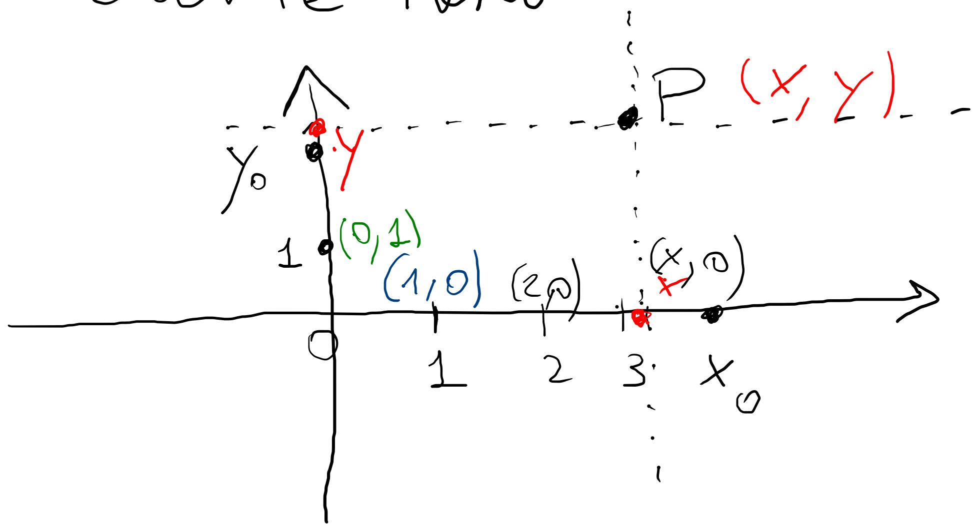
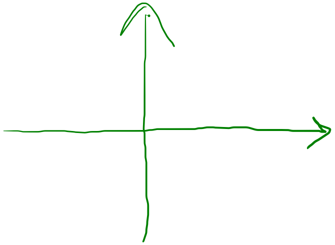
Ciccagna Analisi I Industriali e Navali

Moodle

Piano Cartesiano



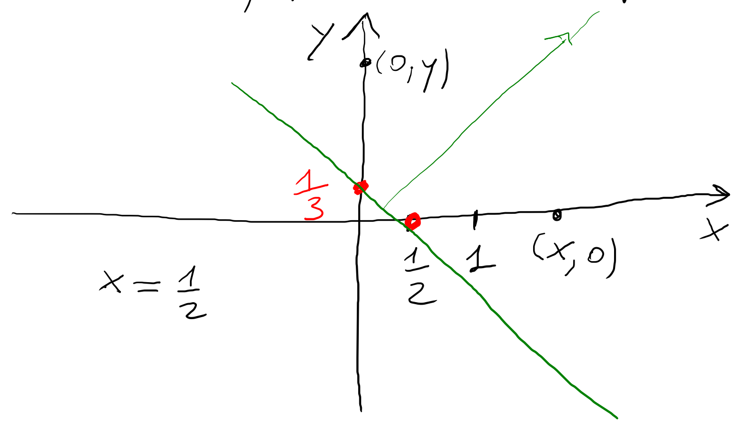


Rette. I punti delle rette sono i punti del piano che risolvono equazioni della forma

$$ax+by=c$$

dove a,b,c sono tre numeri reali, $(a,b)\neq(0,0)$ ed a,b,c sono coefficienti dell'equazione.

$$2x+3y=1$$



$$\begin{cases} 2x+3y=1 \\ y=0 \end{cases} \qquad 2x=1 \qquad x=\frac{1}{2}$$

$$\begin{cases} 2x+3y=1 \\ x=0 \end{cases} \qquad 3y=1 \qquad y=\frac{1}{3}$$

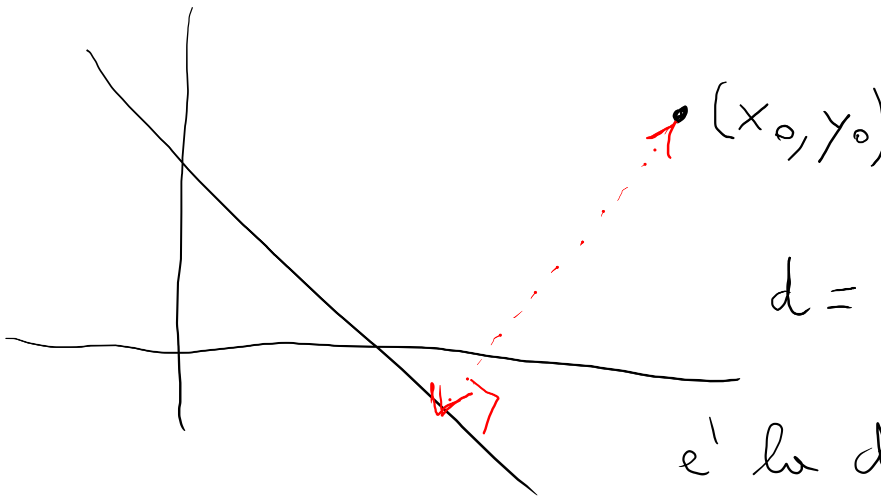
Poi discuteremo del significato geometrico di (a,b,c)

$(2,3,1)$

Data una retta $ax+by=c \iff ax+by-c=0$

il vettore (a,b) è ortogonale alla retta

ed inoltre se considero un punto (x_0,y_0) del piano



allora

$$d = \frac{|ax_0+by_0-c|}{\sqrt{a^2+b^2}}$$

è la distanza di (x_0,y_0) dalla retta

Considerato $2x+3y=1$ $(2,3)$

e preso un numero $\lambda \neq 0$, anche

$$2\lambda x+3\lambda y-\lambda=0 \qquad (2\lambda,3\lambda)=$$

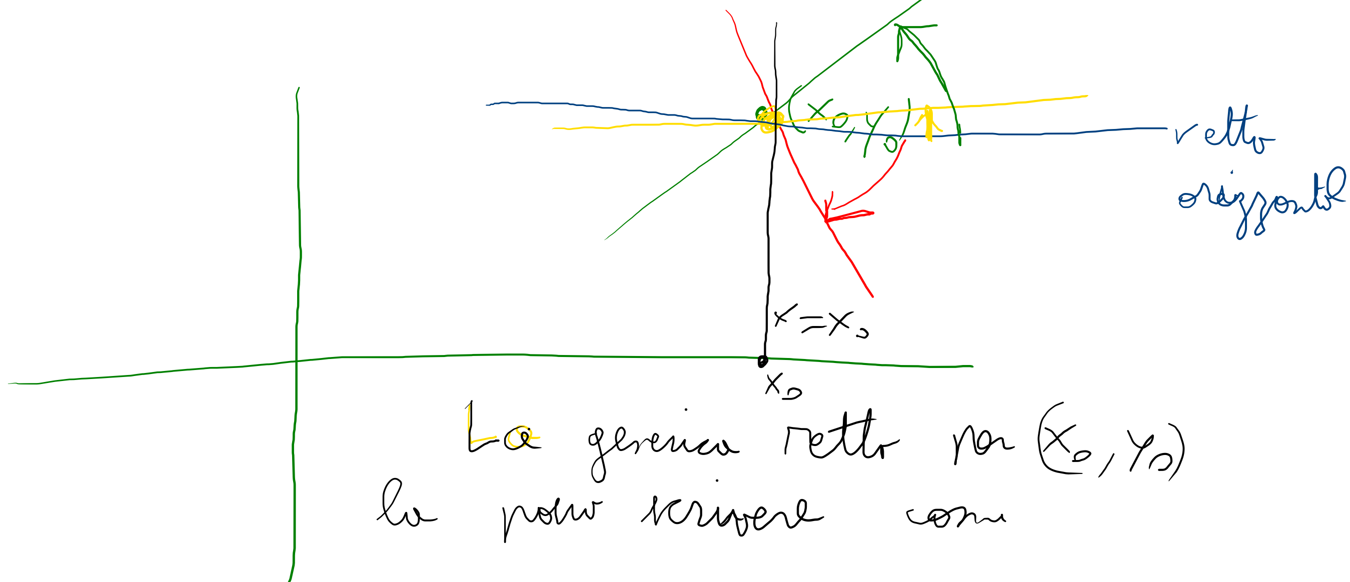
è l'equazione della retta $=\lambda(2,3)$

$$d=\frac{|2x_0+3y_0-1|}{\sqrt{2^2+3^2}}=\frac{|2\lambda x_0+3\lambda y_0-\lambda|}{\sqrt{2^2\lambda^2+3^2\lambda^2}}$$

$$=\frac{|\lambda(2x_0+3y_0-1)|}{\sqrt{\lambda^2(2^2+3^2)}}=\frac{|\lambda|\;|2x_0+3y_0-1|}{\sqrt{\lambda^2}\;\sqrt{2^2+3^2}}$$

$$=\frac{\cancel{|\lambda|}}{\cancel{|\lambda|}}\;\frac{|2x_0+3y_0-1|}{\sqrt{2^2+3^2}}$$

Se considero il punto (x_0, y_0)



La generica retta per (x_0, y_0) la posso scrivere come

$$y - y_0 = m(x - x_0)$$

$$\frac{1}{m}(y - y_0) = x - x_0$$

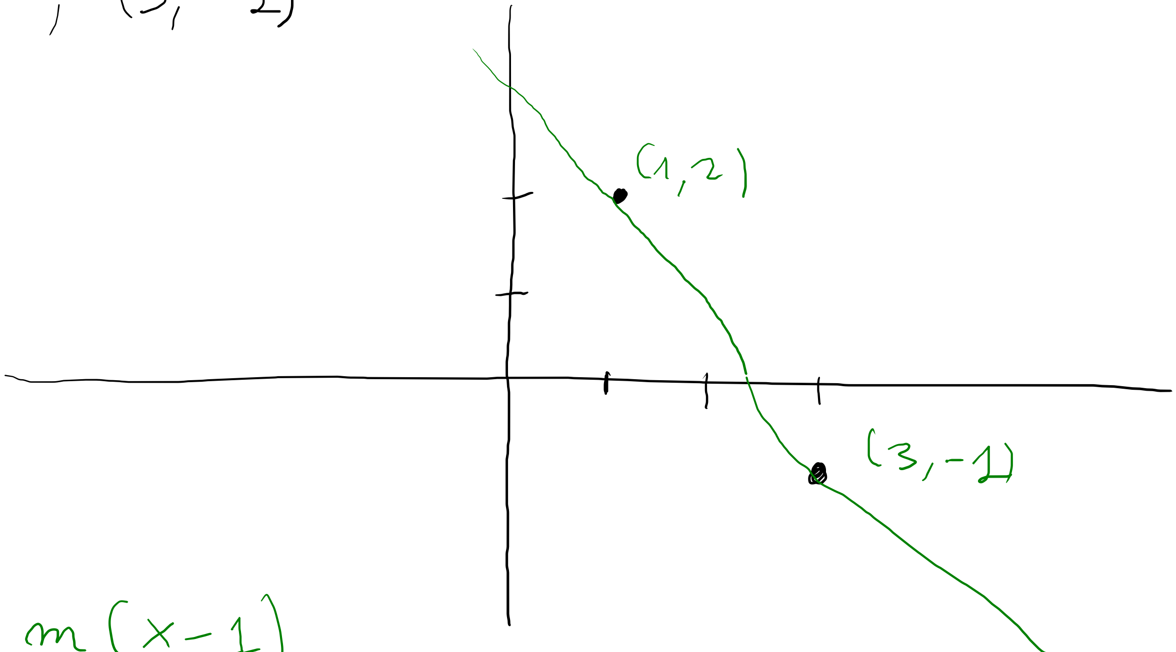
$$x - x_0 = 0$$

m è il coefficiente angolare

$$y = mx + k$$

$$2x + 3y = 1 \qquad 3y = -2x + 1 \qquad y = -\frac{2}{3}x + \frac{1}{3}$$

$(1, 2)$, $(3, -1)$



$y - 2 = m(x - 1)$

$-1 - 2 = m(3 - 1)$

$-3 = m \ 2$

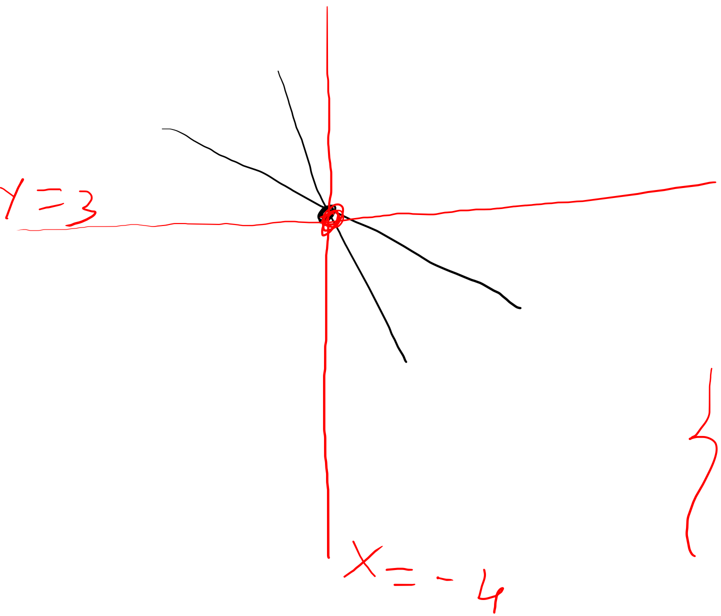
$\boxed{m = -\frac{3}{2}}$

$$\begin{cases} 2x+3y=1 \\ x+2y=2 \end{cases}$$

$$\begin{cases} x=2-2y \\ 2(2-2y)+3y=1 \end{cases}$$

$$4-4y+3y=1$$

$$4-y=1 \qquad y=3$$



$$\begin{cases} x=2-2y \\ y=3 \end{cases}$$

$$x=2-6=-4$$

$$\begin{cases} x=-4 \\ y=3 \end{cases}$$

$$\begin{cases} x+2y=1 \\ 2x+4y=5 \end{cases}$$

$$x = 1-2y$$

$$2(1-2y)+4y=5$$

$$2 - \cancel{4y} + \cancel{4y} = 5$$

$$2 = 5 \quad \text{falso}$$

$$\begin{cases} x+2y=1 \\ 2x+4y=2 \end{cases}$$

$$x = 1-2y$$

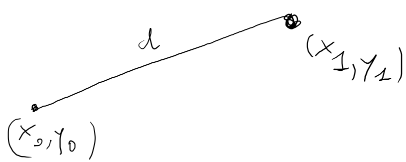
$$2(1-2y)+4y=2$$

$$\cancel{2} - \cancel{4y} + \cancel{4y} = \cancel{2}$$

$$0 = 0$$

$$\forall y$$

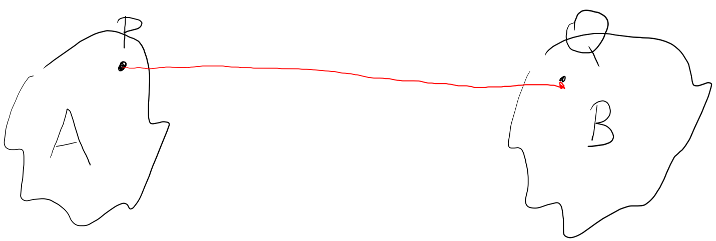
Dati due punti del piano (x_0, y_0) e (x_1, y_1)



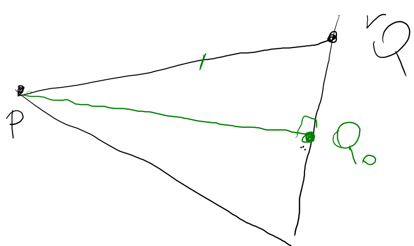
$$d = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2} = \overset{d}{\text{distanza}}\left((x_0, y_0), (x_1, y_1)\right)$$

$$d(P, Q)$$

$d(A, B)$



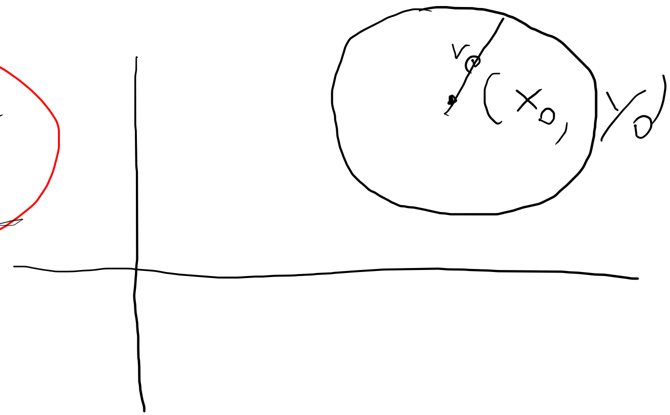
$$d(A, B) = \inf\{d(P, Q) : P \in A, Q \in B\}$$



Una circonferenza di centro (x_0, y_0) e raggio $r_0 > 0$ è l'insieme degli (x, y) la cui distanza da (x_0, y_0) è uguale ad r_0

$$(x - x_0)^2 + (y - y_0)^2 = r_0^2$$

$$\sqrt{(x - x_0)^2 + (y - y_0)^2} = r_0$$



$$ax^2+by^2+cxy+dx+ey+f=0$$

$$a=b\neq 0 \qquad c=0$$

$$2x^2+2y^2+3y+x+1=0$$

$$(x-x_0)^2+(y-y_0)^2=r^2$$

$$x^2+\frac{1}{2}x+y^2+\frac{3}{2}y+\frac{1}{2}=0$$

$$\left(x^2+2\,\frac{1}{4}x\right)+\left(y^2+2\,\frac{3}{4}y\right)+\frac{1}{2}=0$$

$$\left(x+\frac{1}{4}\right)^2-\frac{1}{16}+\left(y+\frac{3}{4}\right)^2-\frac{9}{16}+\frac{1}{2}=0$$

$$\left(x+\frac{1}{4}\right)^2+\left(y+\frac{3}{4}\right)^2=\frac{10}{16}-\frac{1}{2}=\frac{5}{8}-\frac{1}{2}=\frac{5-4}{8}=\frac{1}{8}$$

$$\left(x-\left(-\frac{1}{4}\right)\right)^2+\left(y-\left(-\frac{3}{4}\right)\right)^2=\frac{1}{4}$$

$$\left(x+\frac{1}{4}\right)^2+\left(y+\frac{3}{4}\right)^2=\frac{1}{8} \qquad \left(-\frac{1}{4},-\frac{3}{4}\right) \qquad \frac{1}{\sqrt{8}}$$

$$(x-x_0)^2+(y-y_0)^2=r^2 \qquad (x_0,y_0) \qquad r$$