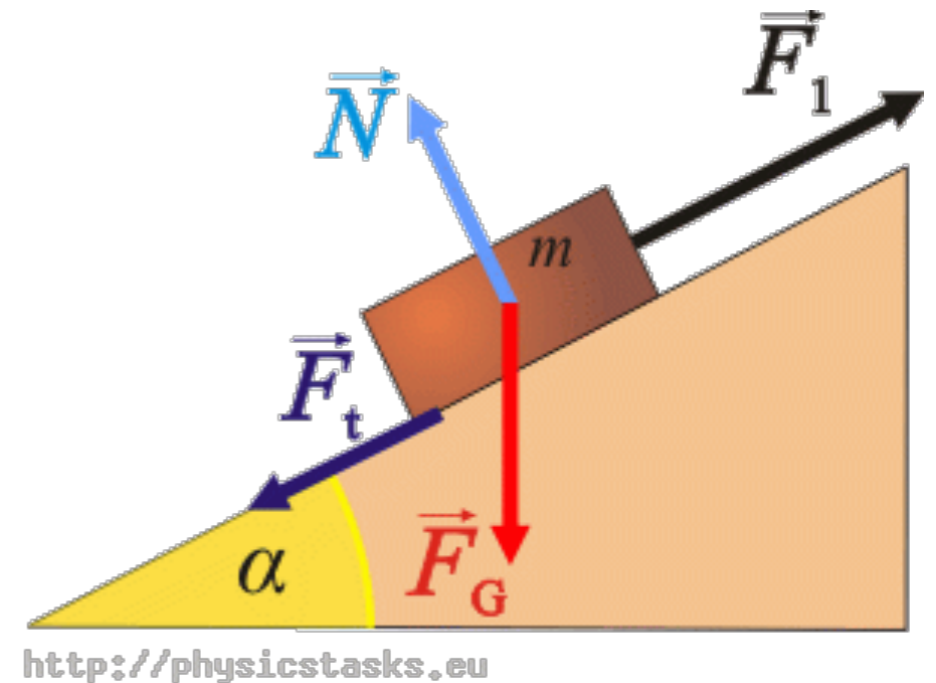
Fluid Statics

- Fluid is at rest
- A static fluid can have no shearing force acting on it.
- The only forces are due to pressure.
- Any force between fluid and boundary must be acting at right angles (normal to).
- Fluid at rest is in equilibrium: sum of components of forces in any direction must be zero.



does pressure have a direction?

- FORCE is a vector (forces on box have different directions and magnitude)
- is PRESSURE a vector too?



Isotropy of Pressure

- In a fluid at rest, the tangential viscous stresses are absent and the only force is normal to the surface.
- The surface force per unit area (PRESSURE) is equal in all directions.
- Pressure at any point in a fluid at rest has a single value (is a scalar). This is known as **Pascal's Law**.

Gauss theorem (or the divergence theorem)

- relates the flow flux of a vector field through a surface to the behavior of the vector field inside the surface
- The outward flux of a vector field through a closed surface is equal to the volume integral of the divergence over the region inside the surface
- The sum of sources and sinks (divergence) will give you the outward flux

$$\iiint_v (\nabla \cdot F) dV = \iint_A F dA$$

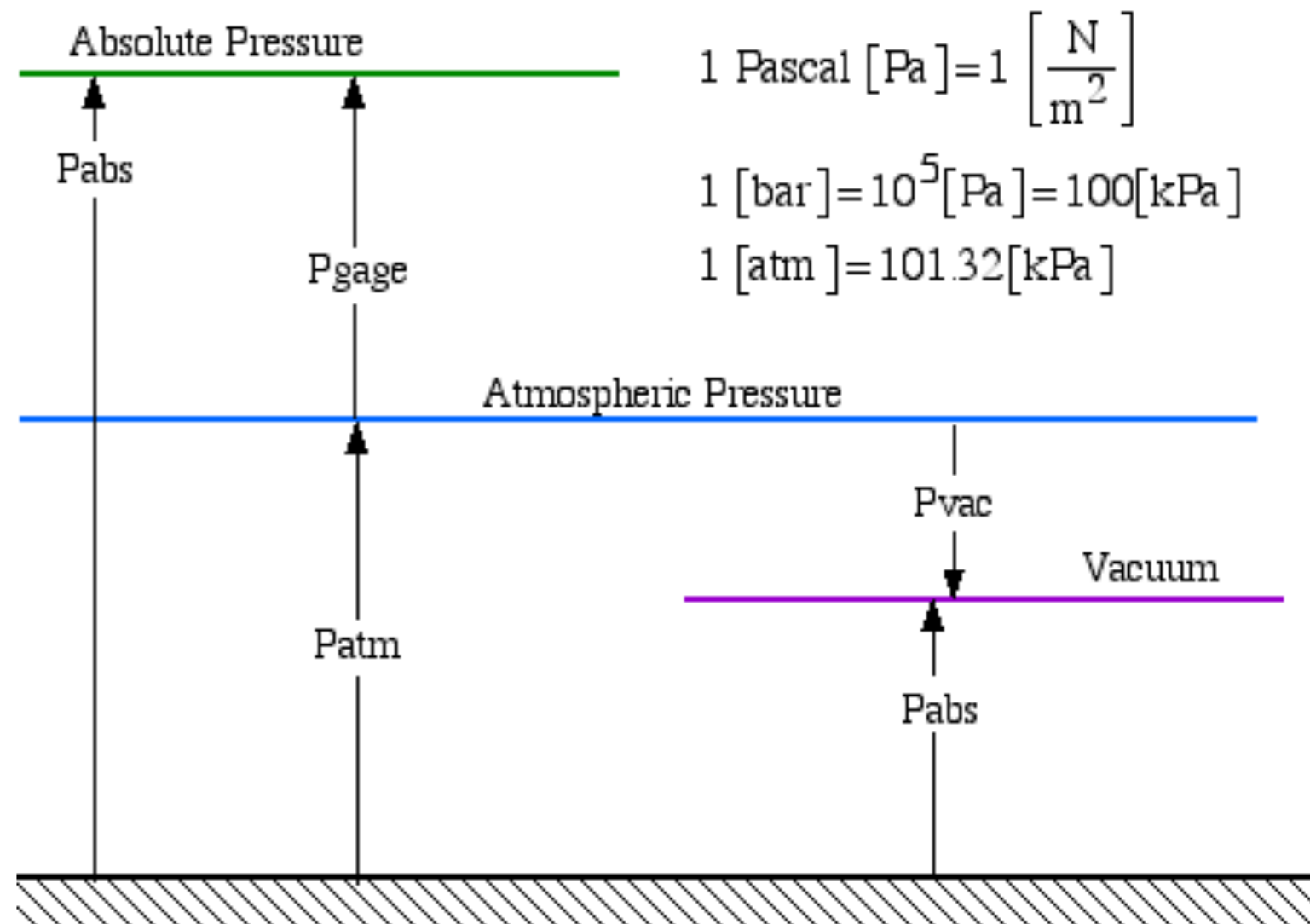
Pressure variations for incompressible fluids

- $P - P_0 = -\rho g (z - z_0)$
- Applies to liquids (no need to consider compressibility unless dealing with large changes in z ... deep in the ocean)
- Applies to gases for small changes in z only
- **$P = -\rho g h$** Pressure related to the height h of a fluid column: *Pressure head*

Absolute and Gage Pressure

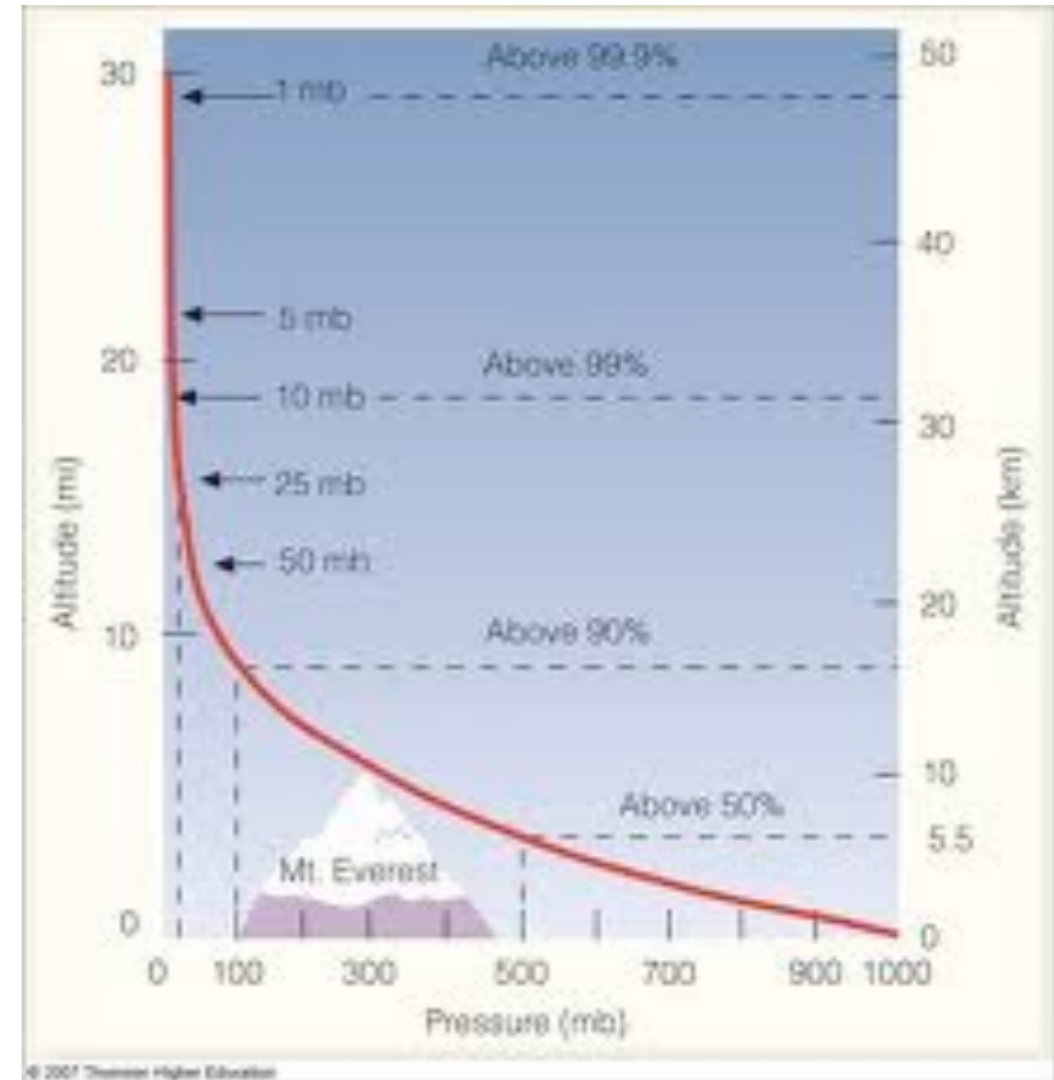
- **Absolute** relative to absolute zero (perfect vacuum)
- **Gage** relative to atmospheric pressure (>0 if $>P_{atm}$; <0 if $<P_{atm}$)
- if $P < P_{atm}$ we call it a vacuum

$$P_{abs} = P_{atm} + P_{gage}$$



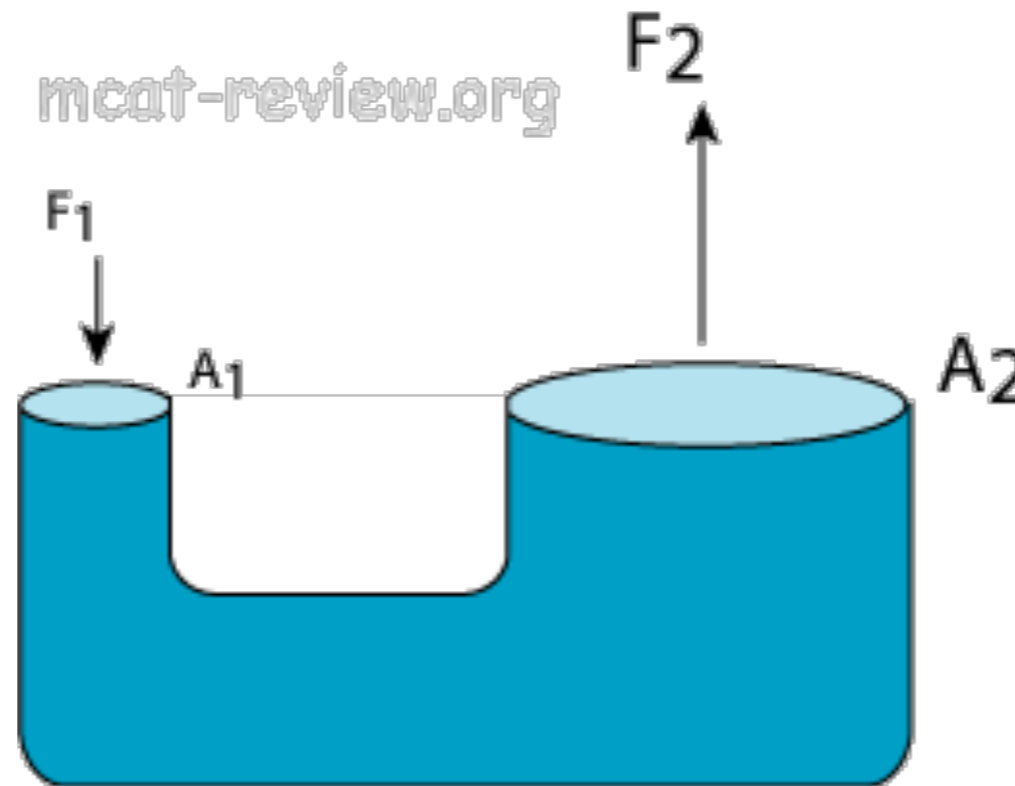
Pressure

- Atmospheric pressure is also called *barometric pressure* (1 bar = 10^5 Pa). It varies with elevation and changes in meteorological conditions
- Absolute pressure used for most problems related to gases/vapor
- Gage pressure related to liquids



Pascal's Law

- *All points in a connected body of constant-density fluid at rest are under the same pressure if they are at the same depth below the liquid surface.*



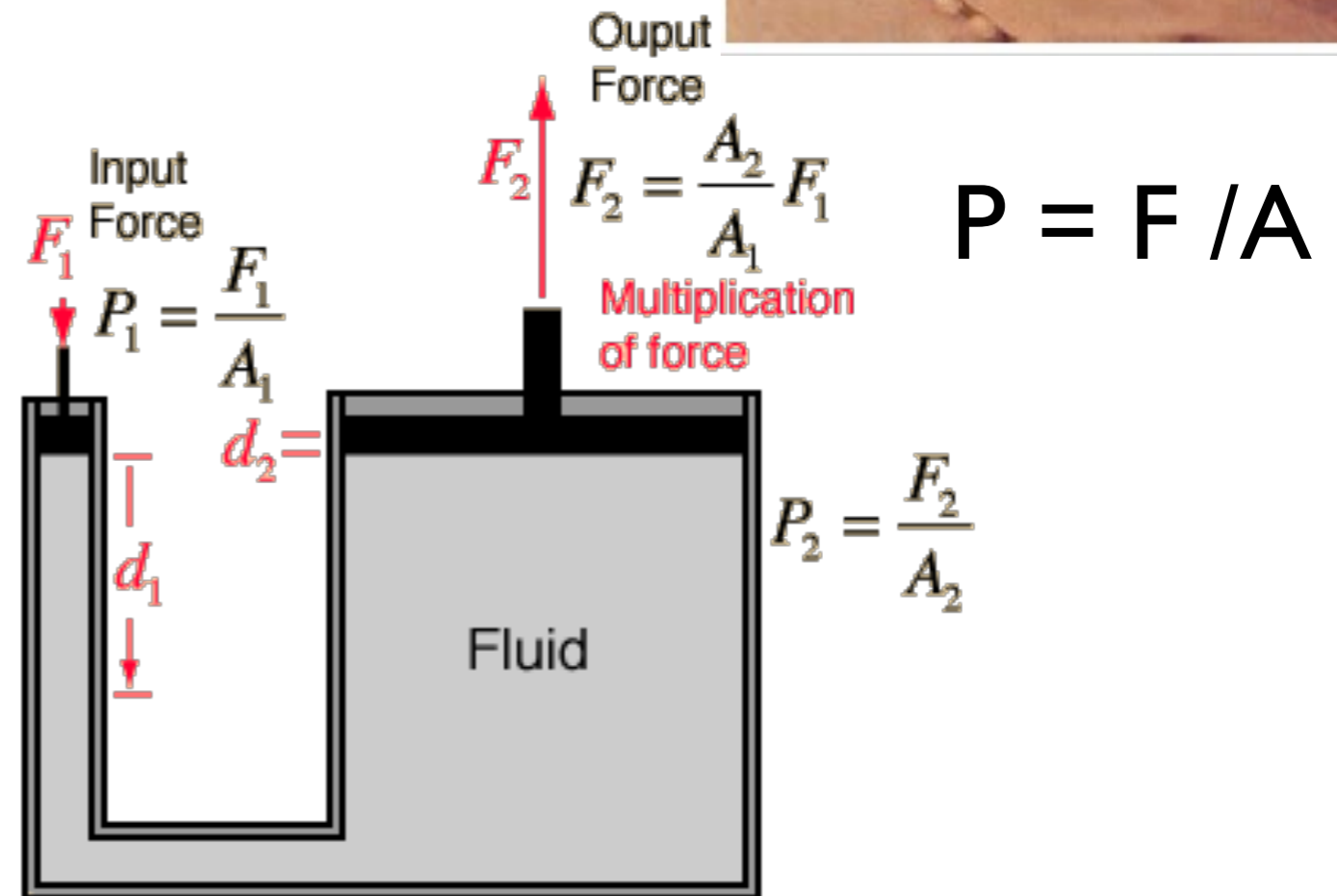
$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Pascal's Law



- if you apply pressure on a liquid, the pressure is transmitted equally and unchanged to all parts of the liquid.

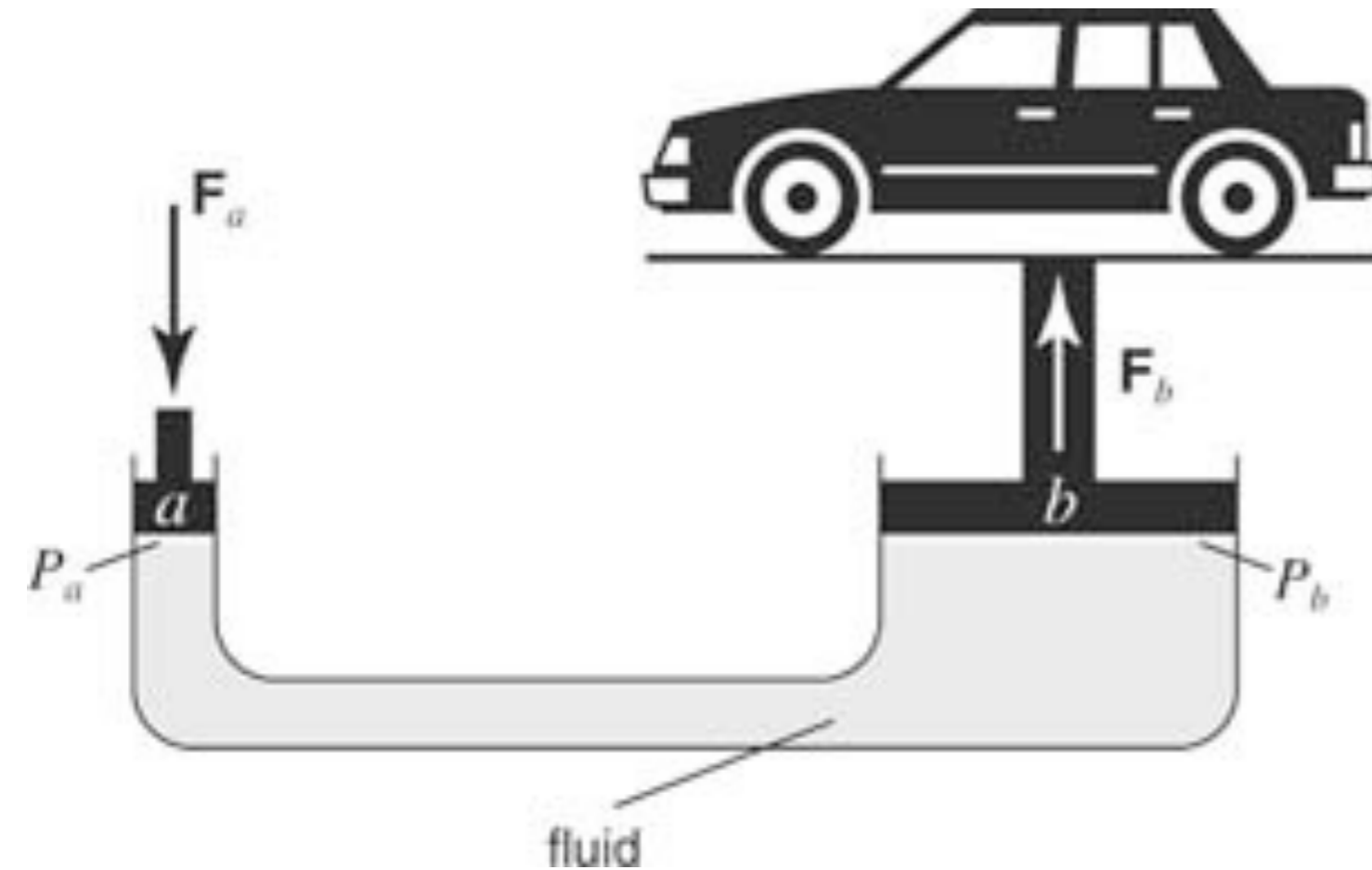


$$F_1 d_1 = F_2 d_2$$

$$d_1 = \frac{F_2}{F_1} d_2 = \frac{A_2}{A_1} d_2$$

You have to pay for the multiplied output force by exerting the smaller input force through a larger distance.

Automobile Hydraulic Lift



diameter $d_1 = 1.25\text{cm}$
diameter $d_2 = 25\text{ cm}$

Areas: $A_1 = 1.22$; $A_2 = 490$

--> $A_2/A_1 = 400$

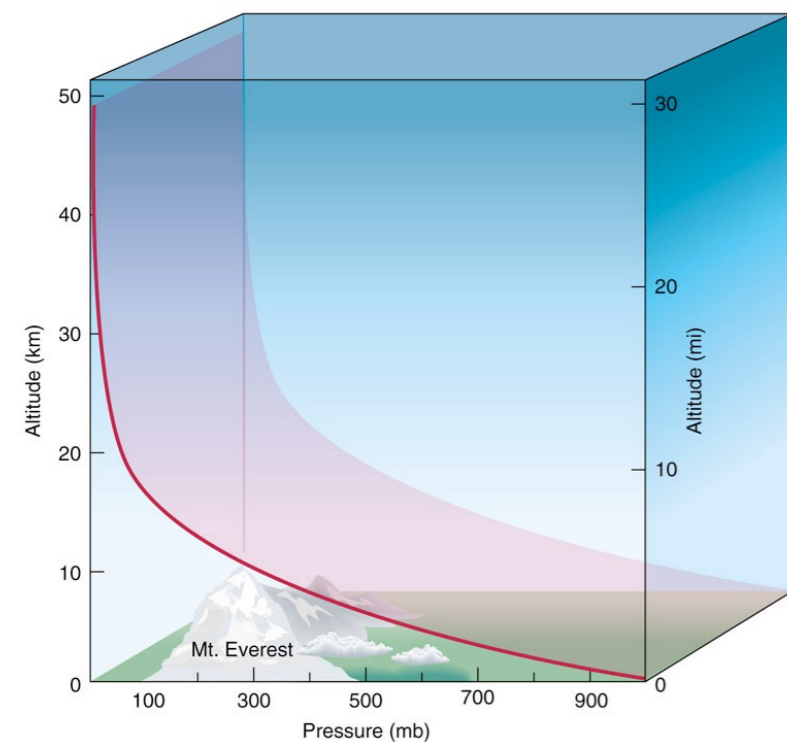
--> $F_2 = 400F_1$

If car is 6000N ---> $F_1 = 6000\text{N}/400 = 15\text{N}$

to lift it 10 cm ---> $400 \times 10 = 40\text{ m}!!$

Buoyancy force

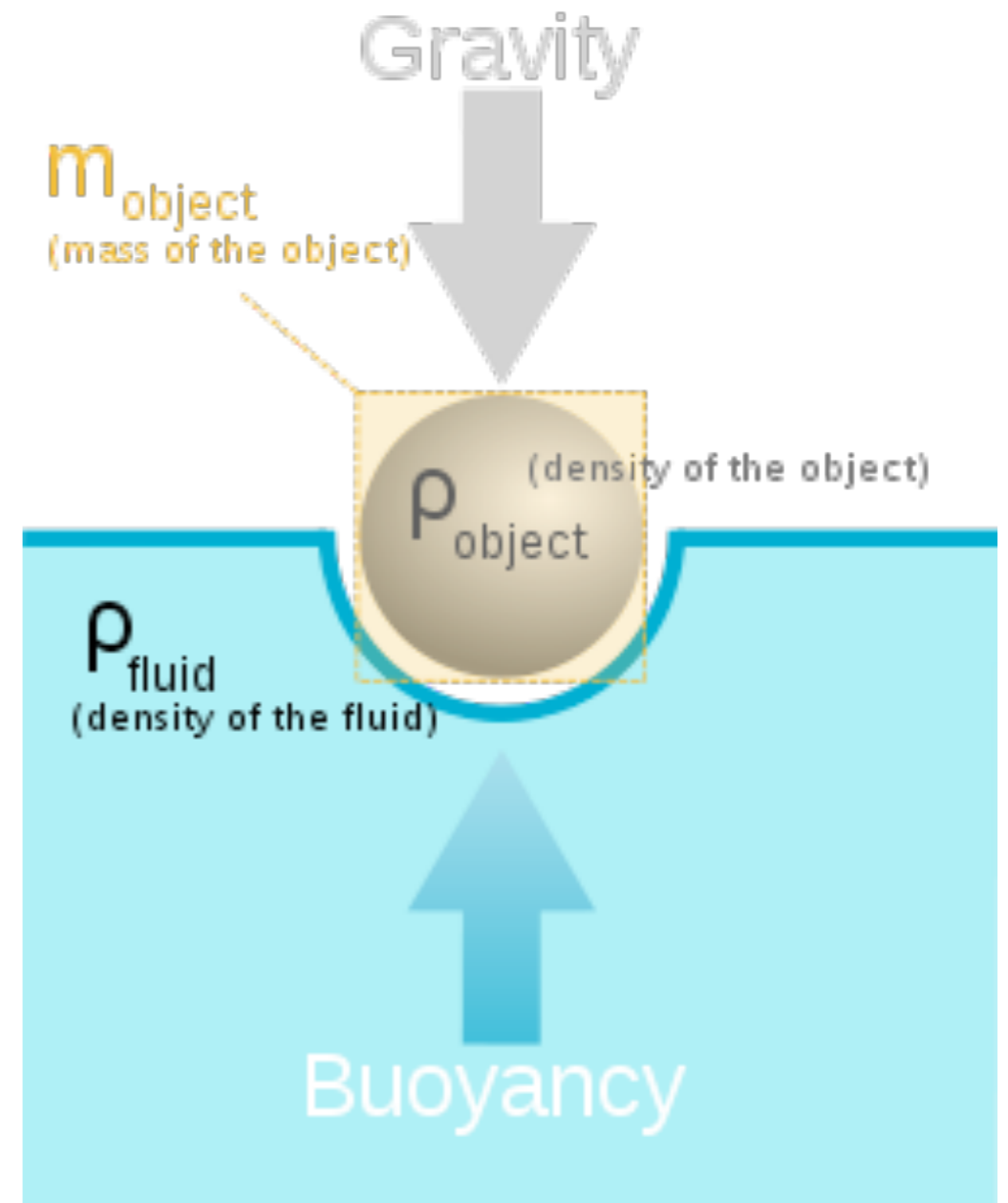
- Pressure in the atmosphere decreases with height (hydrostatics)
- Pressure force on balloon: bottom greater than at top



- Buoyancy force is the difference
- There is always a buoyancy force in a fluid, and it is always positive.

- A force exerted by a fluid that opposes an object's weight
- force is equal to weight of fluid displaced by the object
- $F_b = \rho(\text{fluid}) \times g \times V_{\text{disp}}$
- An object whose density (specific weight) is greater than that of the fluid in which it is submerged tends to sink ...

Buoyancy



is it easier to float in a pool or at sea?

- In equilibrium, the net Force must be zero, so that:

$$m g = \rho V_{disp} g = 0$$

*If the buoyancy of an object exceeds its weight, it tends to rise.
An object whose weight exceeds its buoyancy tends to sink.*

Archimedes' principle indicates that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces.

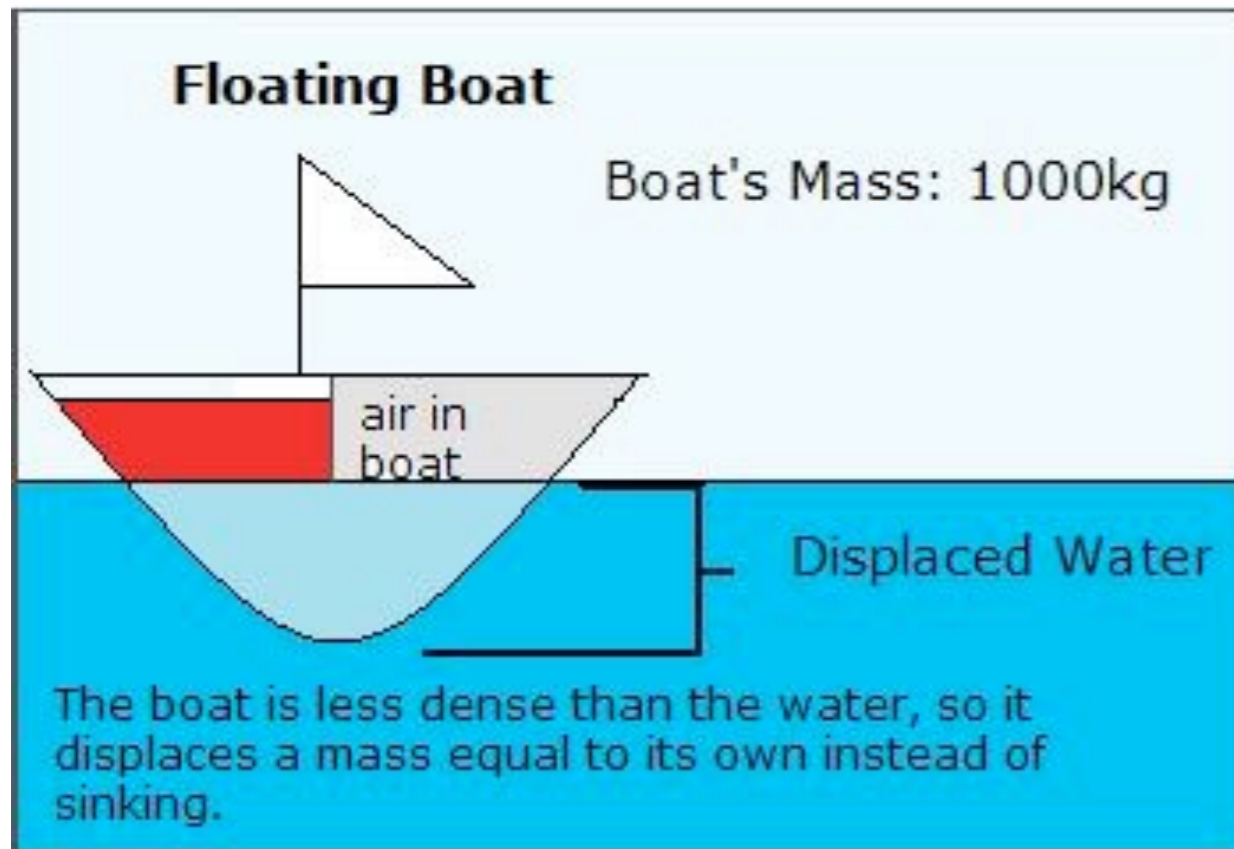
Materials of human body (density Kg/l):

muscle = 1.1; bone = 1.5; air = 0.0012

In fresh water (with air out): MEN all sink - WOMEN some float

In fresh water (with air in): MEN some sink - WOMEN all float

Buoyancy and floating



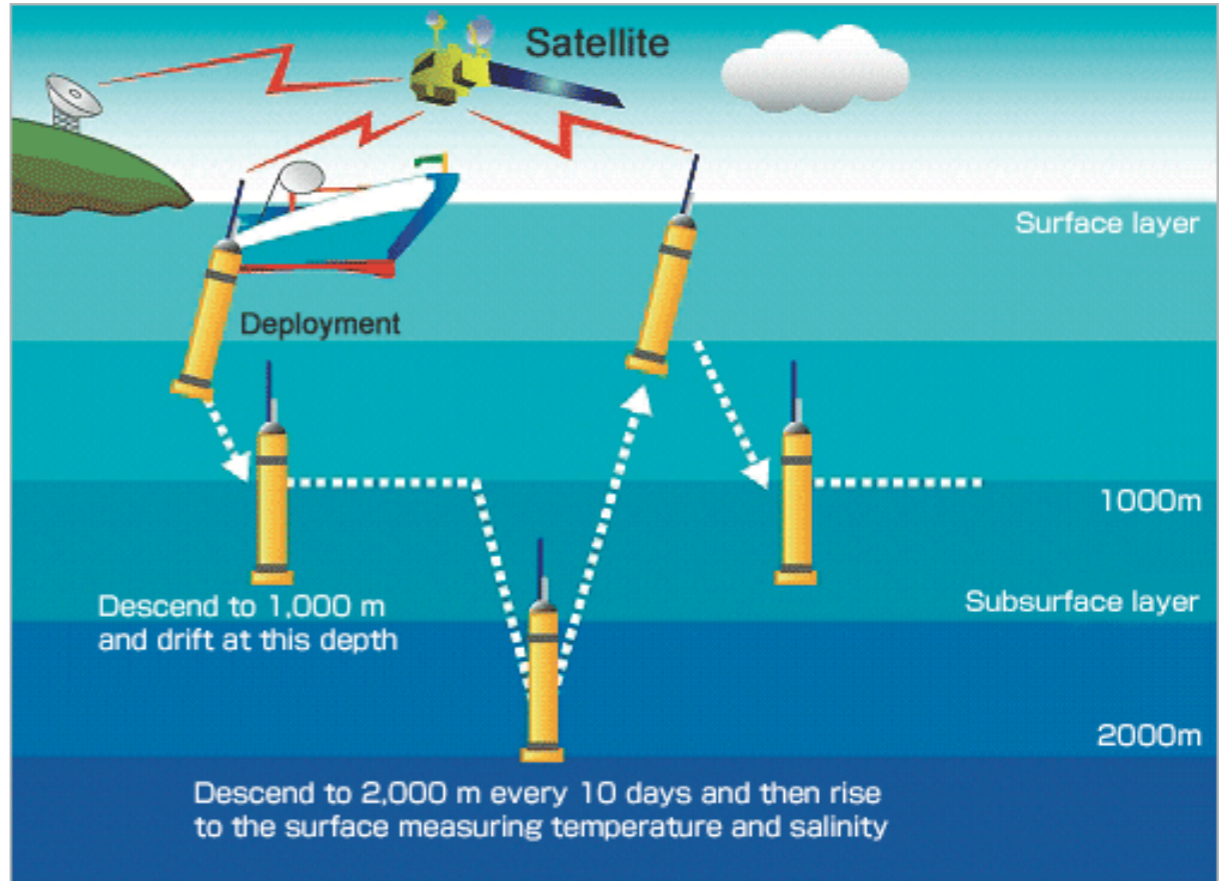
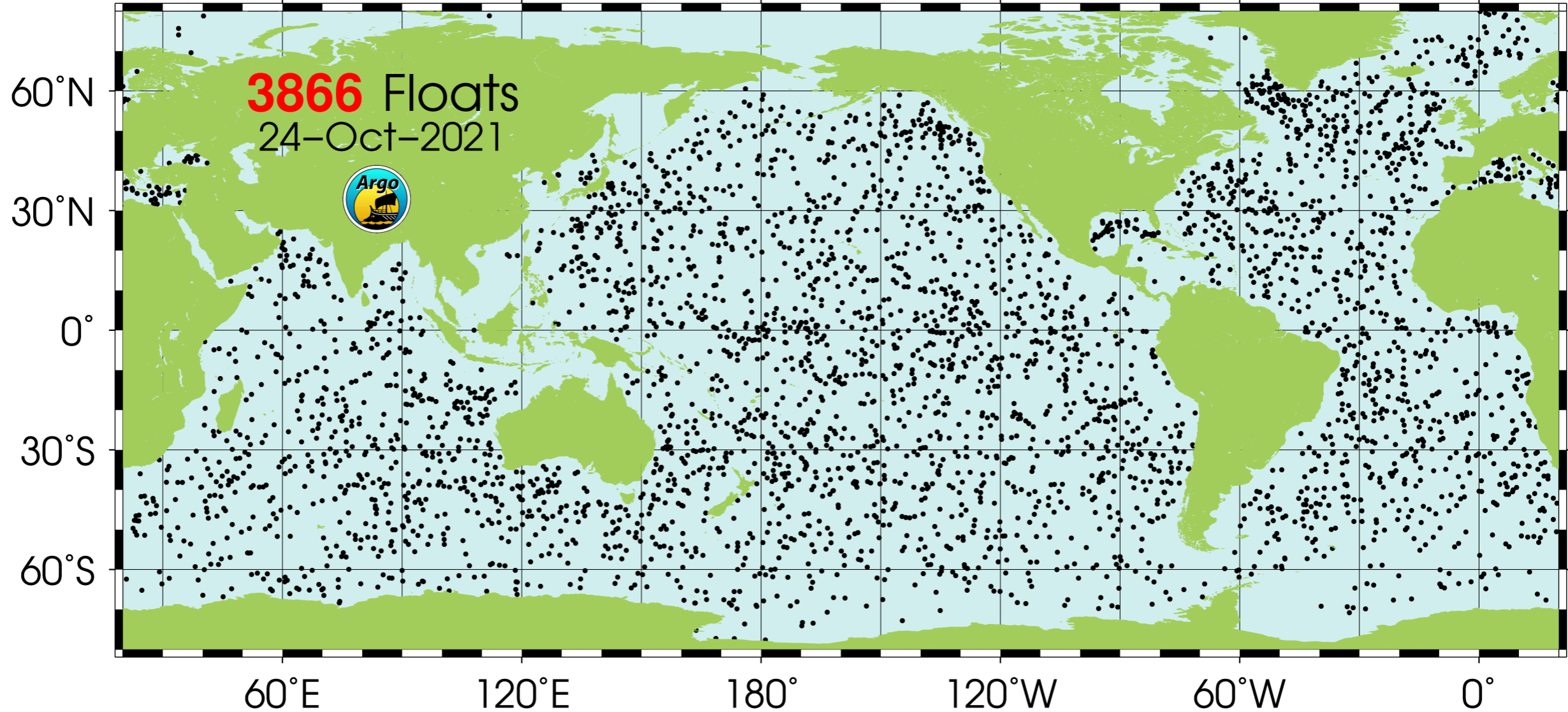
A block of iron dipped in water will sink, while the same metal block shaped like a boat will float.

Buoyancy is thus related to the **density**, **volume** and **shape** of the immersed body.

If $F_b = \rho \times g \times V_{disp}$, what is volume of the displaced water?

Stationarity $\rightarrow W_{boat} = F_b = \rho \times g \times V_{disp}$

$V_{underwater} = W_{boat} / (\rho \times g)$





A curiosity ... (the Iceberg)

- Roughly: $\rho_{\text{ice}} = 92\% \rho_{\text{water}}$

(another curiosity in itself ...)

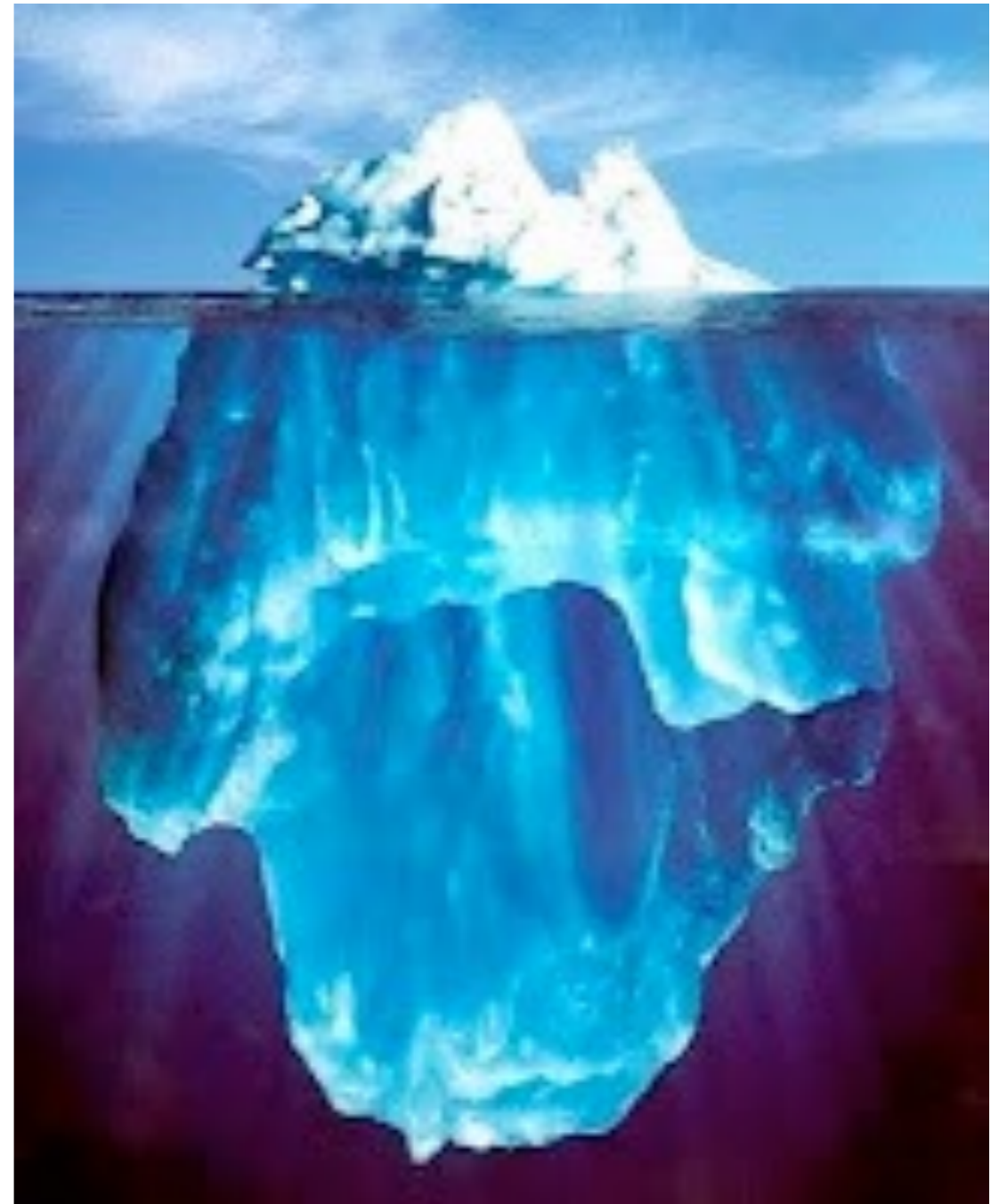
- It is in equilibrium,
so that $mg = F_b$
- how much of the
iceberg is
submerged?

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(another curiosity in itself ...)

- It is in equilibrium, so that $mg = F_b$
- how much of the iceberg is submerged?



- 92% (... “you only see the tip of the iceberg ...”)

Summary

- Fluid Statics
- Pascal's Law
- Absolute and Gage Pressure
- Buoyancy and Archimedes' Principle