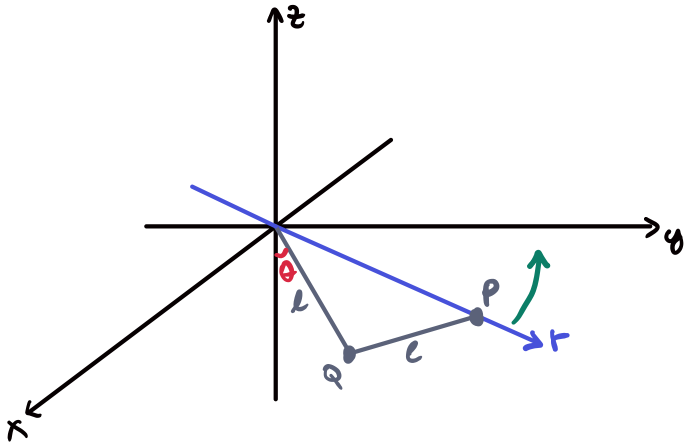

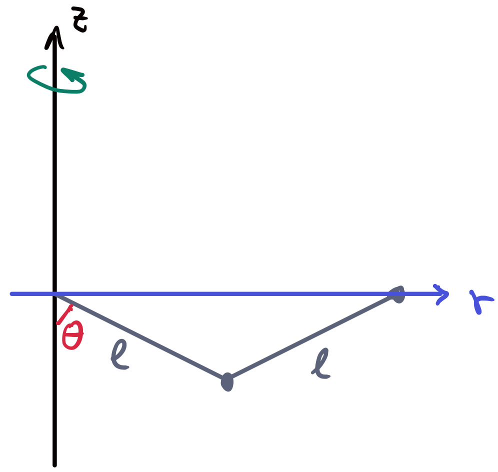

La retta r ruota in modo uniforme nel piano xy con velocità angolare cost. Ω (a $t=0$ $\hat{r}$ coincide con $\hat{x}$)

$$x = r\cos\Omega t \qquad r_P = \ell\,\text{sen}\,\theta \qquad z_P = -\ell\cos\theta$$

$$y = r\,\text{sen}\,\Omega t \qquad r_Q = 2\ell\,\text{sen}\,\theta \qquad z_Q = 0$$

$$x_P = \ell\,\text{sen}\,\theta\cos\Omega t \qquad x_Q = 2\ell\,\text{sen}\,\theta\cos\Omega t$$

$$y_P = \ell\,\text{sen}\,\theta\,\text{sen}\,\Omega t \qquad y_Q = 2\ell\,\text{sen}\,\theta\,\text{sen}\,\Omega t$$

$$z_P = -\ell\cos\theta \qquad z_Q = 0$$

$$\dot{x}_P = \dot{\theta}\,\ell\cos\theta\cos\Omega t - \Omega\ell\,\text{sen}\,\theta\,\text{sen}\,\Omega t \qquad \dot{x}_Q = 2\dot{x}_P$$

$$\dot{y}_P = \dot{\theta}\,\ell\cos\theta\,\text{sen}\,\Omega t + \Omega\ell\,\text{sen}\,\theta\cos\Omega t \qquad \dot{y}_Q = 2\dot{y}_P$$

$$\dot{z}_P = \ell\dot{\theta}\,\text{sen}\,\theta \qquad \dot{z}_Q = 0$$

$$T = \frac{m}{2}\left[\ell^2\dot{\theta}^2 + \Omega^2\ell^2\text{sen}^2\theta + 4\dot{\theta}^2\ell^2\cos^2\theta + 4\Omega^2\ell^2\text{sen}^2\theta\right]$$

$$= \frac{m}{2}\ell^2(1+4\cos^2\theta)\dot{\theta}^2 + \frac{5}{2}m\ell^2\Omega^2\text{sen}^2\theta$$

$$V = -mg\ell\cos\theta$$

$$L = \frac{m}{2} l^2 (1+4\cos^2\theta)\dot{\theta}^2 + \frac{5}{2} m l^2 \Omega^2 \operatorname{sen}^2\theta + mgl\cos\theta$$

Eq. Lagrange, scrivendole come $\ddot{\theta} = \dots$:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{d}{dt}\left(ml^2 (1+4\cos^2\theta)\dot{\theta} \right) = ml^2(1+4\cos^2\theta)\ddot{\theta} - $$
$$- 8ml^2 \cos\theta \operatorname{sen}\theta\, \dot{\theta}^2$$

$$\frac{\partial L}{\partial \theta} = -4ml^2 \cos\theta \operatorname{sen}\theta\, \dot{\theta}^2 + 5ml^2\Omega^2 \cos\theta \operatorname{sen}\theta - mgl\operatorname{sen}\theta$$

$$\ddot{\theta} = \frac{4\dot{\theta}^2 \operatorname{sen}\theta\cos\theta + 5\Omega^2 \operatorname{sen}\theta\cos\theta - g/l \operatorname{sen}\theta}{1+4\cos^2\theta}$$

$$\ddot{\theta} = \frac{\left(2\dot{\theta}^2 + \frac{5}{2}\Omega^2\right)\operatorname{sen}2\theta - \frac{g}{l}\operatorname{sen}\theta}{1+4\cos^2\theta}$$

Linearizzare l'equazione di Lagrange attorno $\theta = 0$ e risolvere eq. lin.

$$\ddot{\theta} = -\frac{g}{5l}\theta + \Omega^2\theta = -\left(\frac{g}{5l} - \Omega^2\right)\theta$$

$$\theta(t) = A\cos\left[\left(\frac{g}{5l} - \Omega^2\right)t + \varphi_0\right] \quad \text{se} \quad \frac{g}{5l} - \Omega^2 > 0$$

$$= A\cosh\left(\left(\Omega^2 - \frac{g}{5l}\right)t + \varphi_0\right) \quad \text{se} \quad \frac{g}{5l} - l^2 < 0$$

Configurazioni di equilibrio

$$V_{eff} = -\frac{5}{2} m l^2 \Omega^2 \operatorname{sen}^2\theta - mgl\cos\theta$$

$$V_{eff}' = -5 m l^2 \Omega^2 \operatorname{sen}\theta\cos\theta + mgl\operatorname{sen}\theta = 0$$

$$-5ml^2\Omega^2 \operatorname{sen}\theta \left(\cos\theta - \frac{g}{5l\Omega^2} \right) \qquad \theta = 0, \pi \quad \cos\theta^* = \frac{g}{5l\Omega^2} \leq 1$$

$$V_{eff}'' = 5ml^2\Omega^2 (\operatorname{sen}^2\theta - \cos^2\theta) + mgl\cos\theta =$$

$$= 5ml^2\Omega^2 (1 - 2\cos^2\theta) + mgl\cos\theta$$

$$V_{eff}''(0) = -5ml^2\Omega^2 + mgl \qquad \text{stabile se} \quad \frac{g}{5l\Omega^2} > 1$$

$$V_{eff}''(\pi) = -5ml^2\Omega^2 - mgl < 0 \qquad \text{instab}$$

$$V_{eff}''(\theta^*) = 5ml^2\Omega^2 \left(1 - 2\frac{g^2}{25l^2\Omega^4} \right) + mgl\frac{g}{5l\Omega^2} =$$

$$= 5ml^2\Omega^2 - \frac{\cancel{2}mg^2}{5\Omega^2} + \cancel{m}\frac{g^2}{5\Omega^2} =$$

$$= 5ml^2\Omega^2 \left(1 - \frac{g}{5l\Omega^2} \right)\left(1 + \frac{g}{5l\Omega^2} \right) \qquad \text{stab. se } \frac{g}{5l\Omega^2} < 1$$

Freq. piccole oscillaz. attorno pti eq. stab.

$$\omega^2 = \frac{V_{eff}''(\theta_0)}{a(\theta_0)} \qquad a(\theta_0) = ml^2(1 + 4\cos^2\theta)$$

$$a(0) = 5ml^2$$

$$a(\theta^*) = ml^2\left(1 + 4\frac{g^2}{25l^2\Omega^4} \right)$$

$$\theta_0 = 0 \qquad \omega^2 = \frac{-5ml^2\Omega^2 + mgl}{5ml^2} = \frac{g}{5l} - \Omega^2$$

$\theta_0 = \theta^*$ $\quad \omega^2 = \dfrac{5\Omega^2\left(1 - \frac{g^2}{25\ell^2\Omega^4}\right)}{1 + 4\frac{g^2}{25\ell^2\Omega^4}}$

Si consideri ora lo stesso sistema, in cui però la retta r è libera di ruotare nel piano xy con coord. libera φ.

- Qual è la coord. ciclica? quale cost. del moto?
- Lagr. ridotta e relative equazioni del moto.

Es. 1

$q_1 \quad p_1$
$q_2 \quad p_2$
$q_3 \quad p_3$

$$\boxed{H = q_1 p_2 - q_2 p_1}$$

$$\dot{p}_1 = -\frac{\partial H}{\partial q_1} = -p_2 \qquad \dot{q}_1 = \frac{\partial H}{\partial p_1} = -q_2$$

$$\dot{p}_2 = -\frac{\partial H}{\partial q_2} = p_1 \qquad \dot{q}_2 = \frac{\partial H}{\partial p_2} = q_1$$

$$\dot{p}_1(t) = -p_1^\circ \sin t - p_2^\circ \cos t$$
$$\dot{p}_2(t) = p_1^\circ \cos t - p_2^\circ \sin t$$

$$p = \alpha \tilde{q} + \ln \tilde{p} \qquad q = \frac{-1}{\alpha}\tilde{p} \qquad + f(\tilde{q}) + g(\tilde{p})$$

$$\{p, q\} = \alpha \cdot \left(-\frac{1}{\alpha} + g'(\tilde{p})\right) + \frac{1}{\tilde{p}} f'(\tilde{q})$$

$$p = \alpha \tilde{q} + \ln \tilde{p} \qquad q = \tilde{p}\, e^{\alpha \tilde{q}} - \tilde{p}/\alpha$$

$$\{q, p\} = \alpha e^{\alpha \tilde{q}} \frac{\tilde{p}}{\tilde{p}} - \left(e^{\alpha \tilde{q}} - \frac{1}{\alpha}\right) \cdot \alpha = 1$$