Corso di Laurea in Fisica - UNITS

ISTITUZIONI DI FISICA PER IL SISTEMA TERRA

Wave propagation

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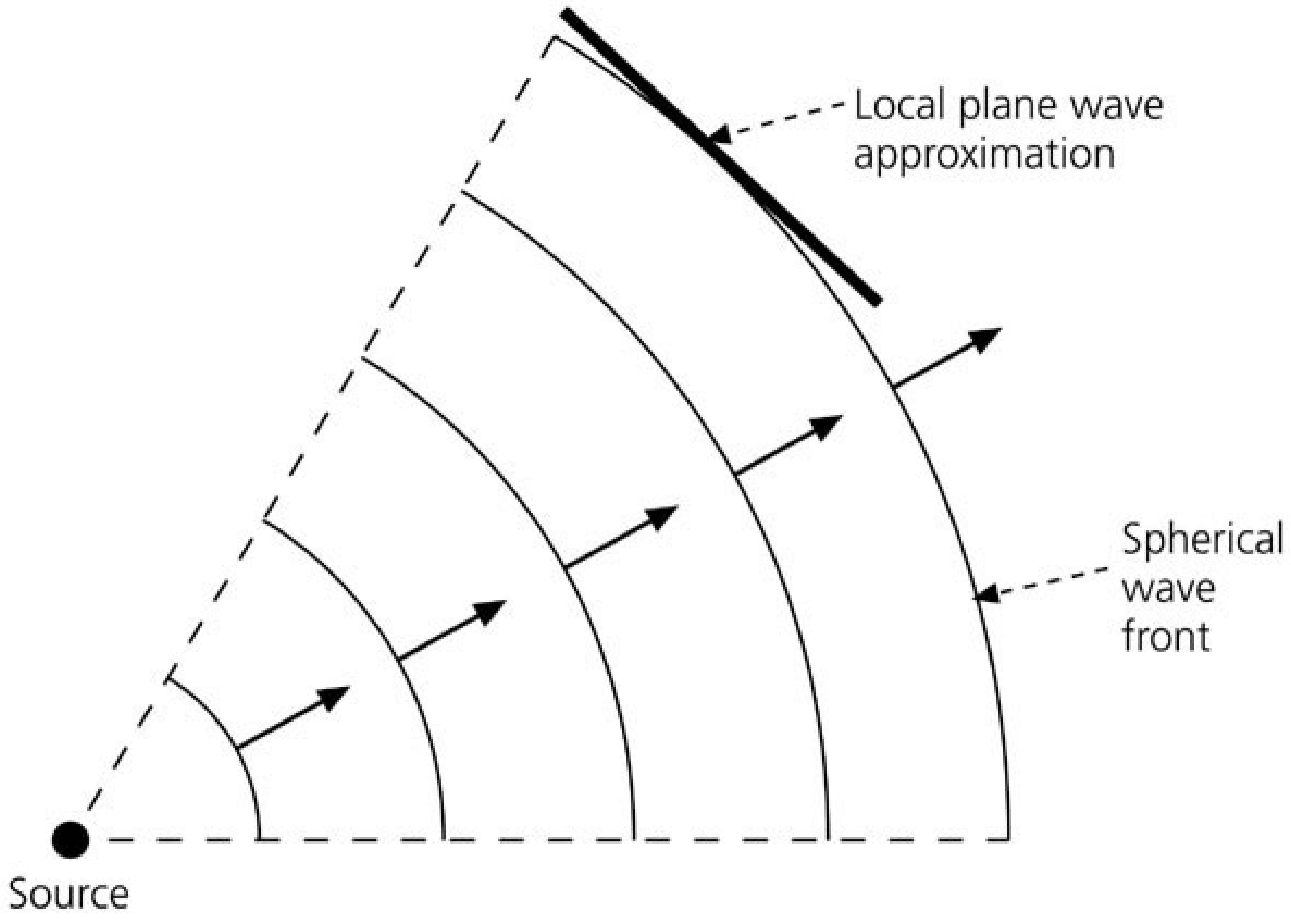
Spherical Waves

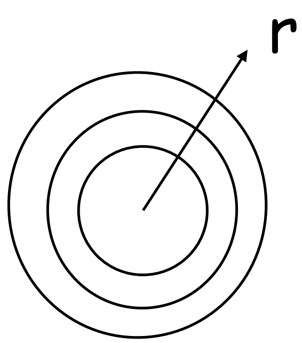
Figure 2.4-2: Approximation of a spherical wave front as plane waves.



Spherical Waves

$$\frac{1}{c^2}\frac{\partial^2 \eta}{\partial t^2} = \Delta\eta$$

Let us assume that η is a function of the distance from the source



$$\Delta\eta = \partial_r^2\eta + \frac{2}{r}\partial_r\eta = \frac{1}{c^2}\partial_t^2\eta$$

where we used the definition of the Laplace operator in spherical coordinates let us define

$$\eta = \frac{\bar{\eta}}{r}$$

to obtain

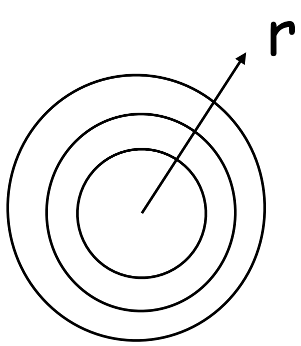
$$\frac{1}{c^2}\frac{\partial^2 \bar{\eta}}{\partial t^2} = \frac{\partial^2 \bar{\eta}}{\partial r^2}$$

with the known solution $\bar{\eta}_r = f\left(\alpha \pm rt\right)$

Geometrical spreading

so a disturbance propagating away with spherical wavefronts decays like

$$\eta = \frac{f(\alpha \pm rt)}{r} \qquad \eta \sim \frac{1}{r}$$



... this is the geometrical spreading for spherical waves, the amplitude decays proportional to 1/r.

If we had looked at cylindrical waves the result would have been that the waves decay as (e.g. surface waves)

$$\eta \sim \frac{1}{\sqrt{r}}$$

Wave propagation

(Sound) Waves propagation is ruled by:

SUPERPOSITION PRINCIPLE

GEOMETRICAL SPREADING

REFLECTION

REFRACTION

DIFFRACTION

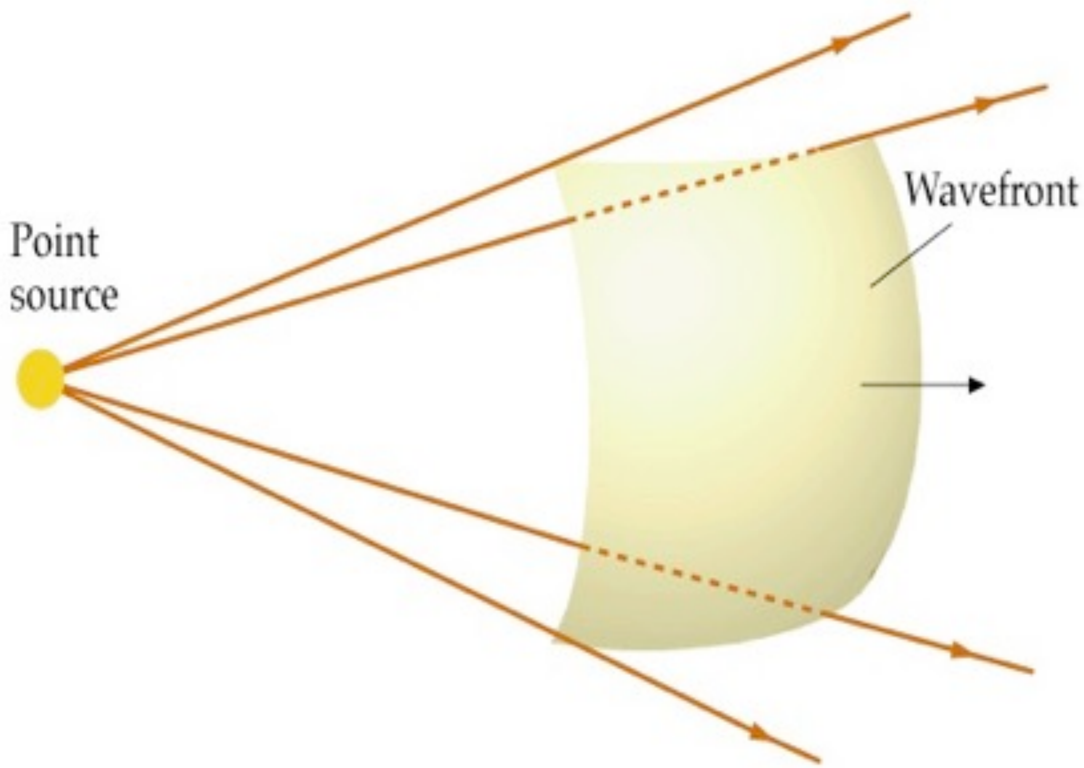
DOPPLER EFFECT

Before Maxwell's equations were developed Huygens and Fermat could describe the propagation of light using their empirically determined principles.

Huygens Principle (Traité de la Lumière, 1690)

All points on a given wavefront are taken as point sources for the production of spherical secondary waves, called wavelets, which propagate outwards with speeds characteristic of waves in that medium.

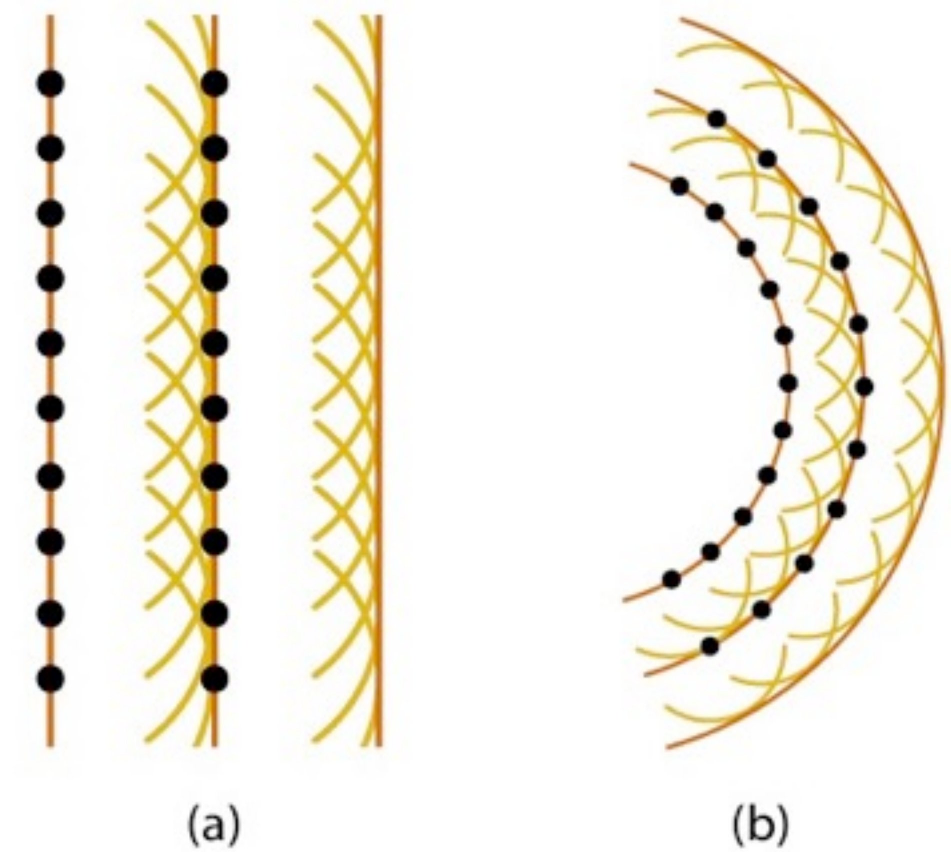
After some time has elapsed the new position of the wavefront is the surface tangent to the wavelets.



Spherical wavefront from a point source

Huygens construction for

plane wave   spherical wave



This is a general principle for determining the paths of light rays.

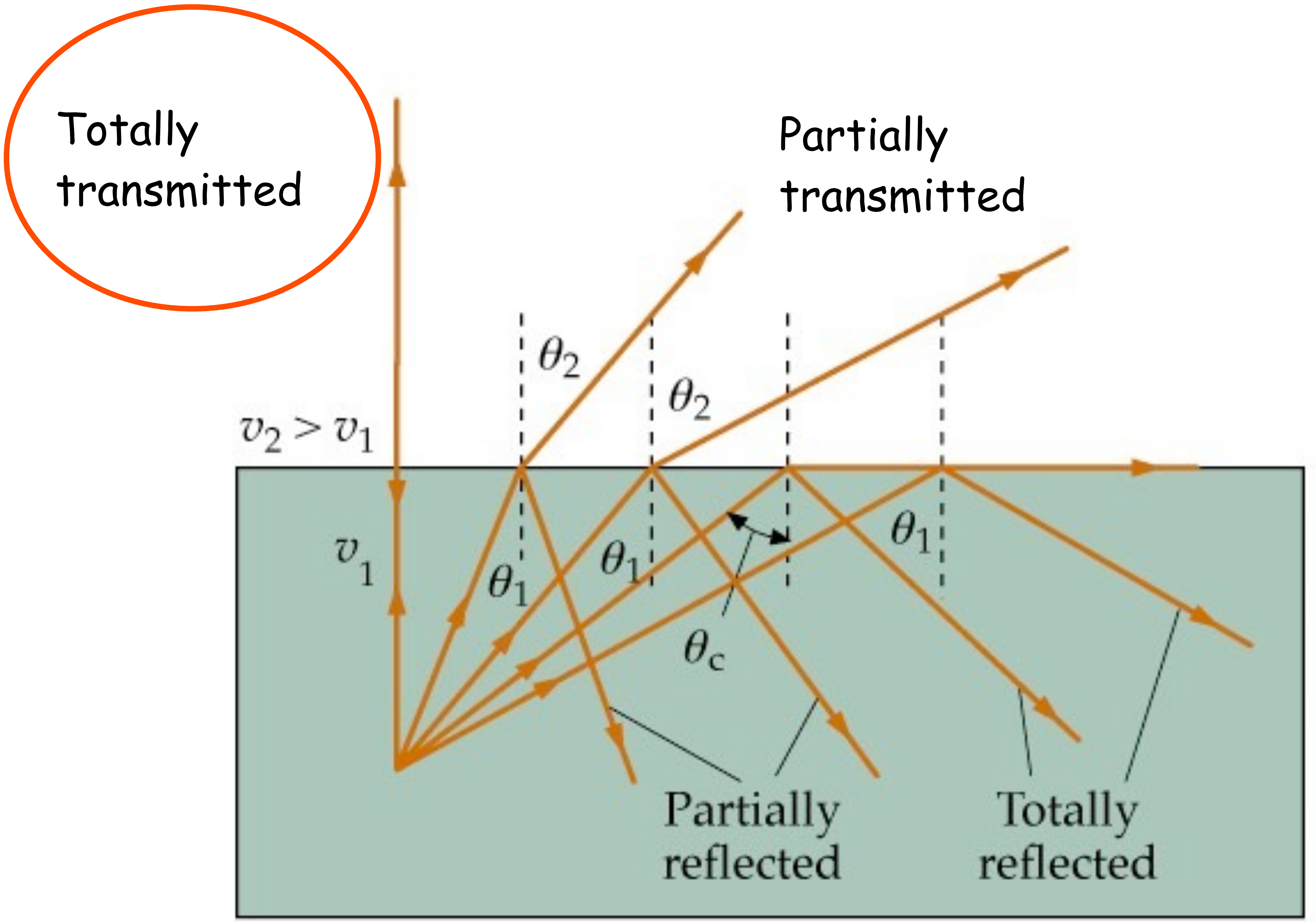
Fermat's Principle (Analyse des réfractions, 1662)

When a light ray travels between any two points P and Q, its actual path will be the one that takes the least time.

Fermat's principle is sometimes referred to as the "principle of least time".

An obvious consequence of Fermat's principle is that when light travels in a single homogeneous medium the paths are straight lines.

Reflection and refraction of light



Index of refraction :

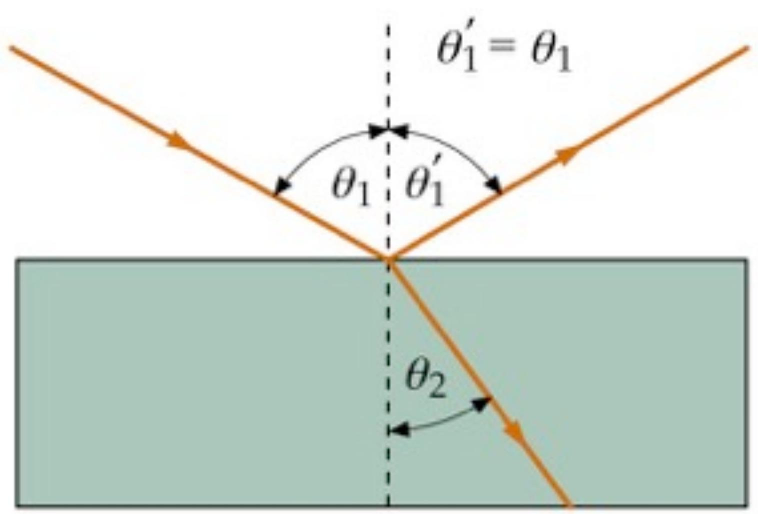
A transparent medium is characterised by the **index of refraction** n, where n is defined as the ratio of the speed of light in a vacuum c, to the speed of light in the medium v

$$n = \frac{c}{v}$$

air	n=1.0003
water	n=1.33
diamond	n=2.4

Law of reflection :

$$\theta_1 = \theta_1'$$



Intensity of transmitted and reflected light

For normal incidence at a boundary (ie $\theta_1 = \theta_1' = 0$)

the reflected intensity I is given by:

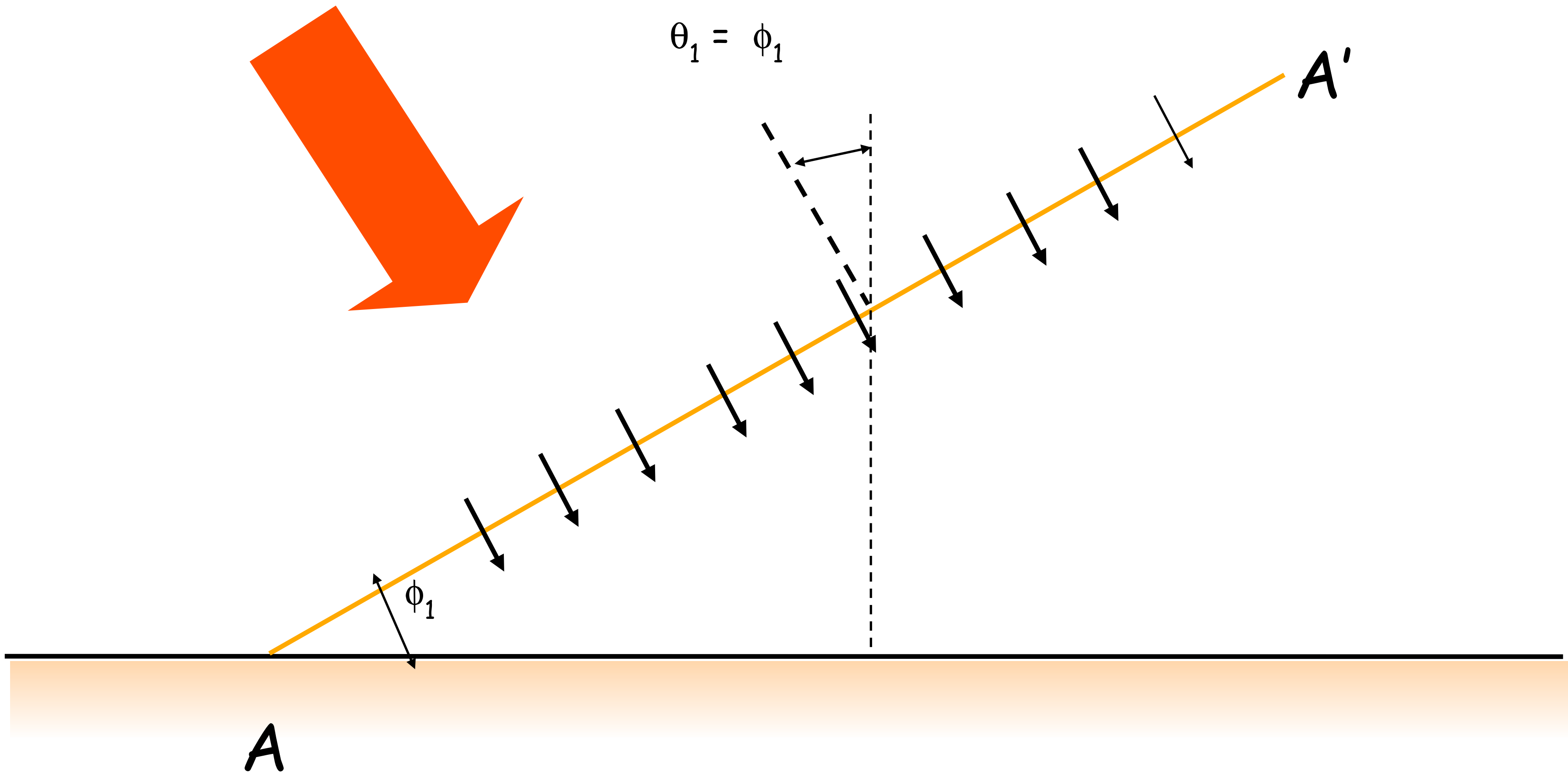
$$I = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 I_o$$

I_o = incident intensity, n_1 and n_2 are the refractive indexes of medium 1 and 2

eg: for an air-glass interface, $n_1 = 1$ and $n_2 = 1.5$

giving $I = I_o/25$

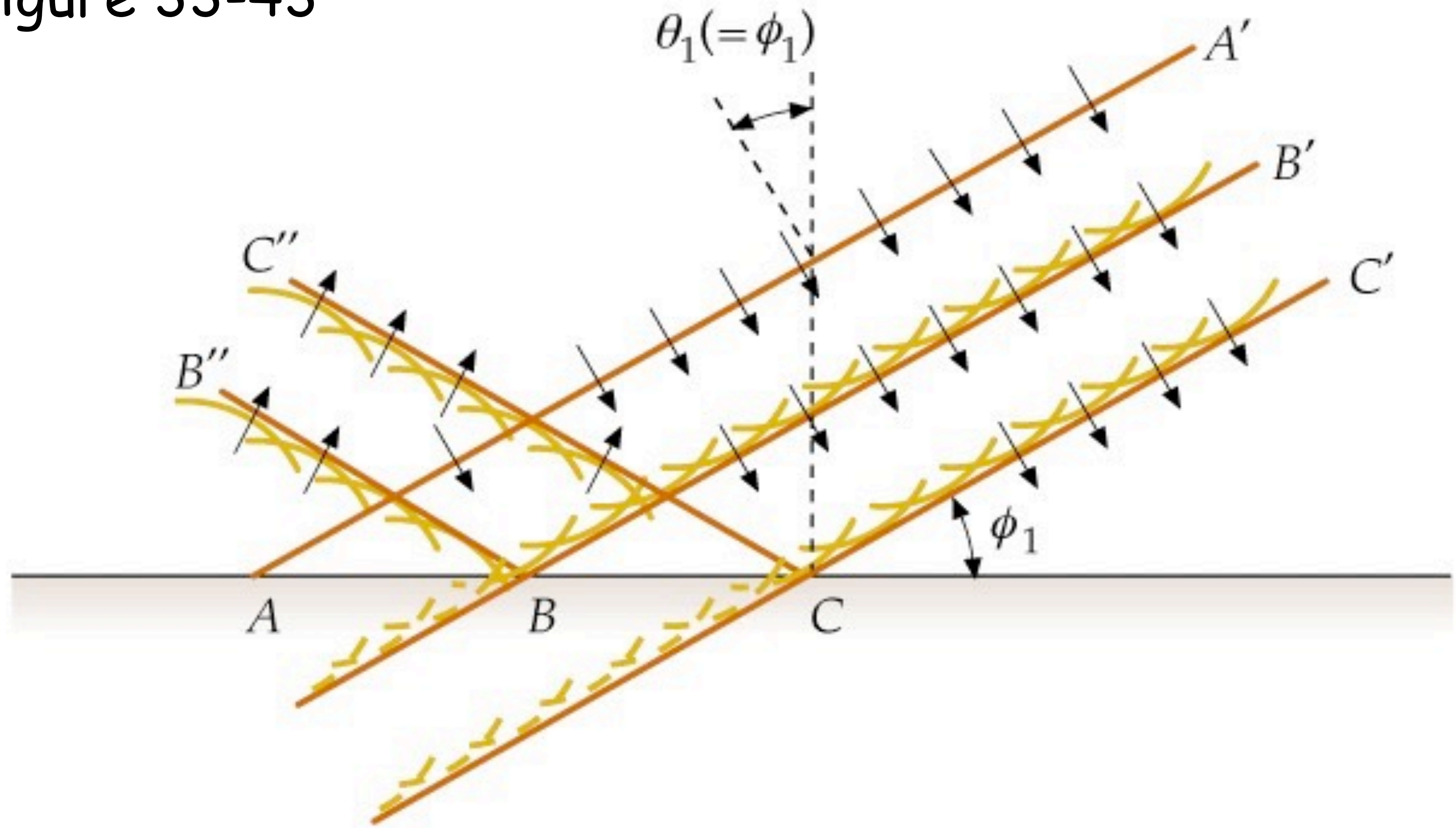
Derivation of law of reflection (Huygens)



AA' is a wavefront of incident light striking a mirror at A

The angle between the wavefront and the mirror = the angle between the normal to the wavefront and the vertical direction (normal to the mirror)

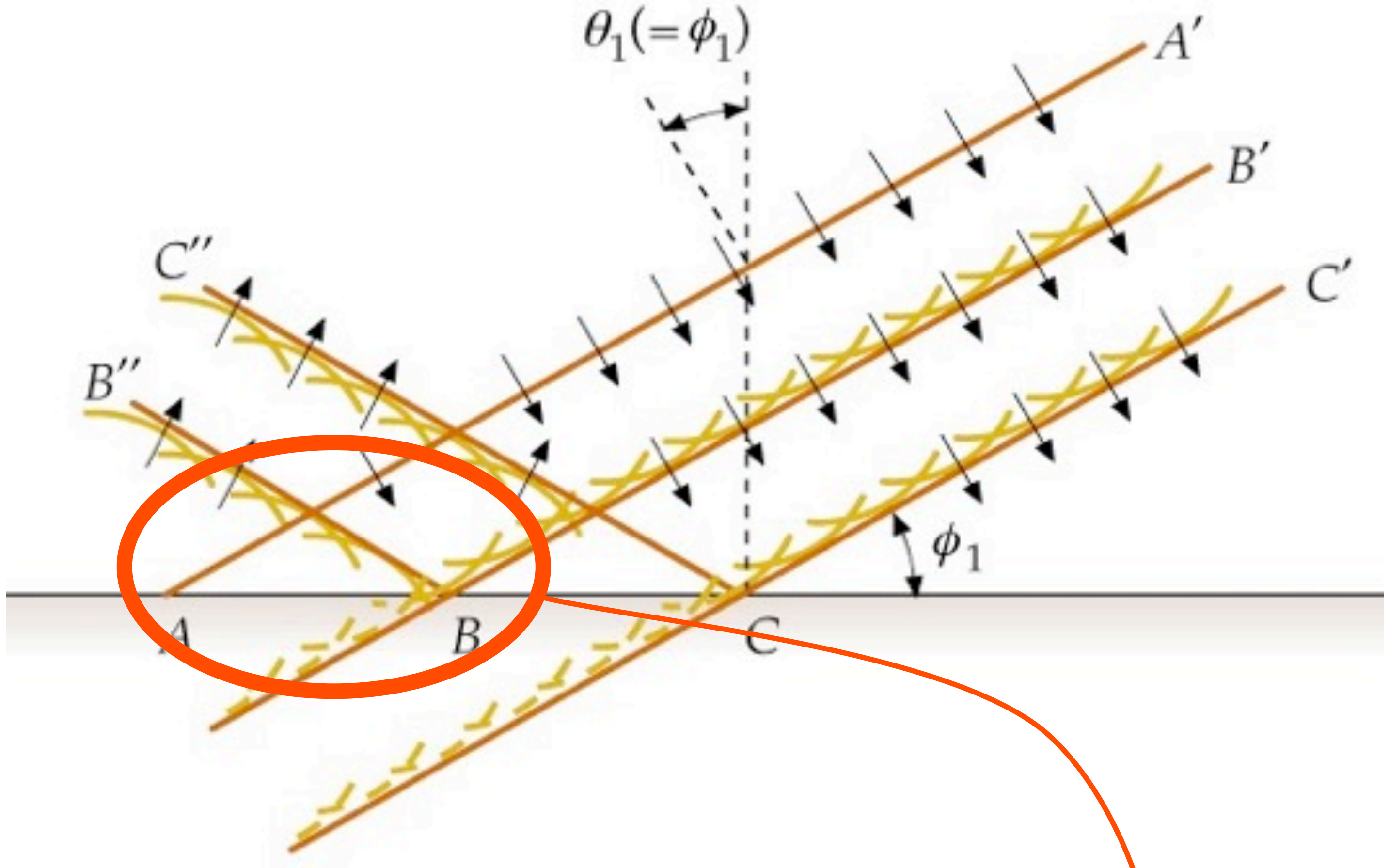
Tipler figure 33-43



According to Huygens each point on the wavefront can be thought of as a point source of secondary wavelets

Derivation of law of reflection (Huygens)

The position of the wavefront after a time t can be found by constructing wavelets of radius ct with centres on AA′



Waves that do not strike the mirror form the new wavefront BB′ (and similarly CC′)

Consider a small portion of these waves........

Derivation of law of reflection (Huygens)

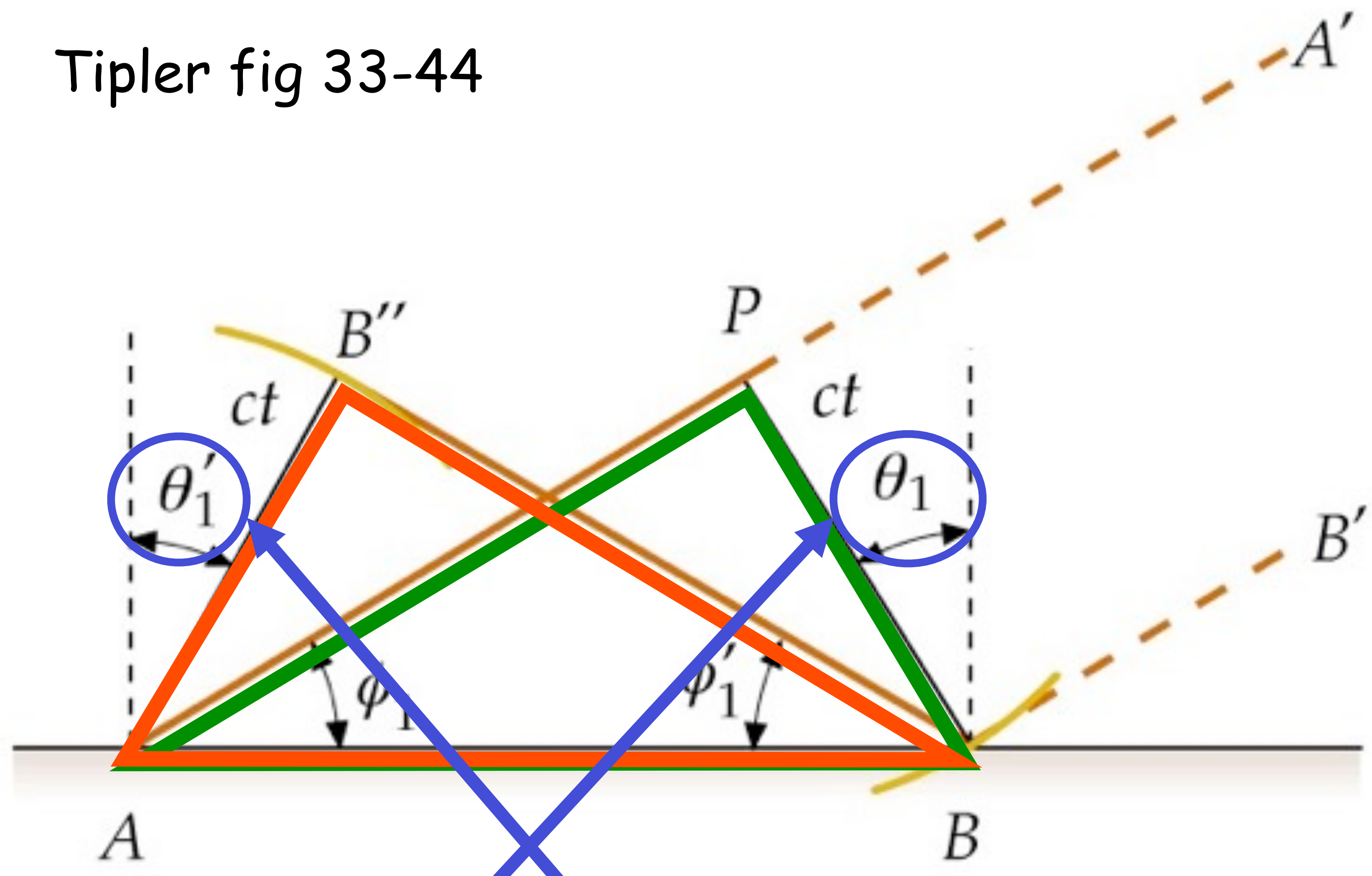
AP is part of AA′

in time t P moves to B

and the wavelet from A reaches B″

BB″ makes an angle ϕ_1' with the mirror, $\phi_1' = \theta_1'$

Tipler fig 33-44



Now have 2 right angle triangles ABP and BAB″

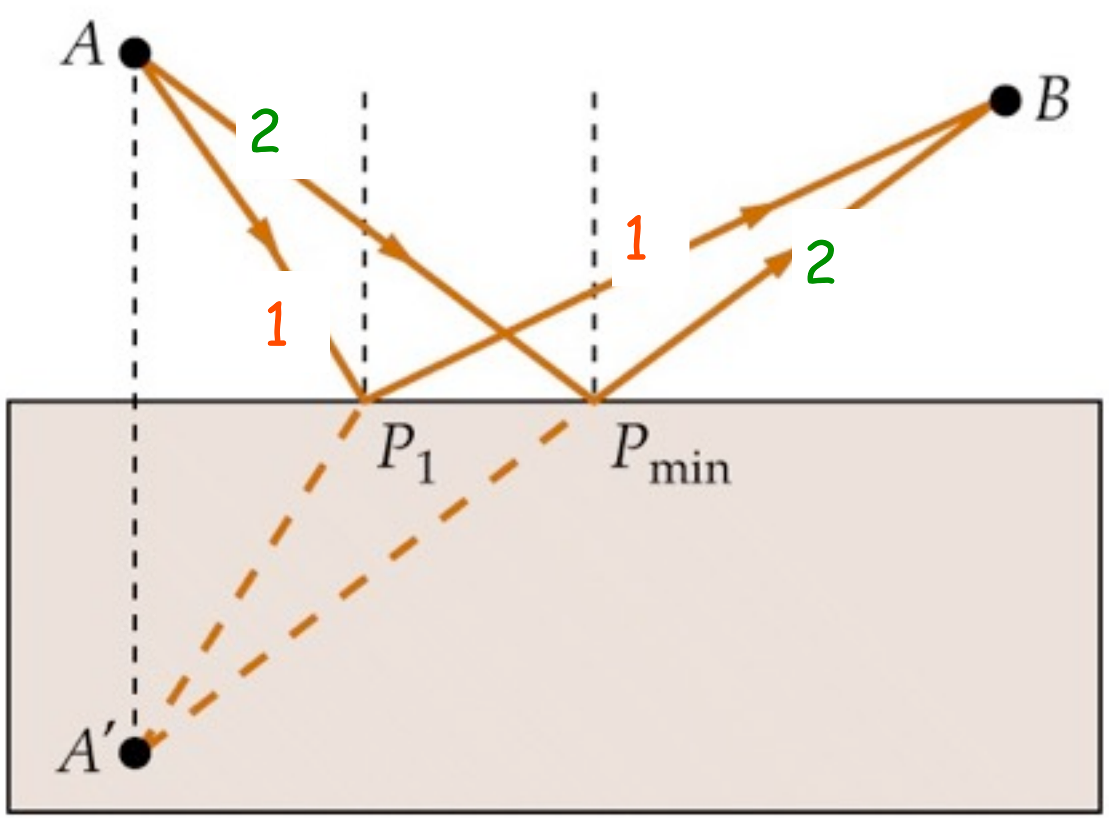
AB is common   AB″ = BP = ct   triangles are congruent

$\phi_1 = \phi_1'$ ie angle of incidence θ_1 = angle of reflection θ_1'

Derivation of law of reflection (Fermat)

Light can travel from A to B (via mirror) on path 1 or path 2.

If we want to apply Fermat's principle we need to know at which point P the wave must strike the mirror so that APB takes the least time.



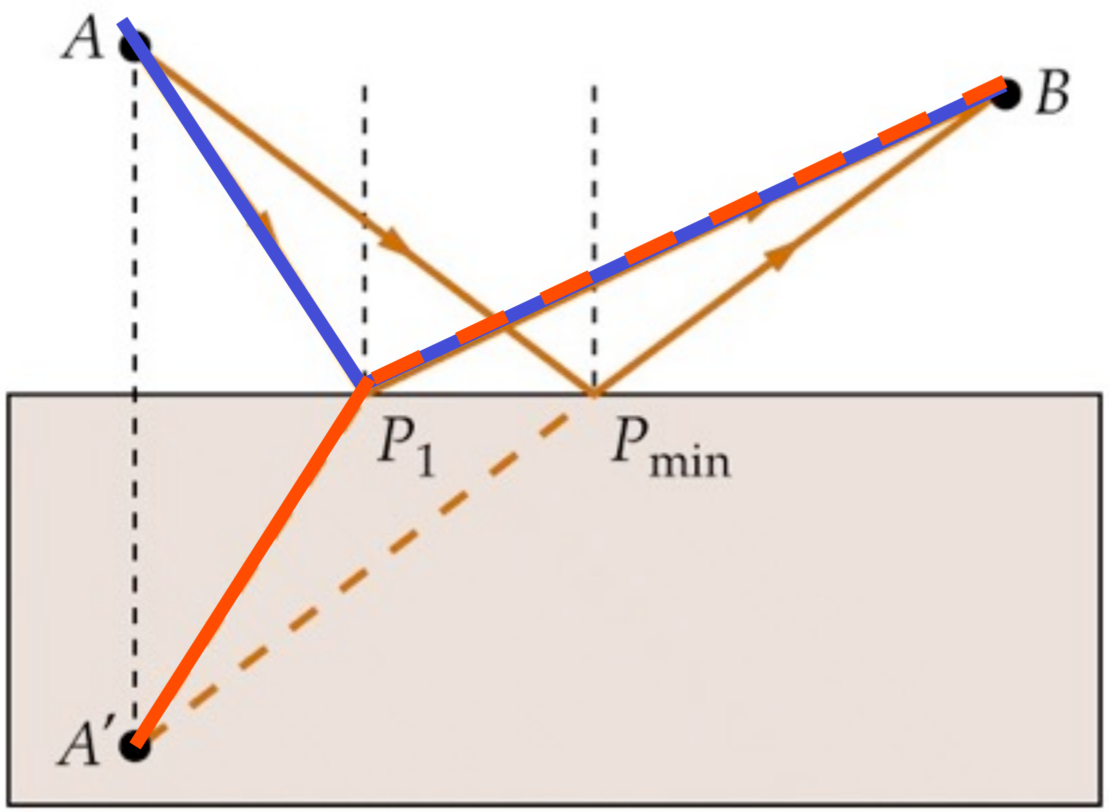
Tipler figure 33-46

As light is travelling in the same medium at all times the shortest time will also be the shortest distance.

Derivation of law of reflection (Fermat)

Where ever P is located the distance APB = A'PB where A' is the position of the image of the light source.

As the position of P varies the shortest distance will be when $A(P=P_{min})B$ lie in a straight line.

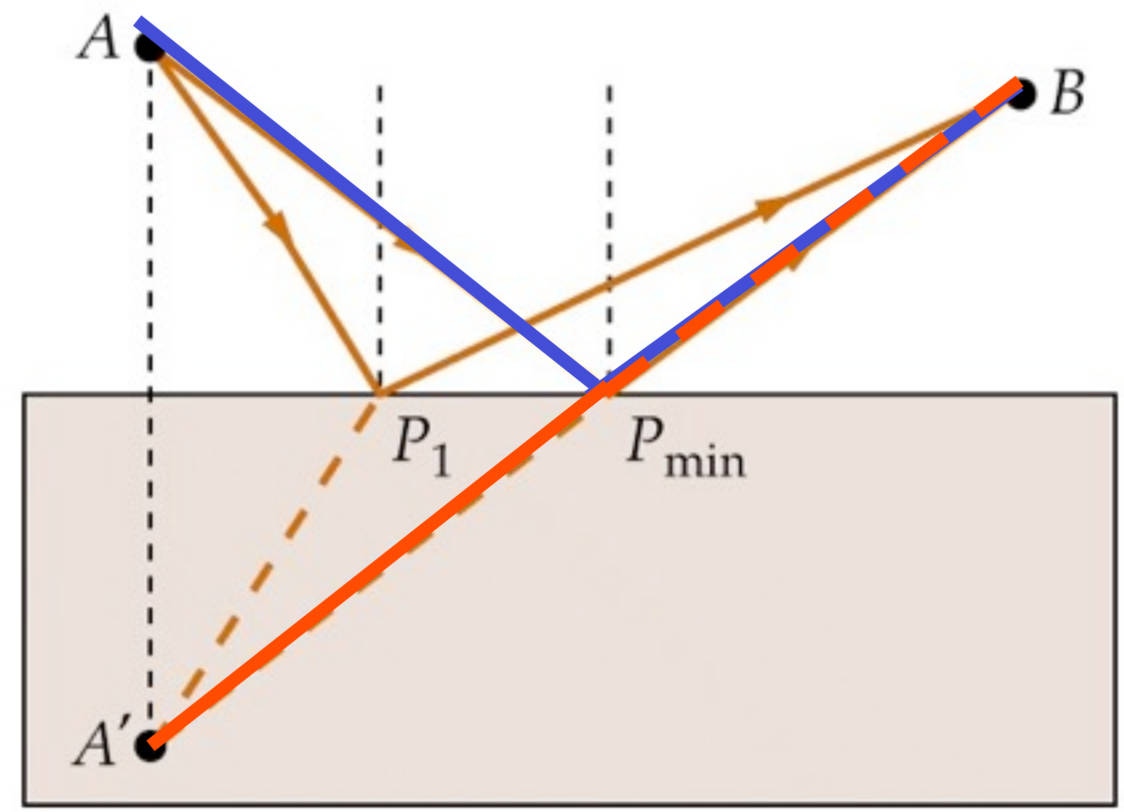


Tipler figure 33-46

Derivation of law of reflection (Fermat)

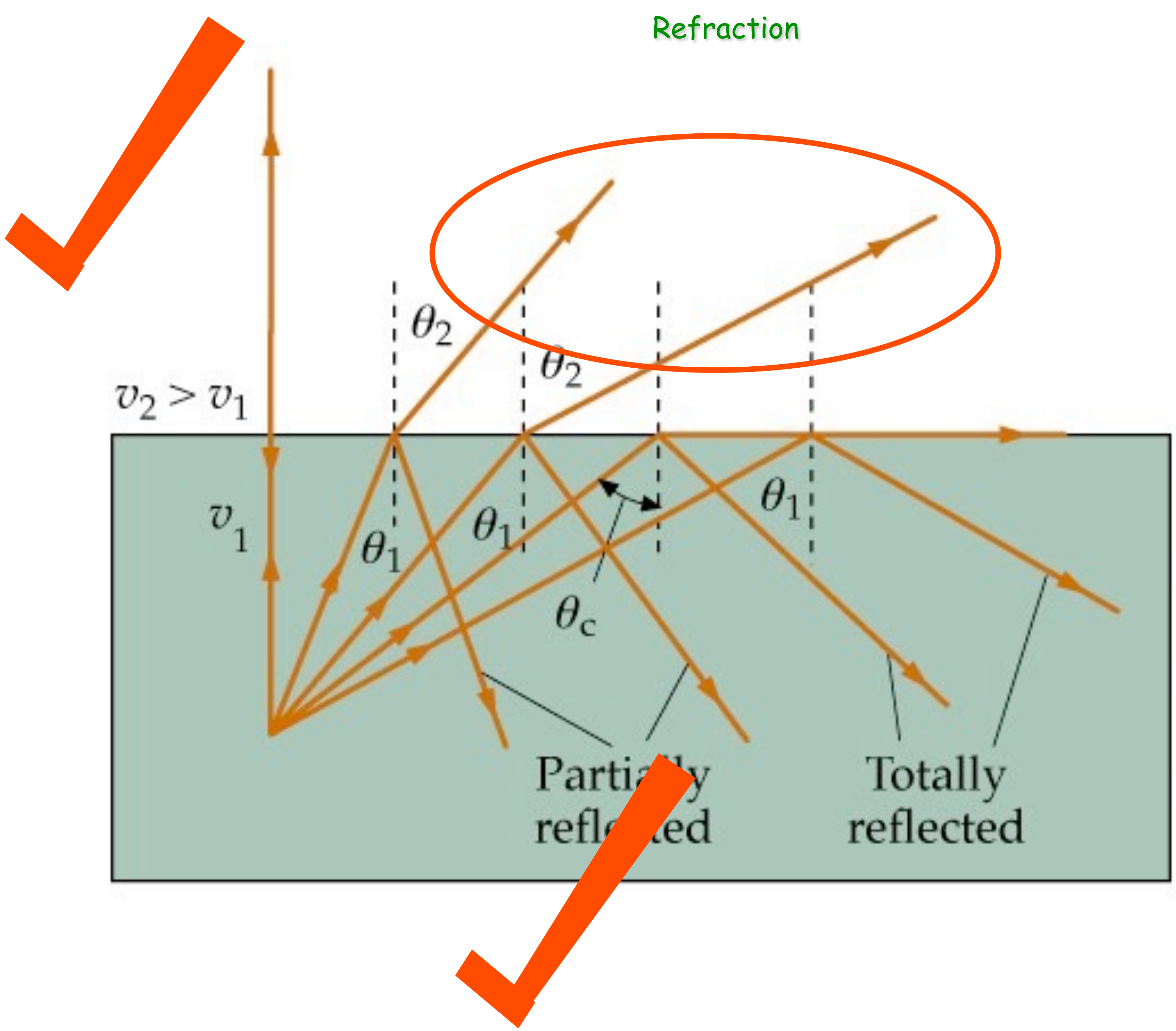
Where ever P is located the distance APB = A'PB where A' is the position of the image of the light source.

As the position of P varies the shortest distance will be when $A'(P=P_{min})B$ lie in a straight line.



Tipler figure 33-46

This will be when the angle of incidence = angle of reflection


Refraction
θ_2
θ_2
$v_2 > v_1$
v_1
θ_1
θ_1
θ_1
θ_c
Partially reflected
Totally reflected

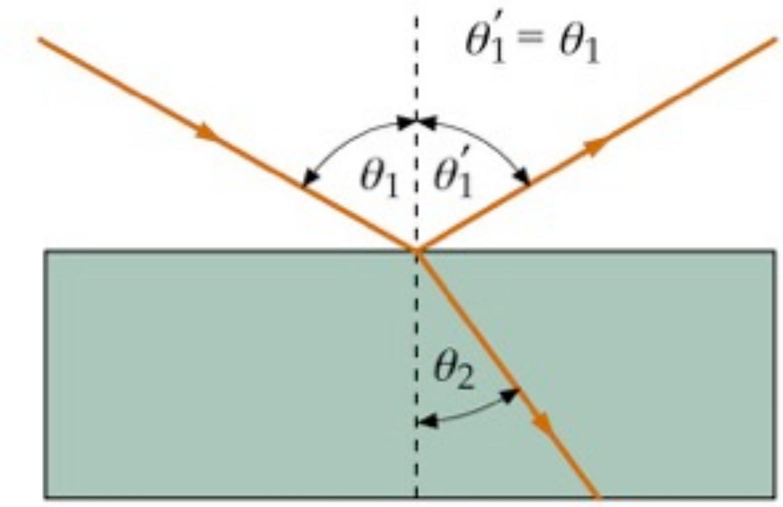
Refraction of light

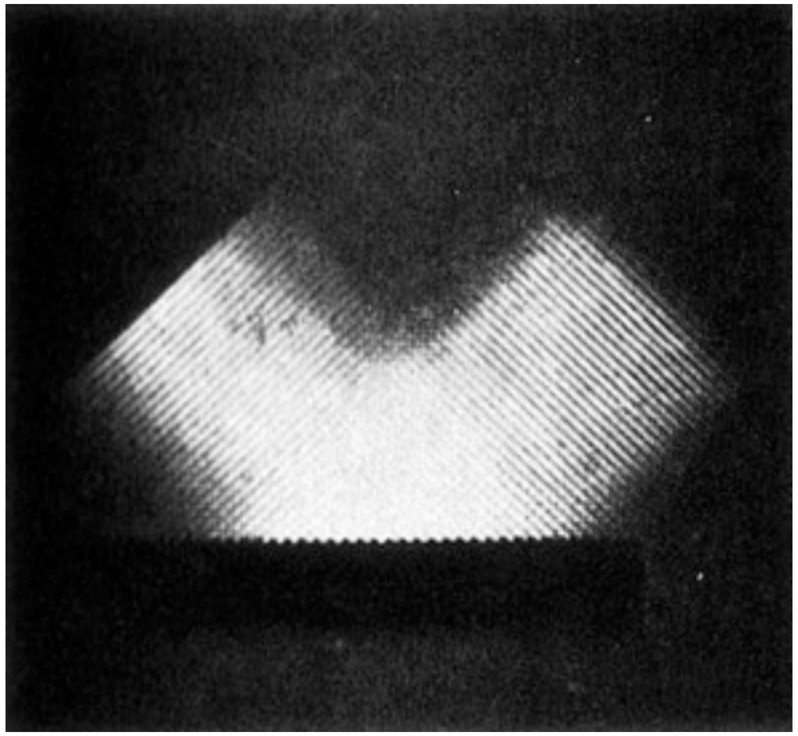
Snell's law of refraction :

if v is the speed of the light in the medium

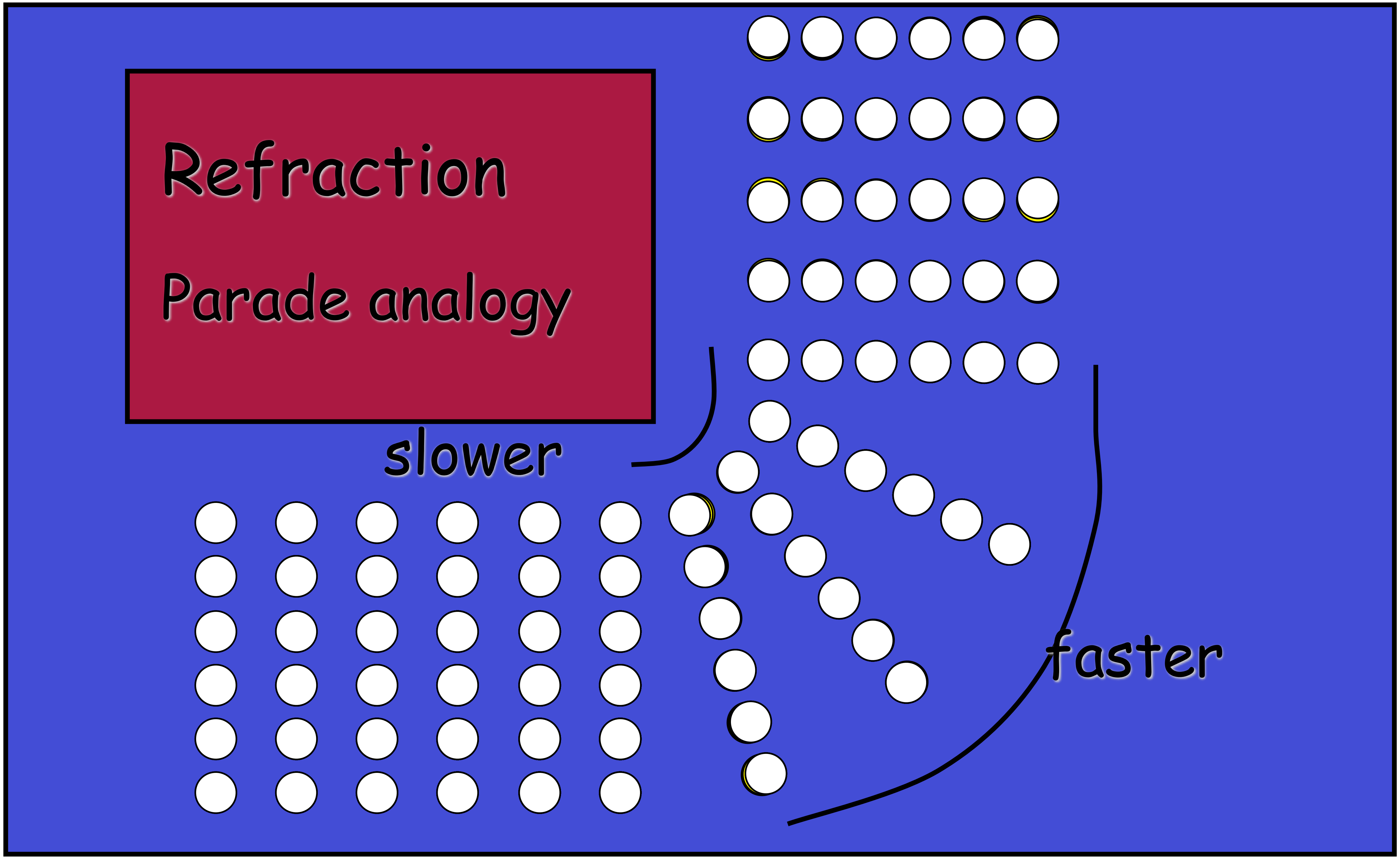
$$\frac{1}{v_1}\sin\theta_1 = \frac{1}{v_2}\sin\theta_2$$

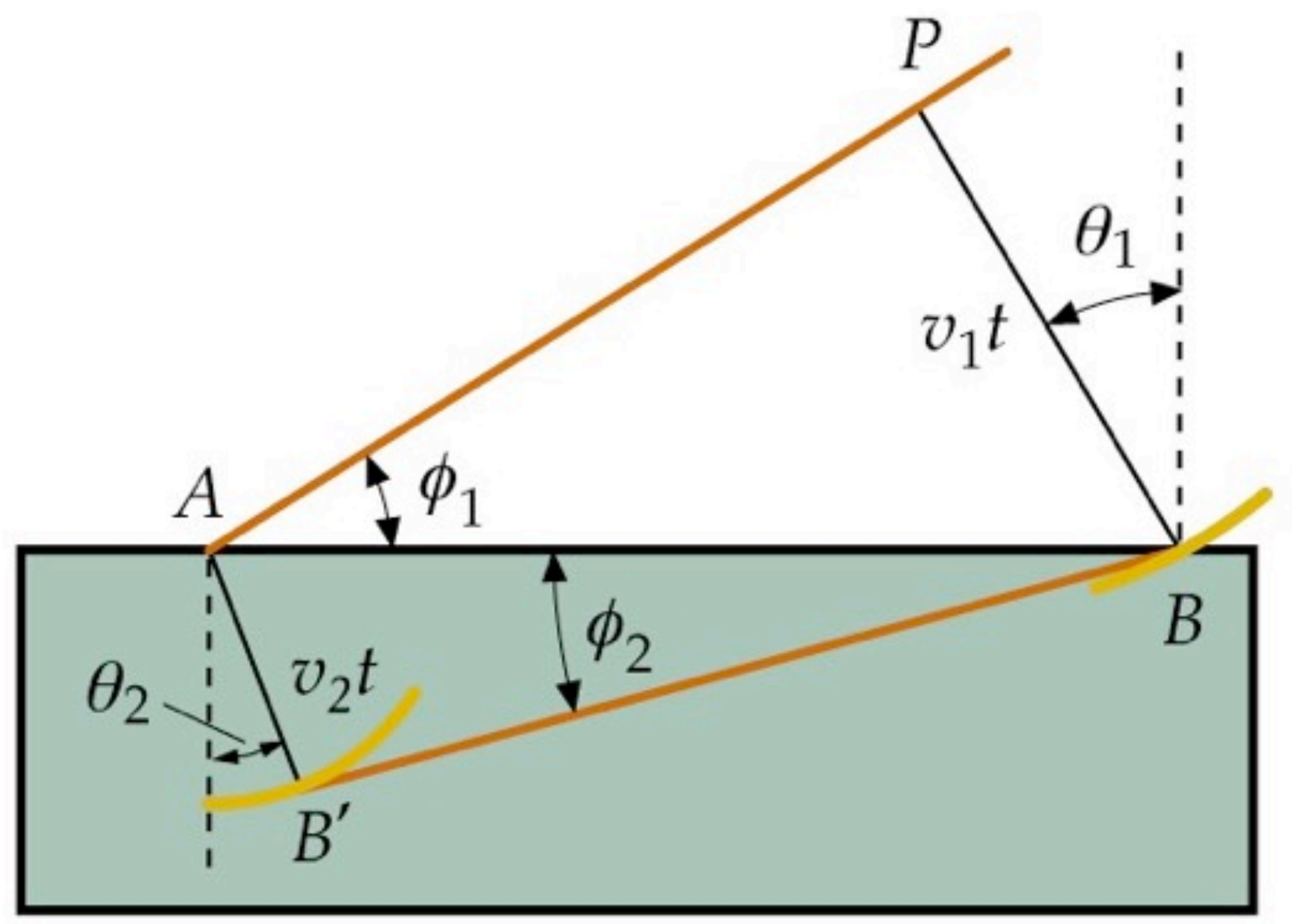
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$





Visualization of refraction





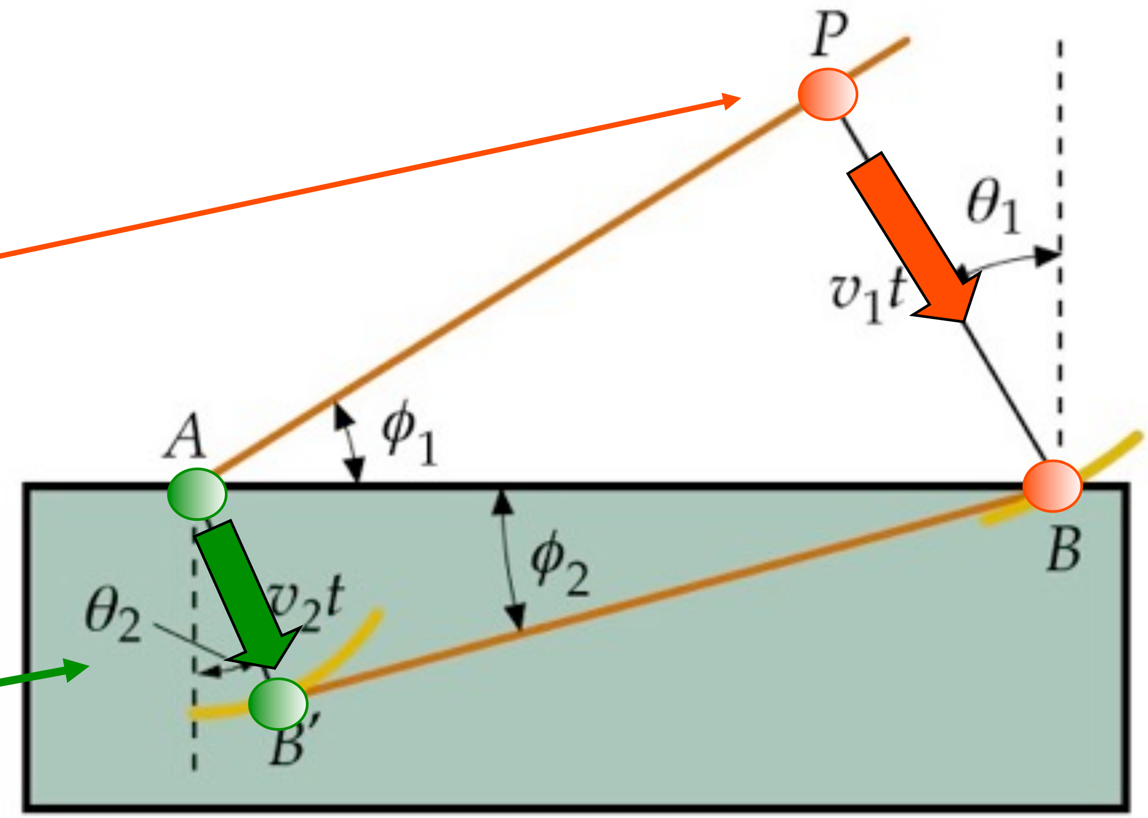
Consider a plane wave incident on a glass interface.

AP represents a portion of the incident wave - we can use Huygens' construction to calculate the transmitted wave.

AP hits the glass surface at an angle ϕ_1.

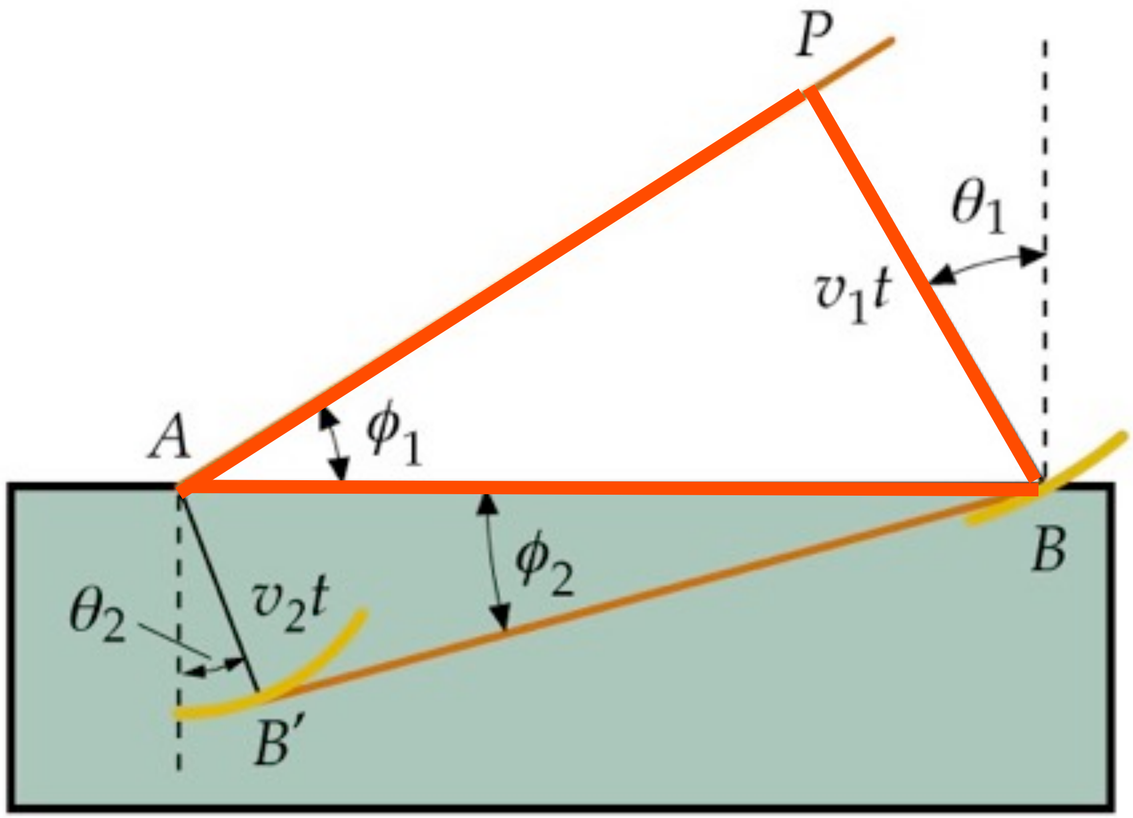
In a time t a wavelet from P travels v_1t to point B

In the same time a wavelet from A travels v_2t to B′



Derivation of law of refraction (Huygens)

AP hits the glass surface at an angle ϕ_1.



In a time t a wavelet from P travels $v_1 t$ to point B

In the same time a wavelet from A travels $v_2 t$ to B′

BB′ is not parallel to AP because $v_1 \neq v_2$

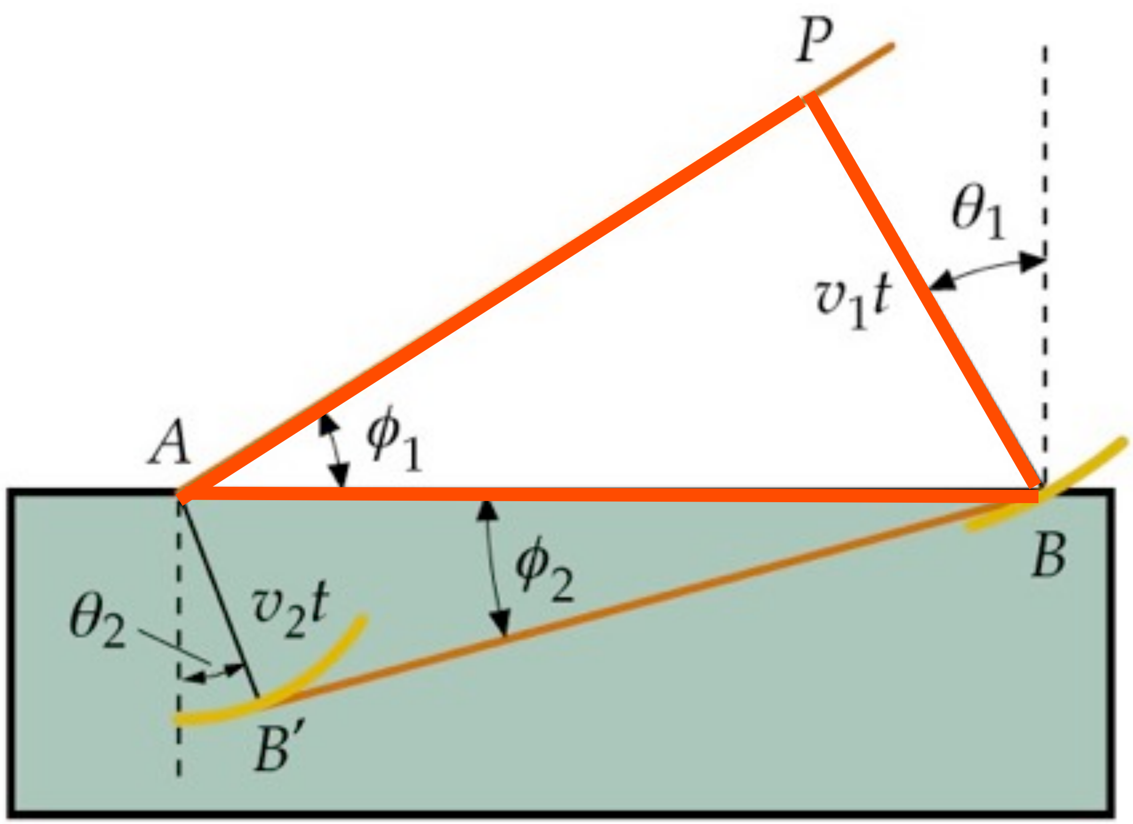
In ΔAPB $\quad \sin\phi_1 = \dfrac{v_1 t}{AB} \quad$ or $\quad AB = \dfrac{v_1 t}{\sin\phi_1}$

Derivation of law of refraction (Huygens)

$$AB = \frac{v_1 t}{\sin\phi_1}$$

but $\quad \phi_1 = \theta_1$

$$\therefore \quad AB = \frac{v_1 t}{\sin\theta_1}$$



Derivation of law of refraction (Huygens)

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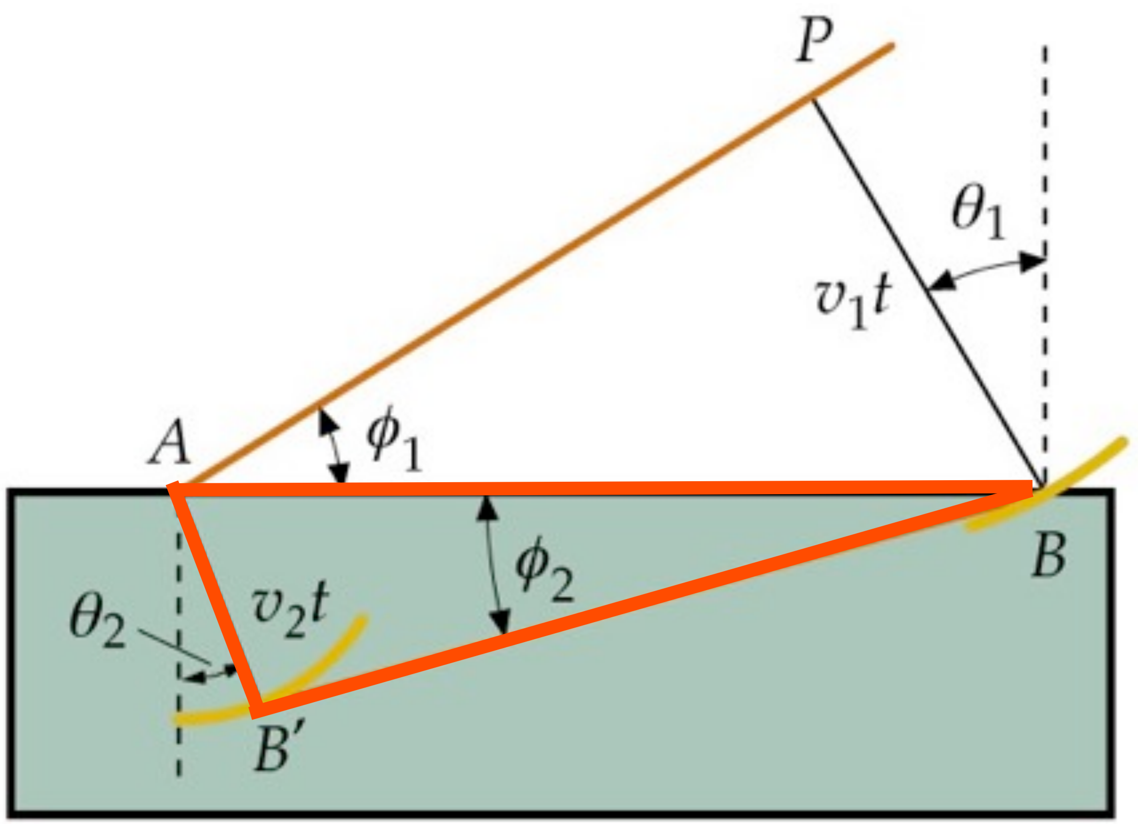


Similarly in $\Delta ABB'$

$$\sin\phi_2 = \frac{v_2 t}{AB}$$

or $$AB = \frac{v_2 t}{\sin\theta_2}$$

Derivation of law of refraction (Huygens)

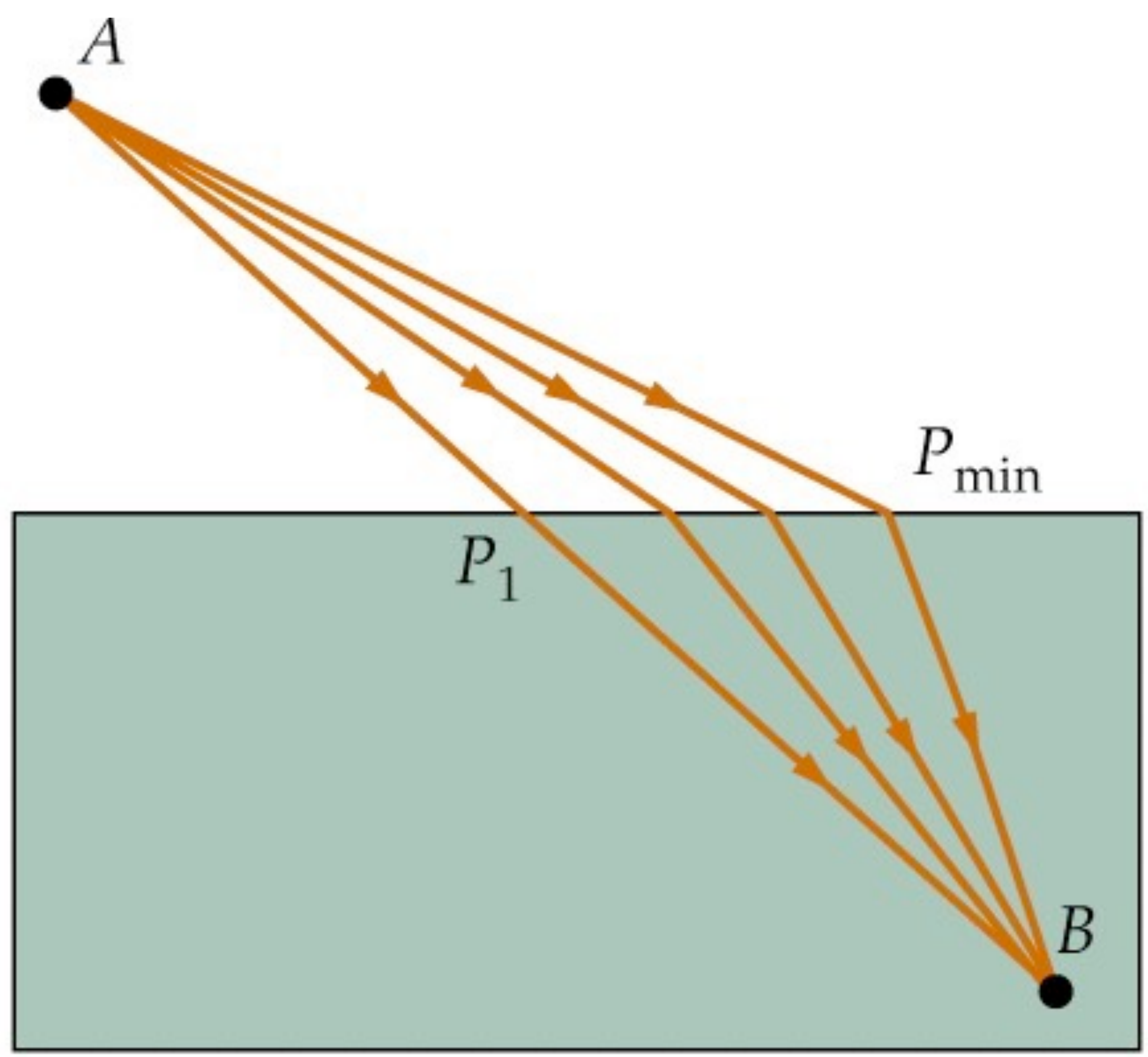
$$\frac{v_1 t}{\sin\theta_1} = \frac{v_2 t}{\sin\theta_2}$$

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

But $v_1 = c / n_1$ and $v_2 = c / n_2$

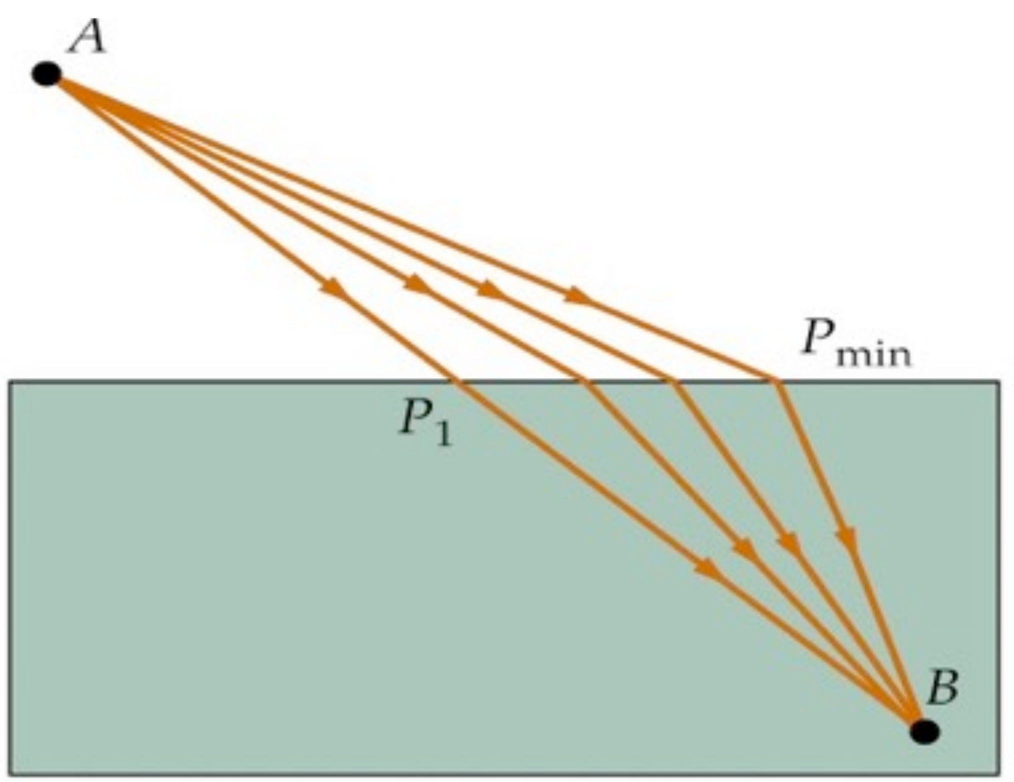
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Derivation of law of refraction (Fermat)



Can also use Fermat's principle to derive Snell's Law (more complicated but important)

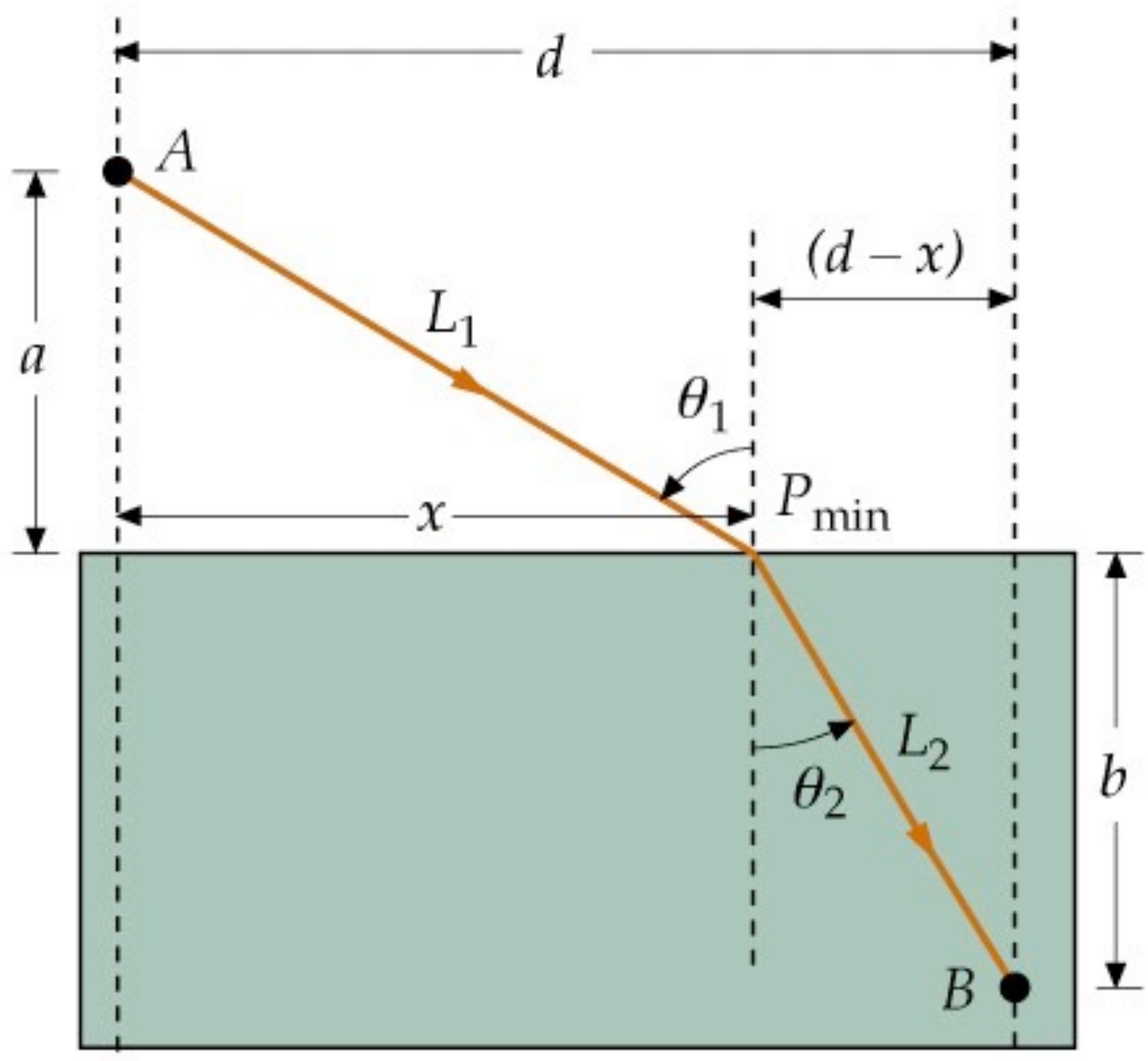
Several possible paths from A (in air) to B (in glass)



Remember that light travels more slowly in glass than in air so A-P_1-B (straight line) will not have the shortest travel time.

If we move to the right the path in the glass is shorter, but the overall path is longer - how do we choose the shortest route ?

Geometry for finding the path of least time



Geometry for finding the path of least time

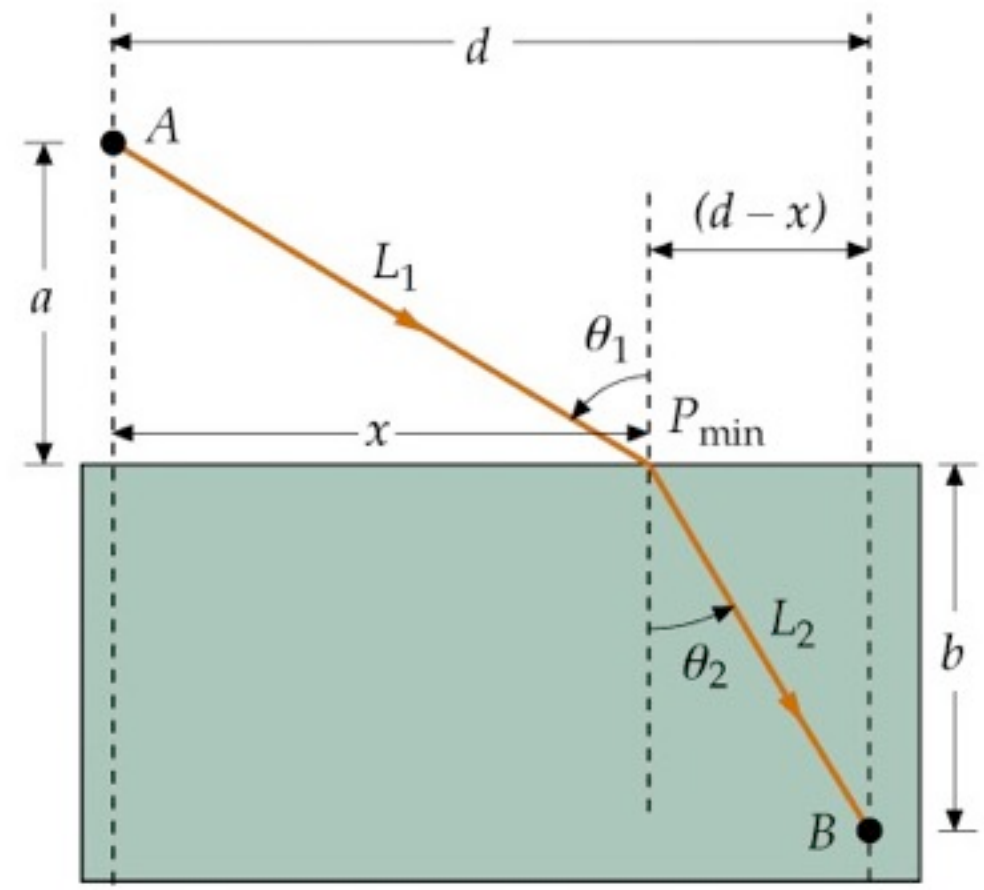
t = time taken to travel A-B

$$t = \frac{L_1}{v_1} + \frac{L_2}{v_2}$$

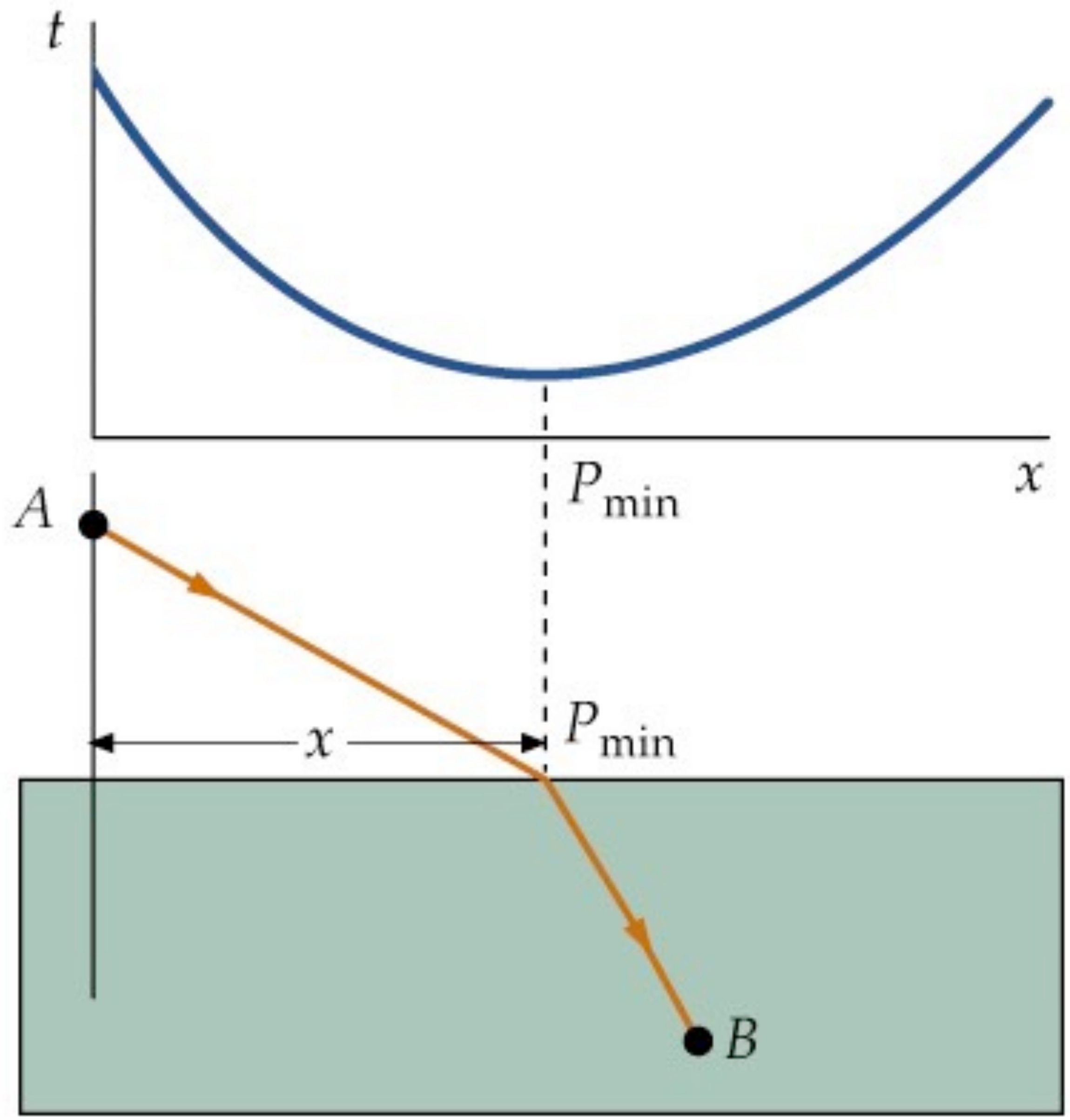
$$= \frac{L_1}{c/n_1} + \frac{L_2}{c/n_2}$$

$$= \frac{n_1 L_1}{c} + \frac{n_2 L_2}{c}$$

$$L_1^2 = a^2 + x^2 \quad \text{and} \quad L_2^2 = b^2 + (d-x)^2$$



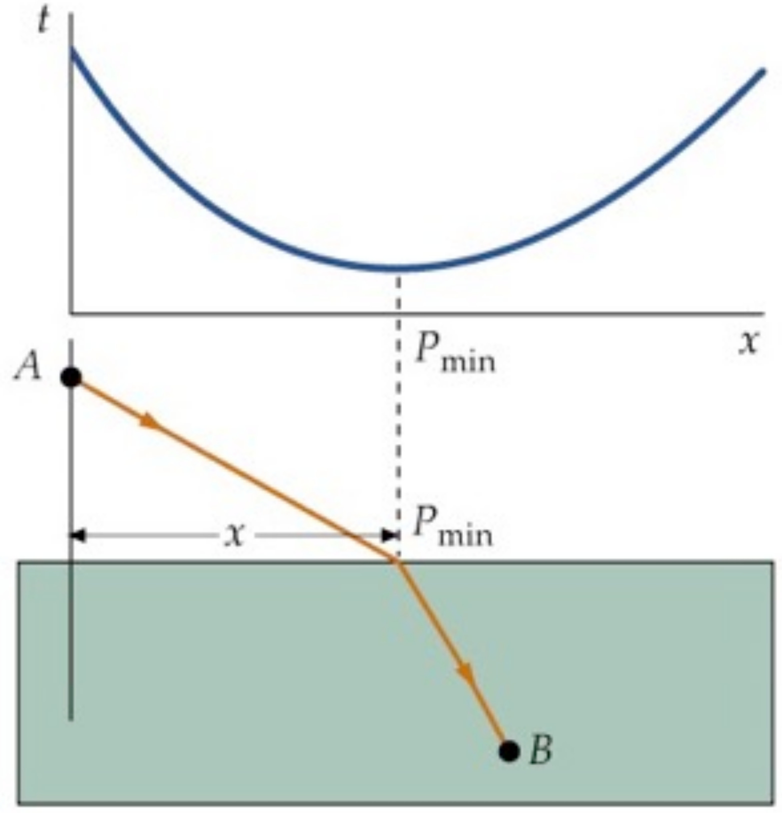
We can combine these three equations and plot the time as a function of x


t
x
P_{min}
A
x
P_{min}
B

Find the minimum time taken by finding when

$$\frac{dt}{dx} = 0$$

$$\frac{dt}{dx} = \frac{1}{c}\left(\frac{n_1\, dL_1}{dx} + \frac{n_2\, dL_2}{dx}\right) = 0$$



$$\frac{dL_1}{dx} = \frac{1}{2}(a^2 + x^2)^{-1/2} \times 2x$$

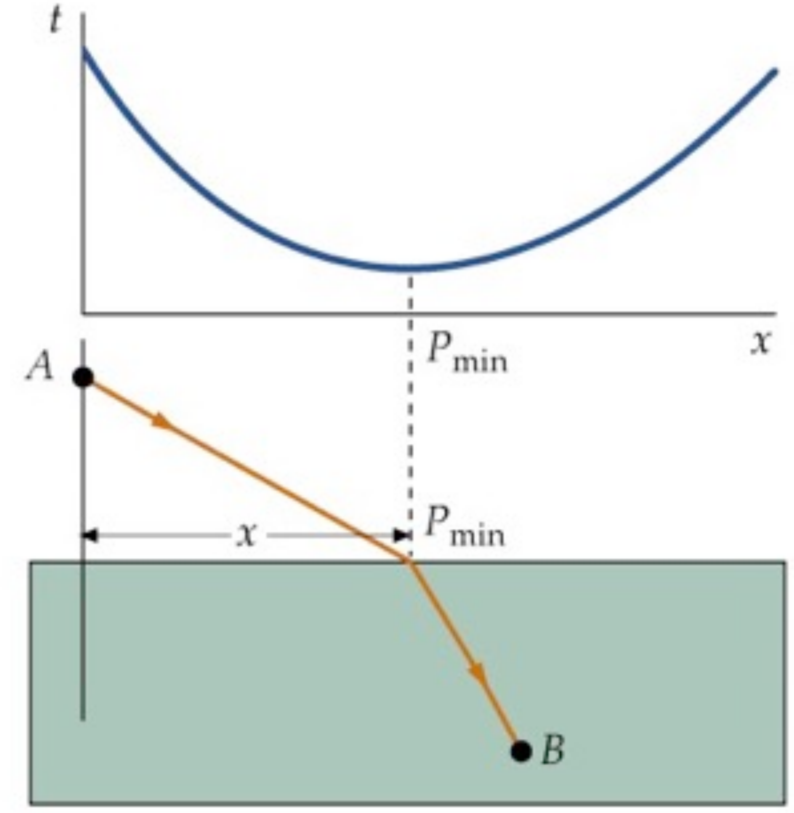
$$= \frac{x}{L_1}$$

but $\frac{x}{L_1} = \sin\theta_1$

Similarly for L_2

$$\frac{dL_2}{dx} = \frac{1}{2}(b^2 + (d-x)^2)^{-1/2} \times -2(d-x)$$

$$= -\frac{(d-x)}{L_2}$$



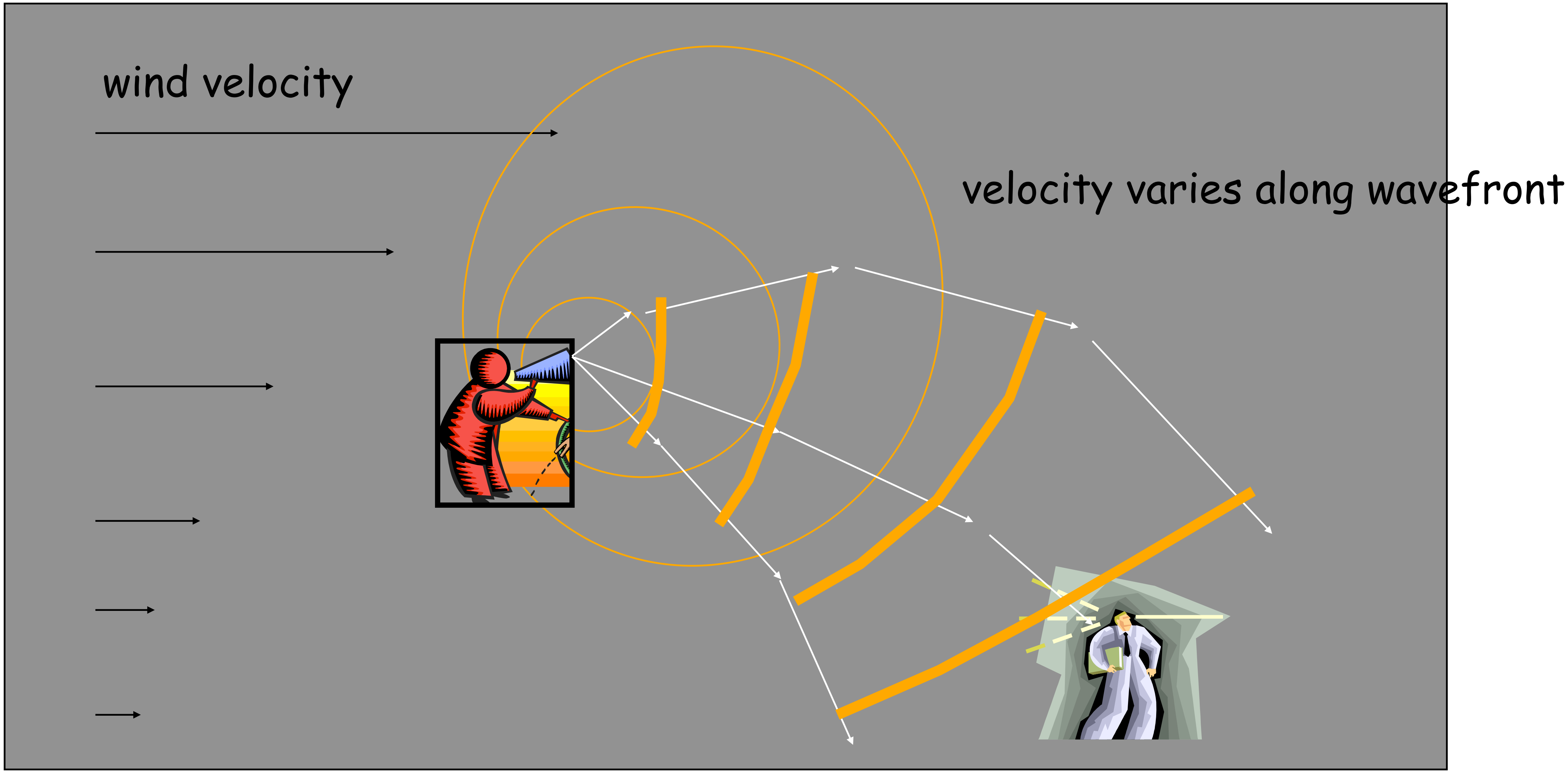
but $\quad -\frac{d-x}{L_2} = -\sin\theta_2$

we want $\quad \frac{dt}{dx} = 0$

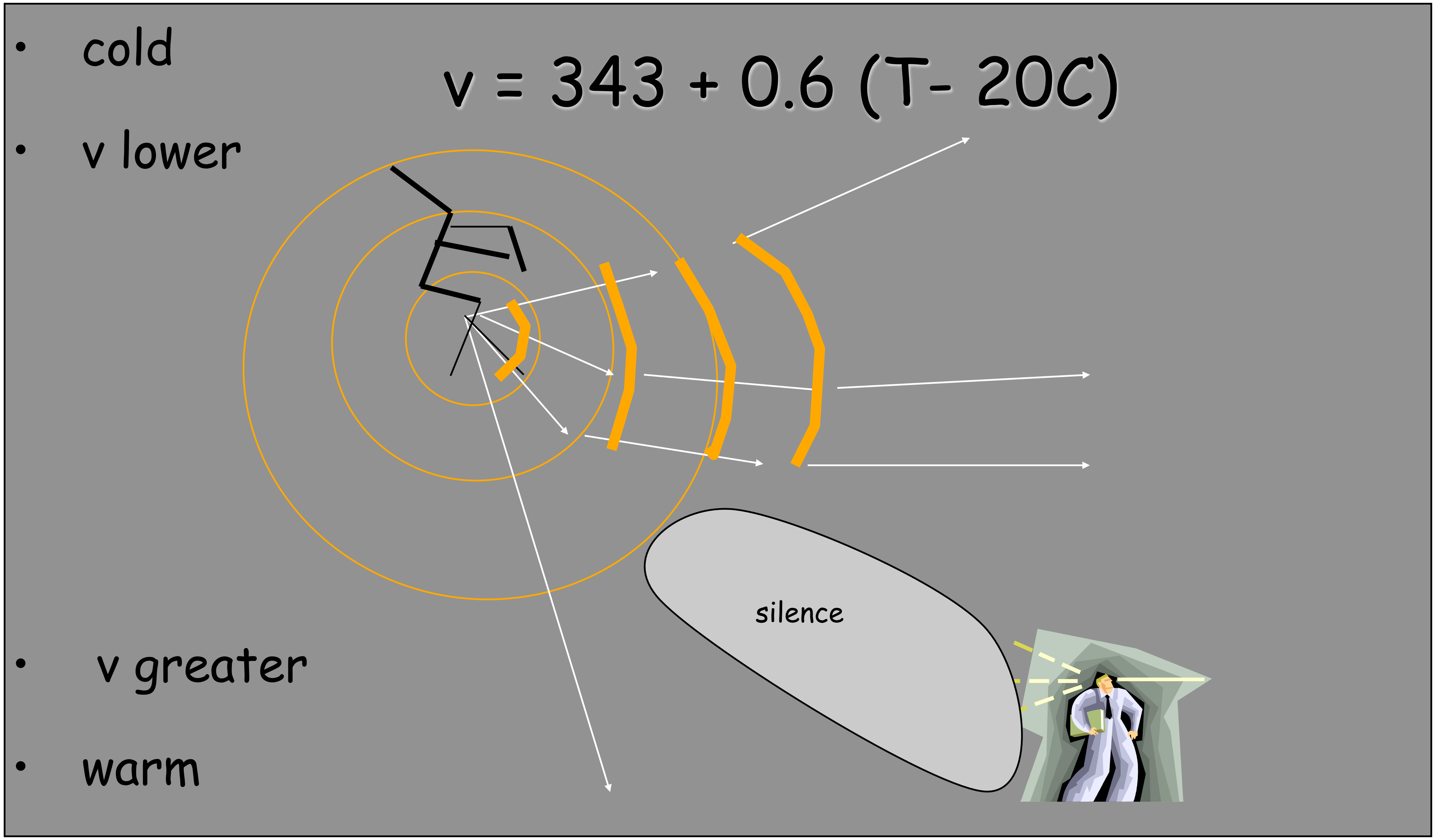
$$n_1 \sin\theta_1 + n_2(-\sin\theta_2) = 0$$

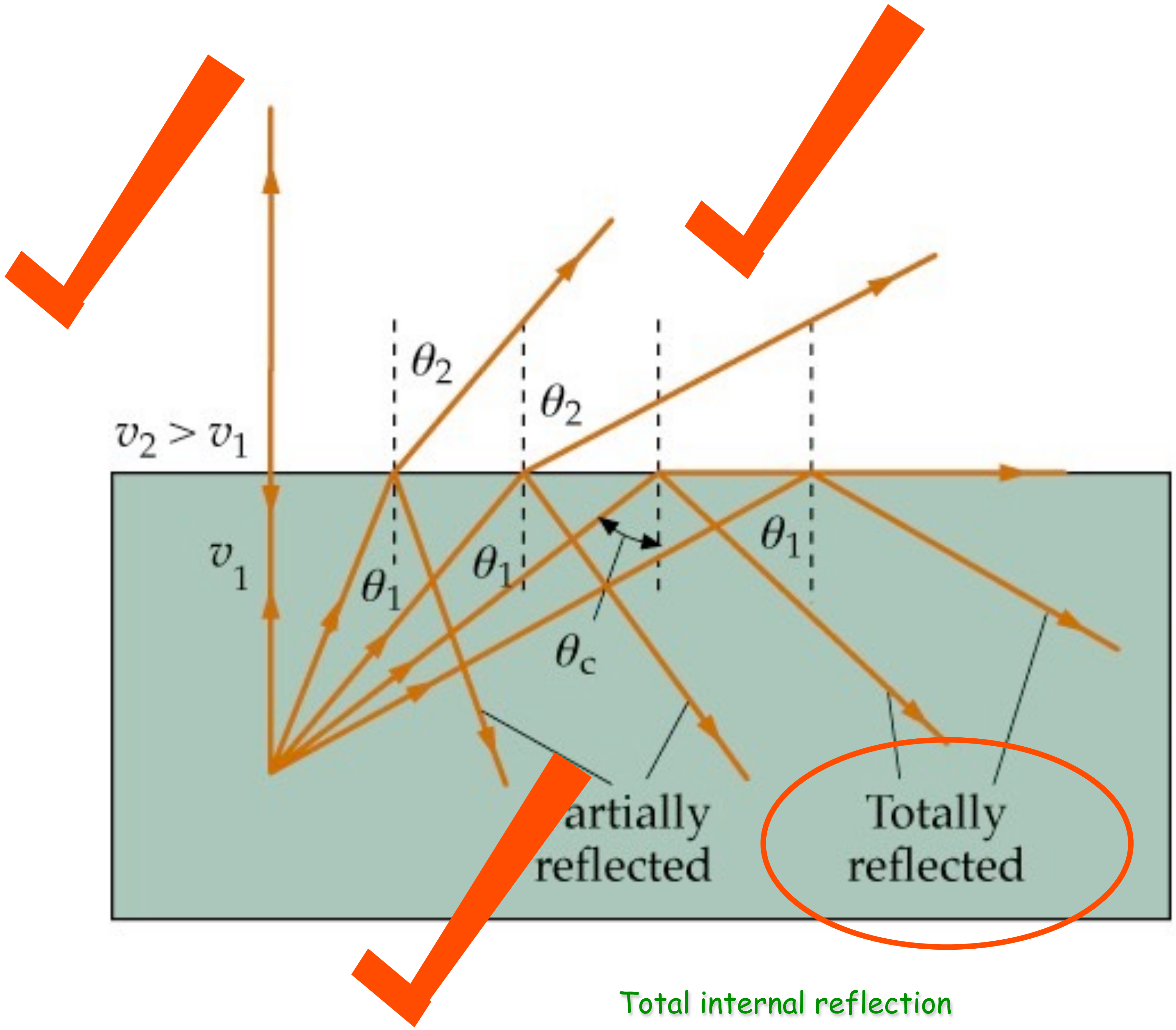
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Example



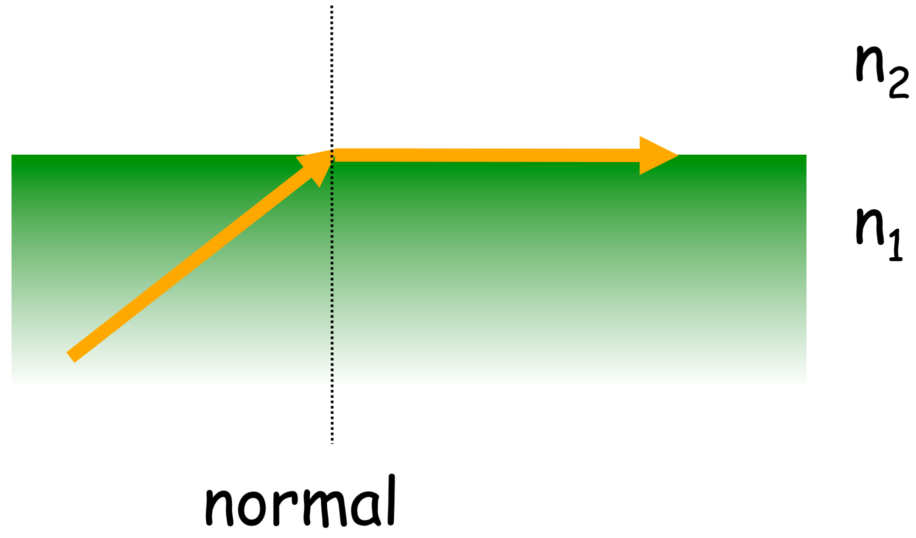
Example





Total internal reflection

Total internal reflection occurs when light attempts to move from a medium of refractive index n_1 to a medium of refractive index n_2 and $n_1 > n_2$



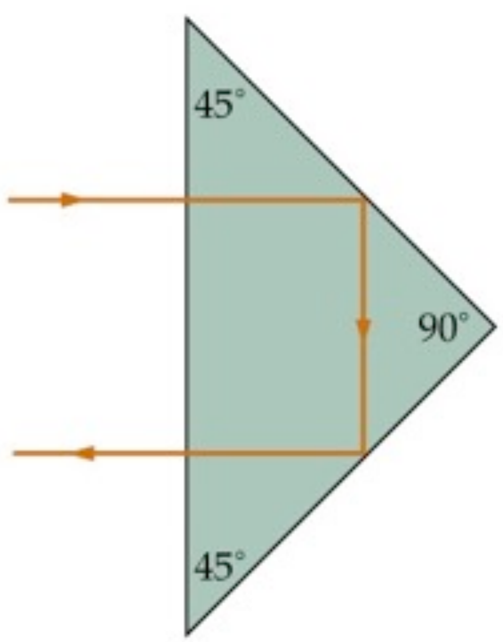
At a particular angle of incidence (the critical angle θ_c) the refracted ray will travel along the boundary ie $\theta_2 = 90^o$

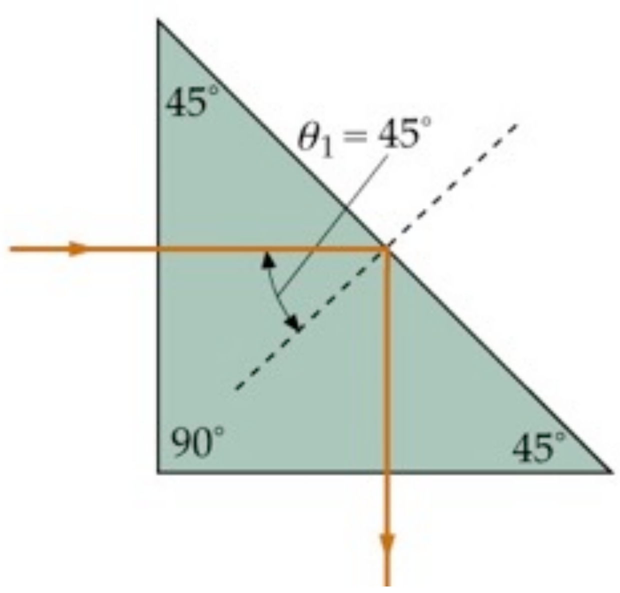
If the angle of incidence > θ_c the light is entirely reflected

From Snell's Law $\qquad n_1 \sin\theta_c = n_2 \sin 90 = n_2$

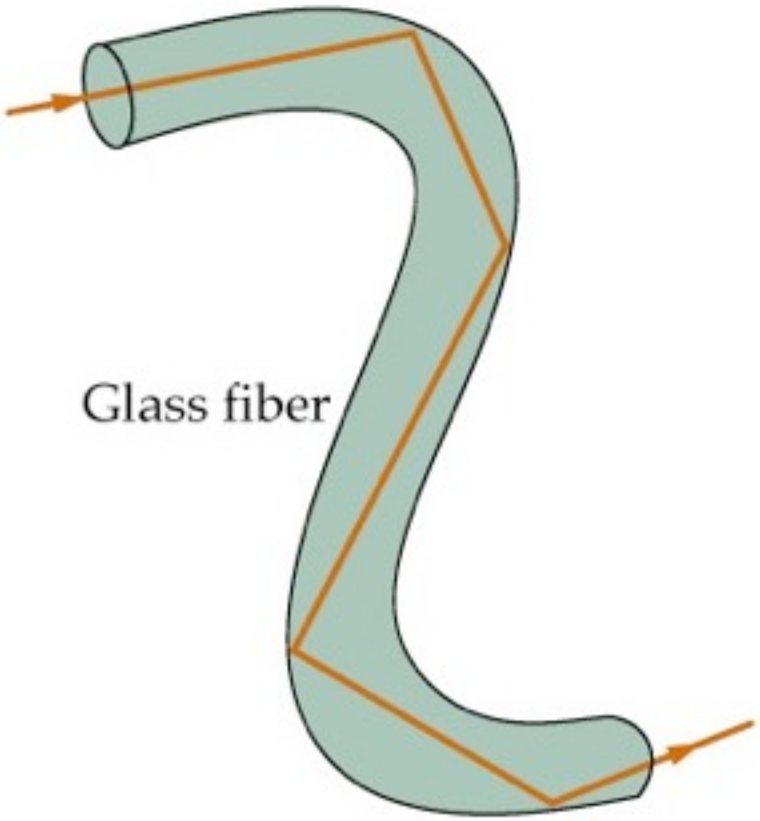
$$\sin\theta_c = \frac{n_2}{n_1} \qquad \text{for } n_1 > n_2$$

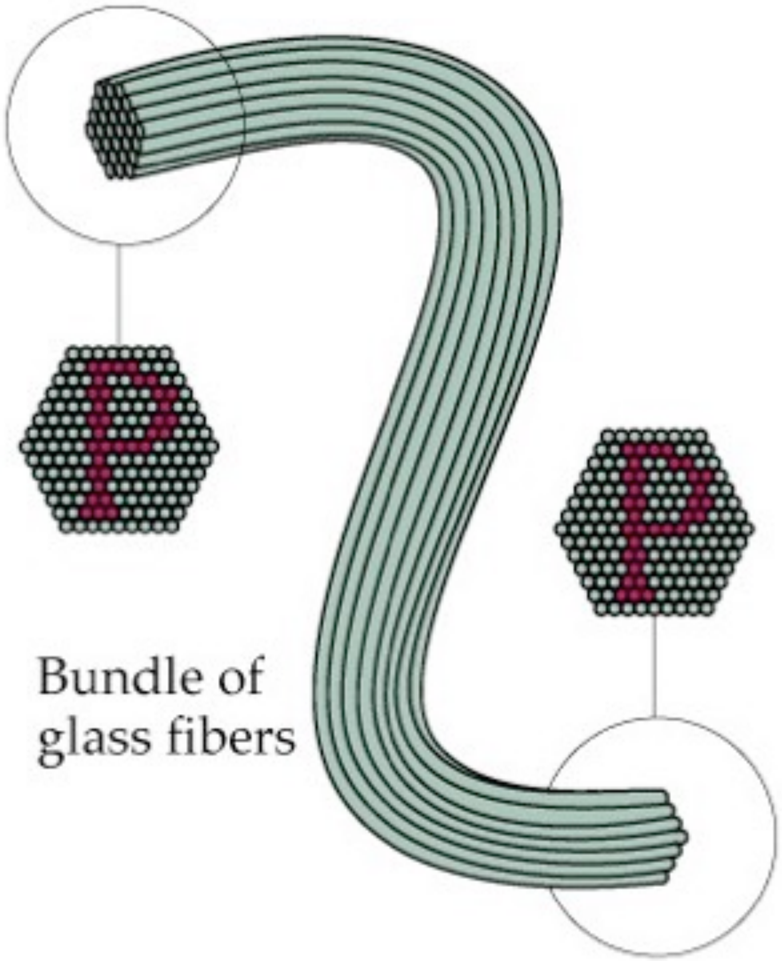
Internal reflection in a prism

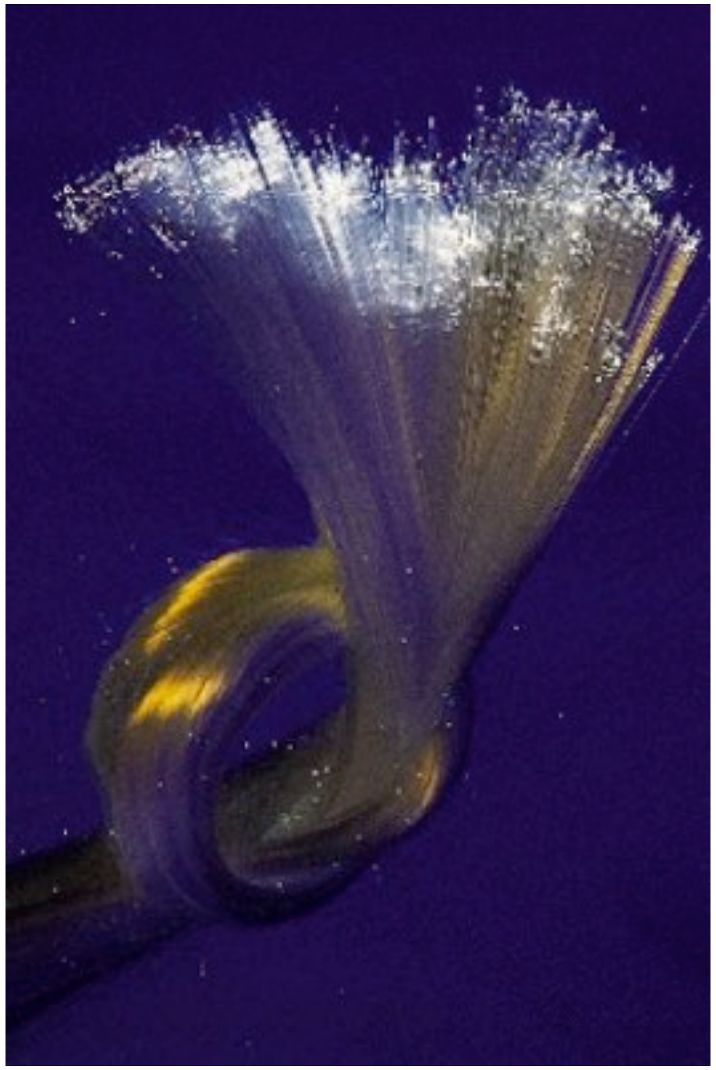




Optical Fibres







Diffraction of sound waves

The phenomenon of sound waves diffraction is part of everyday experience: standing in a room we can hear someone speaking in the nearby corredor.

This happens because the **wavelength of the human voice is comparable with the dimensions of the obstacles encountered along the path.**

In the points where the wave meets the obstacle, spherical waves are generated and they propagate in all directions, allowing the sound to reach points beyond the obstacle itself.

Visualization

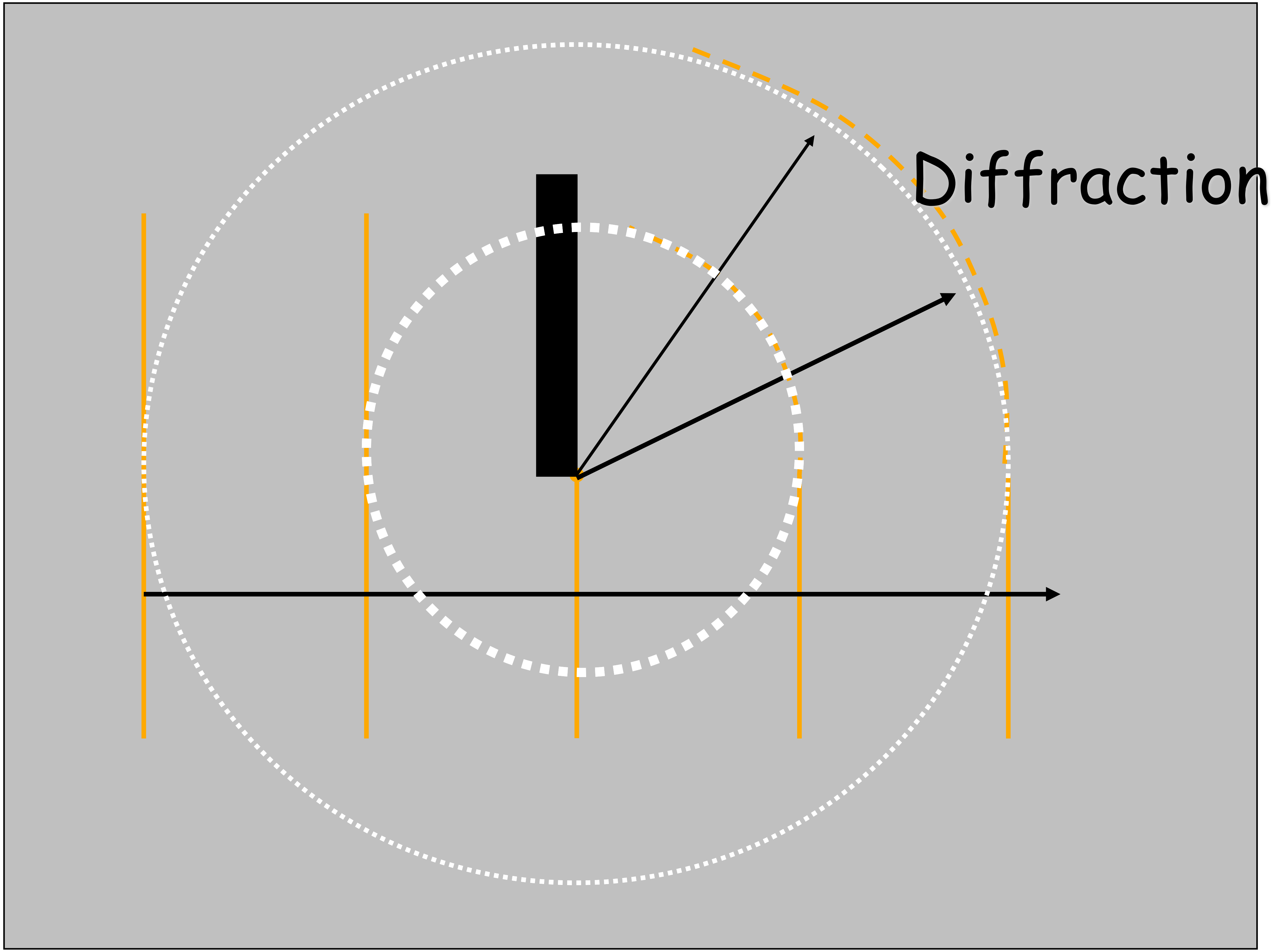
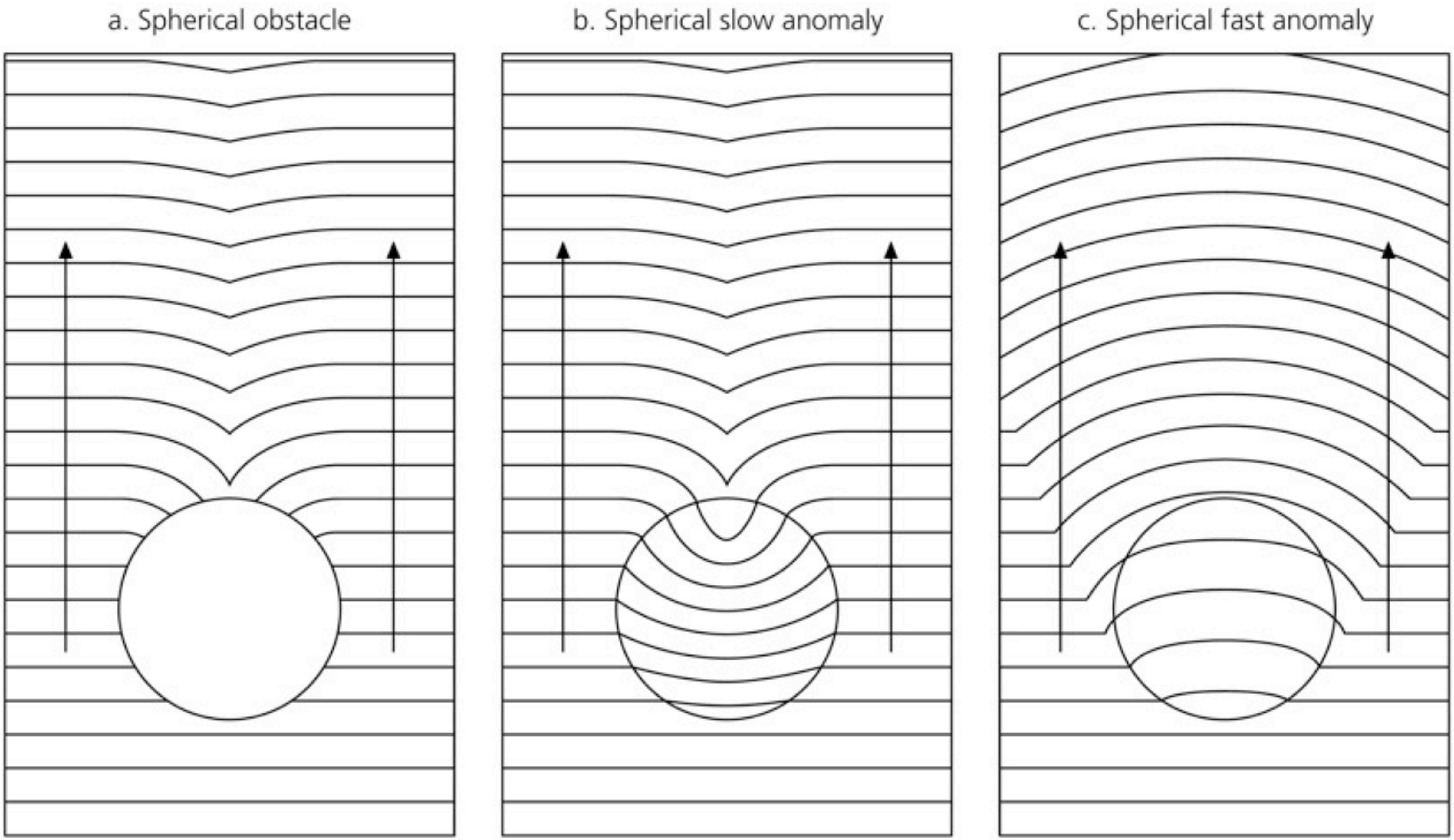


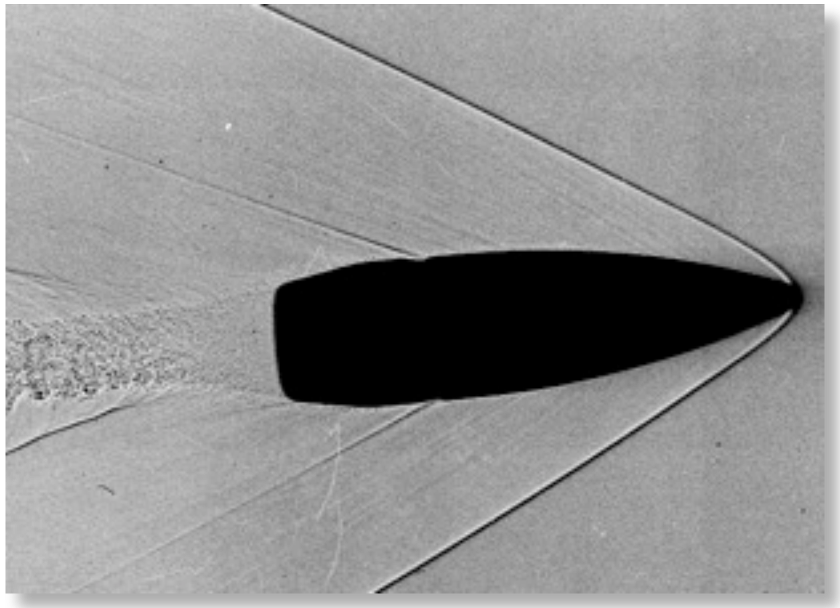
Figure 2.5-19: Waves interacting with a spherical anomaly.



Doppler effect

Shock waves and

Sonic booms



The Doppler effect

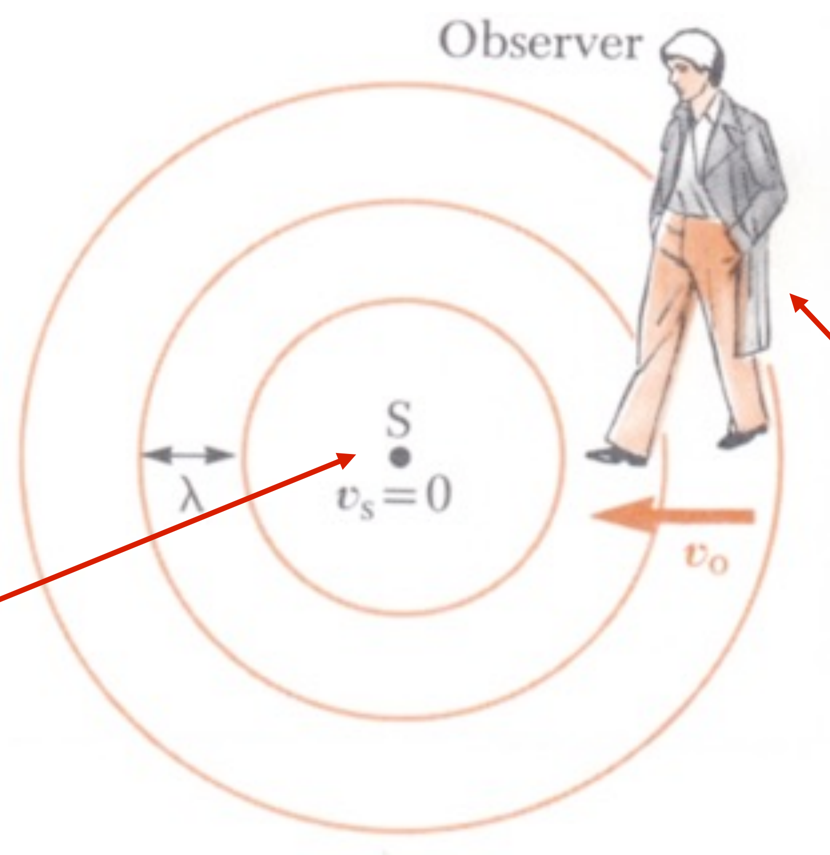
The Doppler effect is experienced whenever there is a relative motion between source and observer.

When the source and observer are moving towards each other the frequency heard by the observer is higher than the frequency of the source.

When the source and observer move away from each other the observer hears a frequency which is lower than the source frequency.

Although the Doppler effect is most commonly experienced with sound waves it is a phenomenon common to all harmonic waves.

Stationary source & observer moving to source



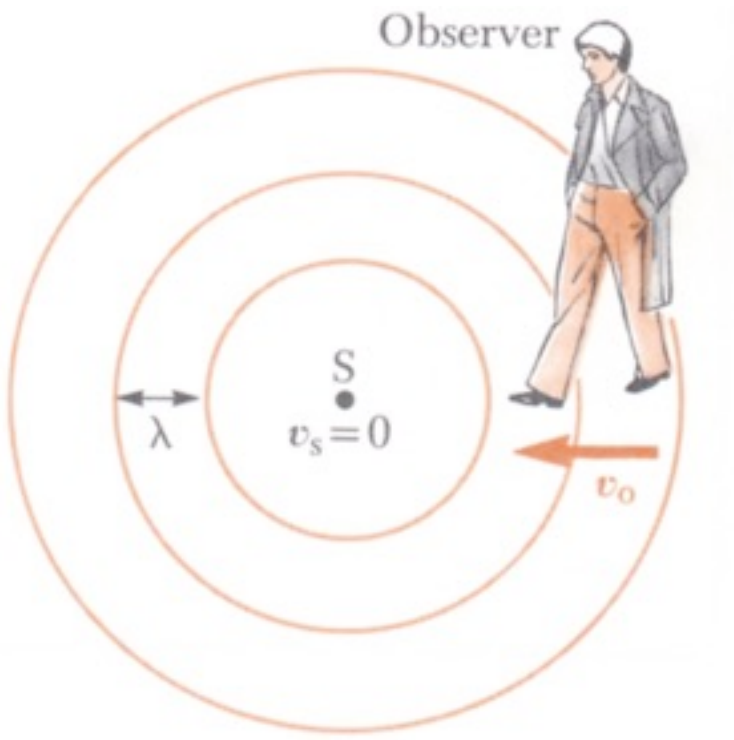
Observer moving **TOWARDS** S at speed v_o

Point source S at rest ie $v_s = 0$

We are dealing with relative speeds ∴ "at rest" = at rest with respect to the air. We assume the air is stationary.

frequency of source = f frequency observed = f′
velocity of sound = v wavelength of sound = λ

Stationary source & observer moving to source



If observer O was stationary he would detect f wavefronts per second

ie: if $v_o = 0$ and $v_s = 0$ $f' = f$

When O ⟶ S O moves a distance $v_o t$ in t seconds

During this time O detects an additional $\frac{v_o t}{\lambda}$ wavefronts

ie: an additional $\frac{v_o}{\lambda}$ wavefronts / second

As more wavefronts are heard per second the frequency heard by the observer is increased.

$$f' = f + \Delta f = f + \frac{v_o}{\lambda}$$

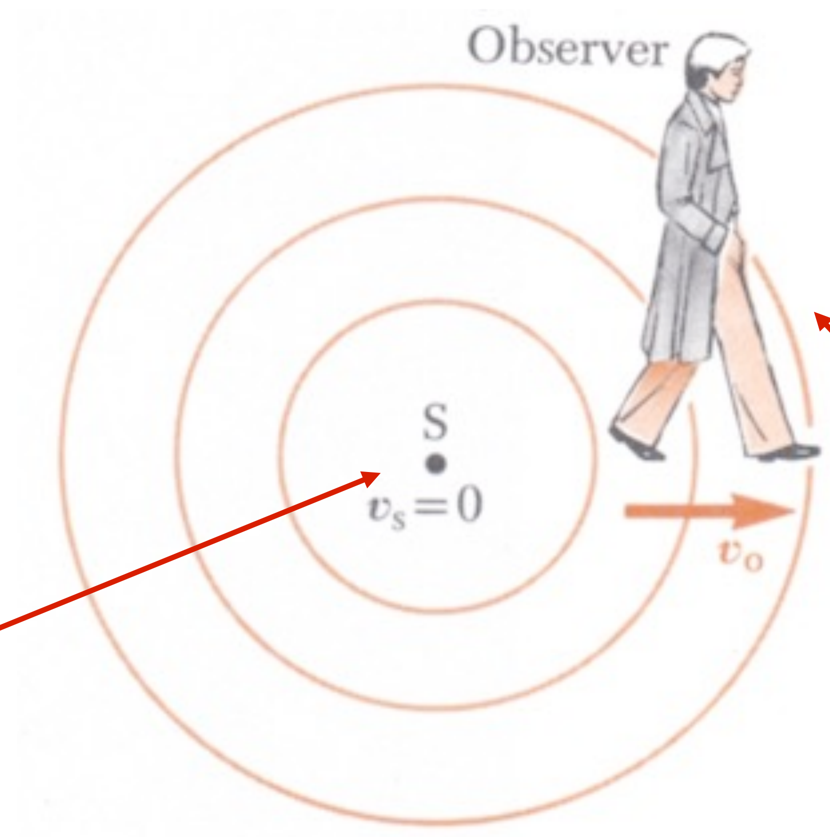
$$\text{but } v = f\lambda \quad \text{or} \quad \frac{1}{\lambda} = \frac{f}{v}$$

$$\therefore \frac{v_o}{\lambda} = \frac{v_o}{v} f$$

$$\therefore f' = f + \frac{v_o}{v} f$$

$$f' = f\left(\frac{v + v_o}{v}\right)$$

$(v + v_o)$ = speed of waves relative to O



Point source S at rest ie $v_s = 0$

Observer moving **AWAY FROM** S at speed v_o

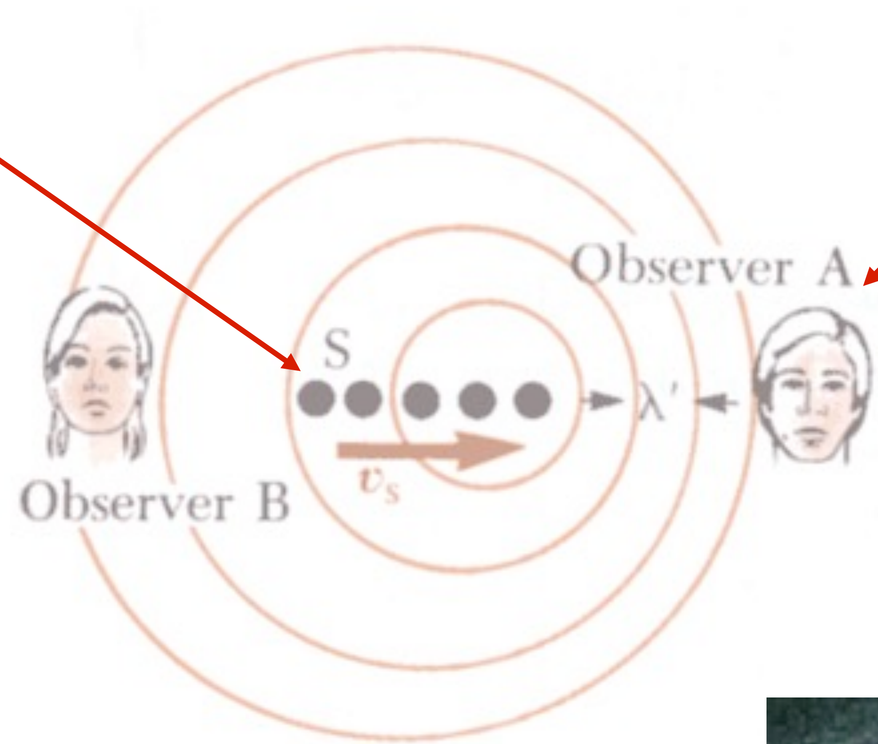
O now detects fewer wavefronts /second and therefore the frequency is lowered.

The speed of the wave relative to O is $(v - v_o)$

$$\therefore \; f' = f\left(\frac{v - v_o}{v}\right)$$

Stationary observer, source moving

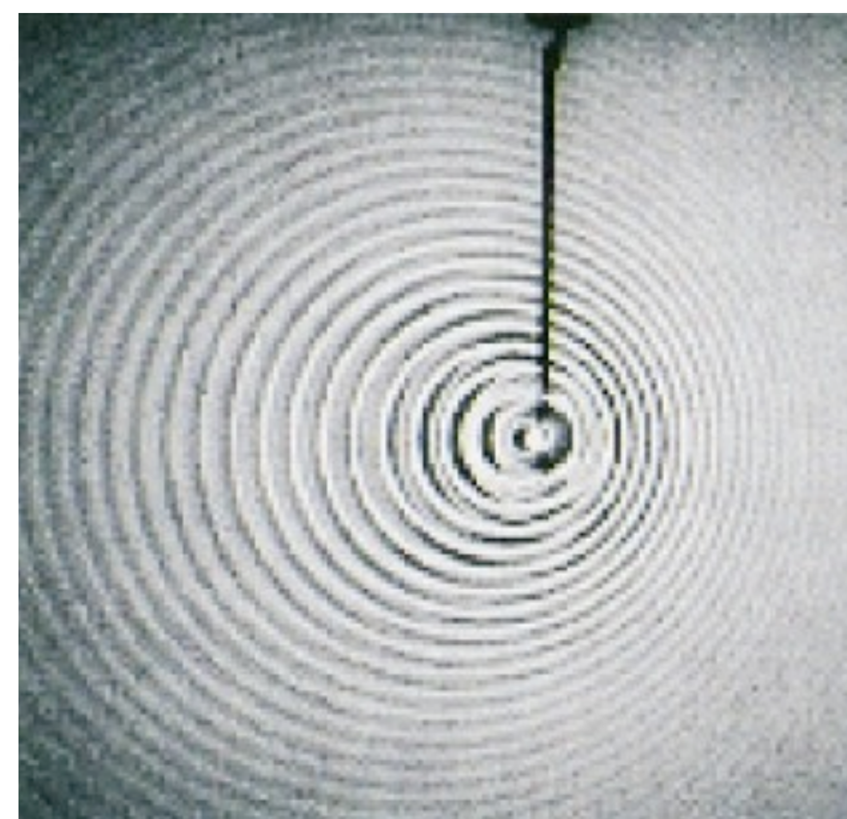
Point source S moving with speed v_s



S moving **TOWARDS** O_A at speed v_s

Source is moving towards O_A at speed v_s.

Wavefronts are closer together as a result of the motion of the source.



Stationary observer, source moving

The observed wavelength λ' is shorter than the original wavelength λ.

During one cycle (which lasts for period T) the source moves a distance $v_s T$ (= v_s/f)

In one cycle the wavelength is shortened by v_s/f

$$\lambda' = \lambda - \Delta\lambda = \lambda - \frac{v_s}{f}$$

$$\text{but} \quad \lambda = \frac{v}{f} \quad \text{and} \quad \lambda' = \frac{v}{f'}$$

$$\therefore \; f' = \left(\frac{v}{\lambda - v_s / f} \right)$$

Stationary observer, source moving

$$f' = \left(\frac{v}{\lambda - v_s/f}\right)$$

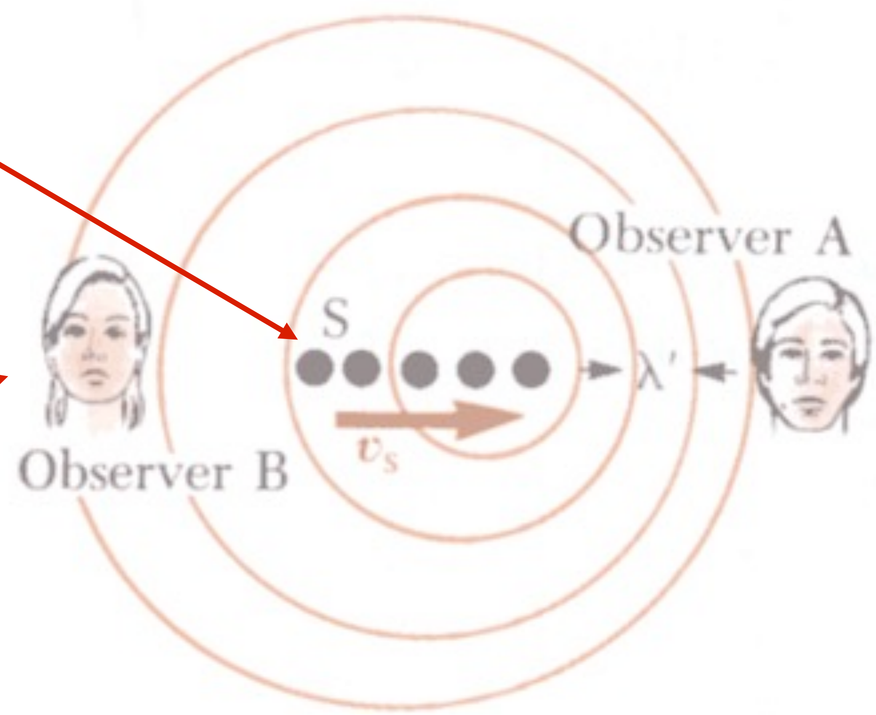
$$= \left(\frac{v}{v/f - v_s/f}\right)$$

$$f' = f\left(\frac{v}{v - v_s}\right)$$

ie: the observed frequency is increased when the source moves towards the observer.

Note - the equation breaks down when $v_s \sim v$. We will discuss this situation later.

Point source S
moving with speed v_s



S moving **AWAY FROM**
O_B at speed v_s

Source is moving away from O_B at speed v_s.

Wavefronts are further apart, λ is greater and O_B hears a decreased frequency given by

$$f' = f\left(\frac{v}{v + v_s}\right)$$

General equations

Frequency heard when observer is in motion

$$f' = f\left(\frac{v \pm v_o}{v}\right)$$

+ O towards S
- O away from S

Frequency heard when source is in motion

$$f' = f\left(\frac{v}{v \mp v_s}\right)$$

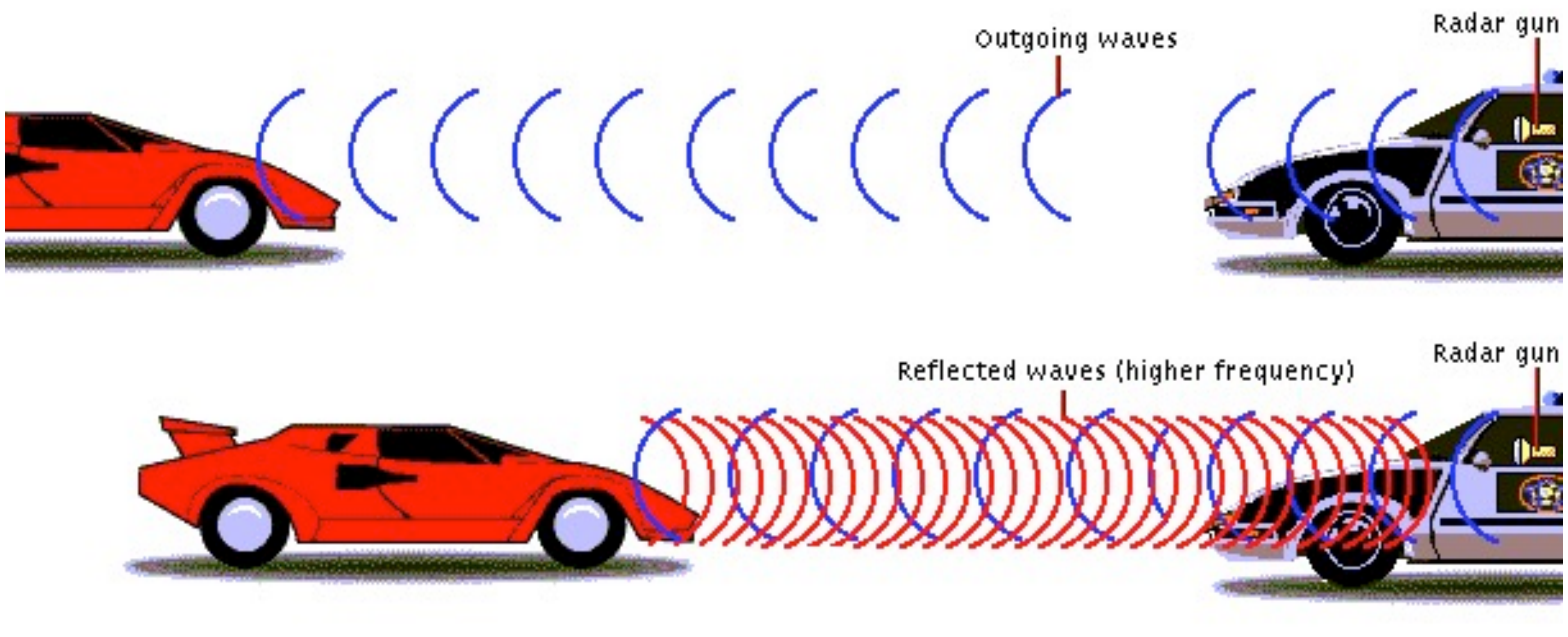
- S towards O
+ S away from O

Frequency heard when observer is in motion

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

upper signs = towards
lower signs = away from

Use of Doppler effect: Radar guns



Gun transmits waves at a given frequency (blue) toward an oncoming car.

Reflected waves (red) return to the gun at a different frequency, depending on how fast the car is moving.

A device in the gun compares the transmission frequency to the received frequency to determine the speed of the car.

Example

A sound source with a frequency of 10kHz moves in the +ve x-direction with a speed of $50ms^{-1}$.

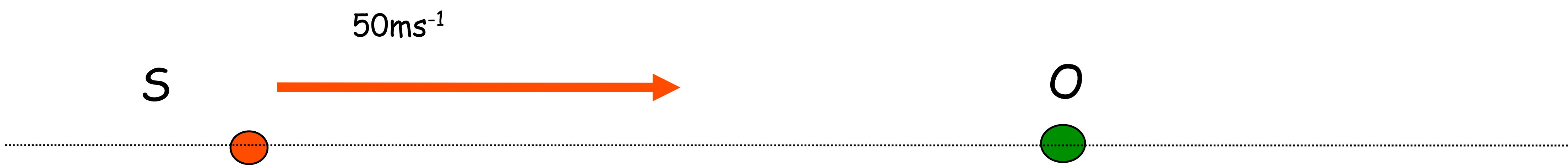
(a) What frequency will be heard by the observer in front of the source along the x-axis ?

(b) What is the wavelength of the sound wave in front of the source along the x-axis ?

(c) What frequency would be heard if the observer were in front of the source and moving in the -ve x-direction with a speed of $5ms^{-1}$ relative to still air.

Assume the speed of sound in air is $330ms^{-1}$

(a) What frequency will be heard by the observer in front of the source along the x-axis ?



$$f' = f\left(\frac{v}{v - v_s}\right)$$

$$f' = 10^4\left(\frac{330}{330 - 50}\right)$$

$$f' = 1.18 \times 10^4 \text{ Hz}$$

(b) What is the wavelength of the sound wave in front of the source along the x-axis ?

$$\lambda' = \frac{v}{f'} = \frac{330}{1.18 \times 10^4} = 0.028\text{m}$$

(c) What frequency would be heard if the observer were in front of the source and moving in the -ve x-direction with a speed of 5ms^{-1} relative to still air ?

$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 10^4\left(\frac{330 + 5}{330 - 50}\right) = 1.2 \times 10^4\text{Hz}$$

Shock Waves

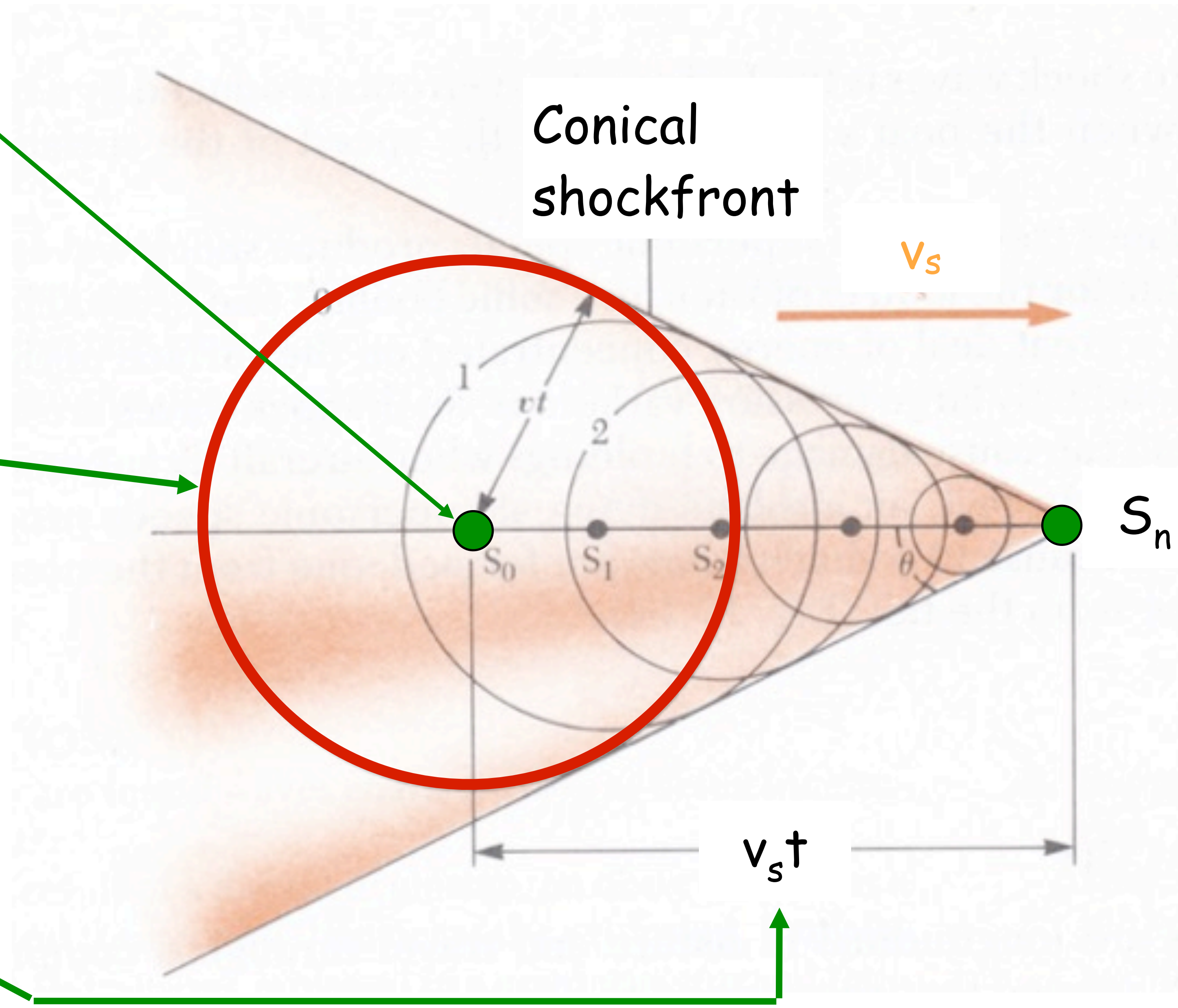
Go back to situation where source is moving with velocity v_s which exceeds wave velocity

At t=0 source is at S_o

At t = t S is at S_n

During this time the wavefront centred on S_o reaches a radius vt

Source has travelled from S_o to $S_n = v_s t$

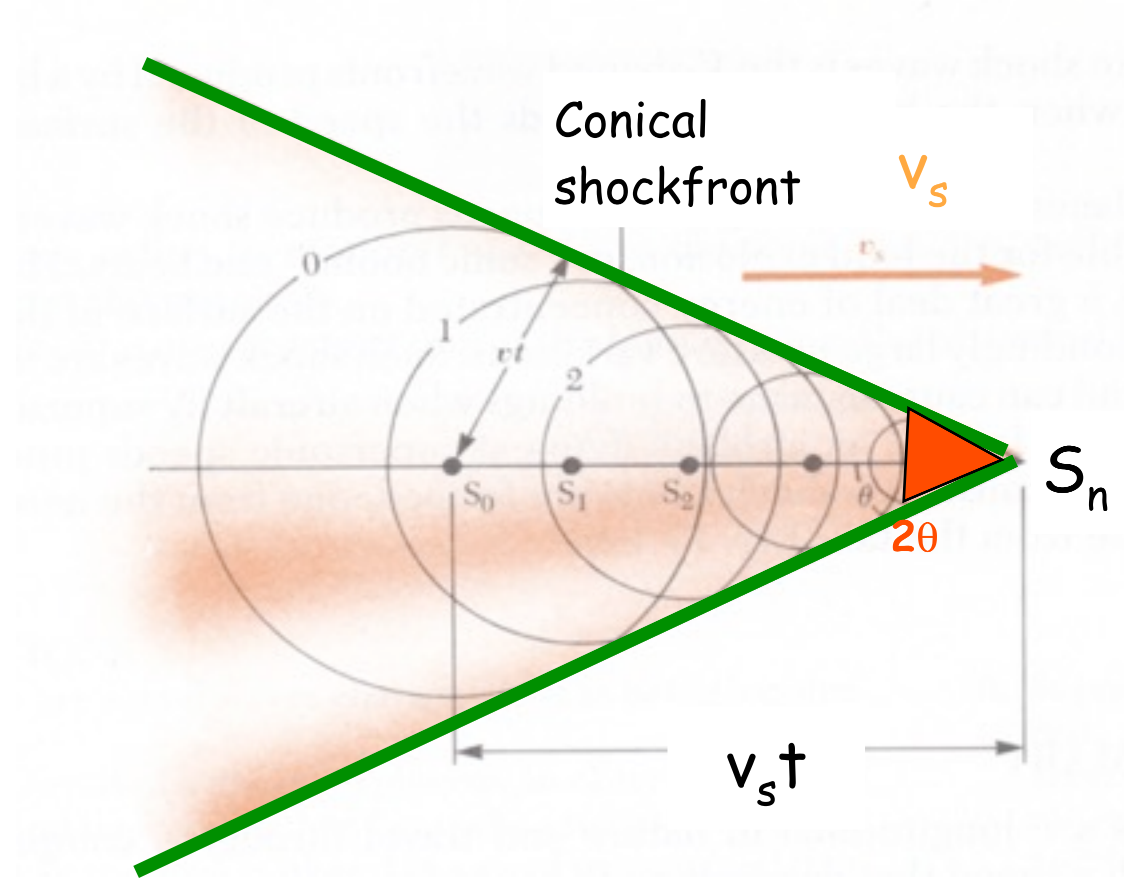


At t=t S is at S_n and waves are just about to be produced here

The line drawn from S_n to the wavefront centred on S_o is tangential to all wavefronts generated at intermediate times

The envelope of these waves is a cone whose apex half angle θ is given by

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$



The ratio $\frac{v_s}{v}$ is known as the **MACH** number.

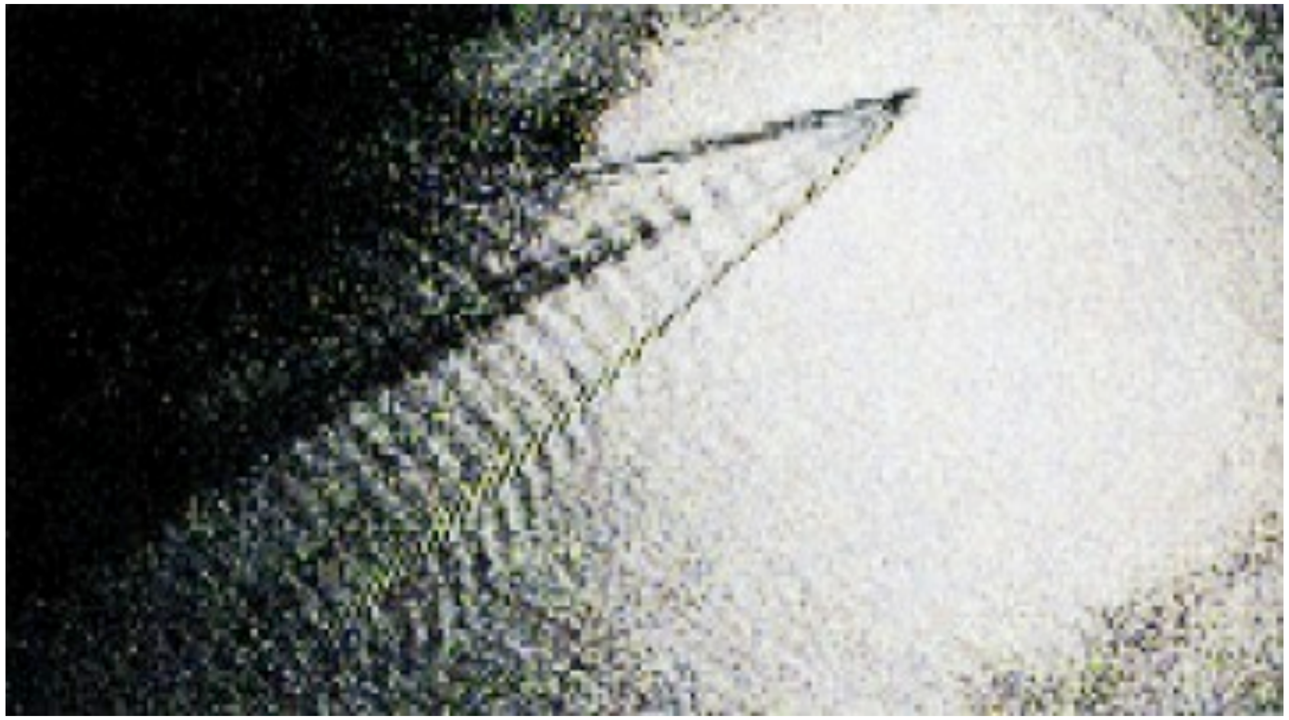
The conical wavefront produced when $v_s > v$ (supersonic speeds) is known as a shock wave.

An aeroplane travelling at supersonic speeds will produce shockwaves.

In this photo the cloud is formed by the adiabatic cooling of the shock wave to the dew point.

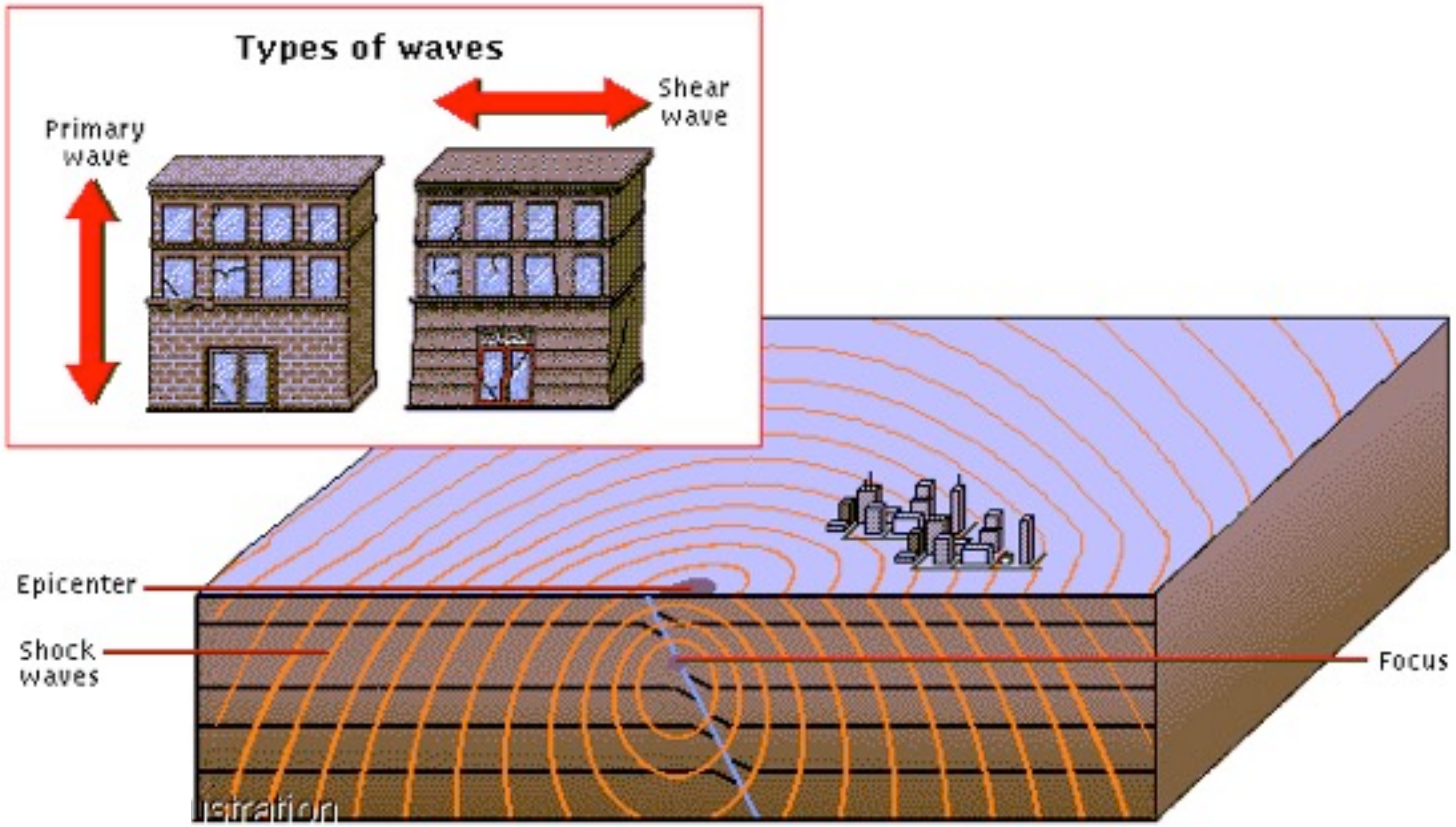
http://www.youtube.com/watch?v=gWGLAAYdbbc

Examples of shockwaves: wavefronts in water



An interesting analogy to shock waves is the V-shaped wavefronts produced when a boat's speed exceeds the speed of the surface water waves.

Examples of shockwaves: earthquakes



Earthquakes occur when a build-up of pressure or strain between sections of rocks within the earth's crust is suddenly released, causing minor or severe vibrations on the surface of the land.

Shock waves propagate like ripples from the focus and epicenter, decreasing in intensity as they travel outward. The main types of seismic waves are primary waves (P waves) and shear waves (S waves). P waves cause particles to vibrate in the same direction as the shock wave. P waves are the first to be recorded in an earthquake because they move faster than S waves, which cause vibrations perpendicular to the direction of travel.