LUMINOSITY FUNCTION

$\boxed{\text{TEO}}$

$$\overset{(2)}{dN} \equiv \phi(M,\bar{x})\, dM\, d^3\bar{x} \quad \text{(eq. 3.20)} \qquad d^3\bar{x} = dx \cdot dy \cdot dz$$

number of stars in $\overset{\text{mag}}{\text{range}}$ $(M+dM, M)$ and in volume $d^3\bar{x}$

→ M (or L) independent of position

$$\Longrightarrow \quad dN = \underbrace{\phi(M)}_{LF}\, dM\, \nu(x)\, d^3\bar{x}$$

↳ "number of stars" per volume (density!)

(general or specific)

↳ x star type

$\boxed{\text{observations}}$ → star counts in the range $(m+dm, m)$ in some region of the sky

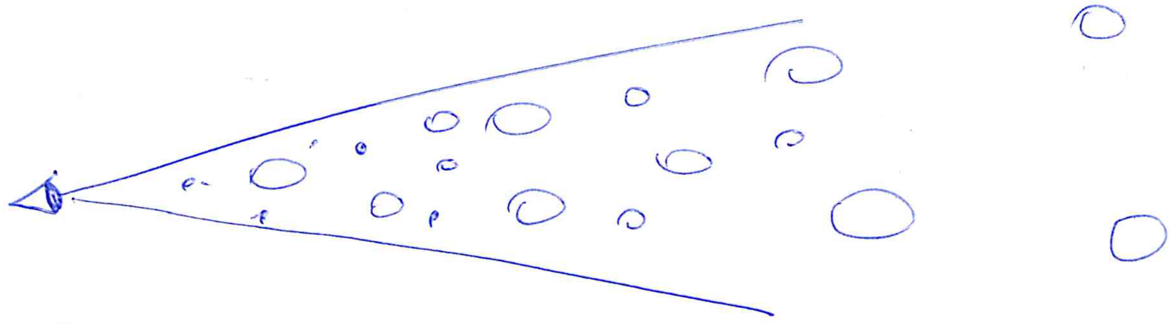
a magnitude Limited sample

average M in the sample

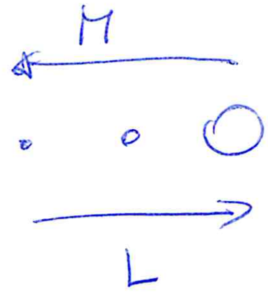
$\langle M_{obs} \rangle < M_{true}$

↳ or in a volume limited sample

MALMQUIST BIAS



in a magnitud sample



FUNDAMENTAL EQUATION OF STELLAR STATISTICS

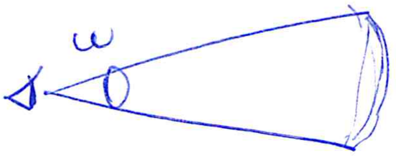
$\boxed{\text{TEO} \leftrightarrow \text{OBS}}$

# of stars in a mag-limited sample in $(M+dM, M)$ and $(m+dm, m)$

$$dN = \phi(M)\, dM\, \underbrace{\nu(x)\, d^3x}_{dm}$$

$$\frac{d^2N}{dm\, dM} = \phi(M) \left.\frac{dm}{dm}\right|_M = \phi(M) \frac{dm}{ds} \left(\frac{\partial s}{\partial m}\right)_M$$

$s \equiv$ star distance



$$dm = \omega s^2 ds\, \nu(s) \qquad \frac{dm}{ds} = \omega s^2 \nu(s)$$

$$A(m) \equiv \frac{dN}{dm} = \int_{-\infty}^{+\infty} dM \frac{d^2N}{dm\, dM} = \omega \int_{-\infty}^{+\infty} dM\, \phi(M) \left(\frac{\partial s}{\partial m}\right)_M s^2 \nu(s)$$

(3.25)

change of the integration variable

$M \to s$ $\qquad M \to -\infty \Rightarrow s \to +\infty$ $\qquad$ for fixed m!

$\qquad M \to +\infty \Rightarrow s \to 0$

distance modulus $\boxed{m - M = 5 \lg s - 5}$

$$\left(\frac{\partial M}{\partial s}\right)_m = -\left(\frac{\partial m}{\partial s}\right)_M$$

$$dM \left(\frac{\partial s}{\partial m}\right)_M = -dM \left(\frac{\partial s}{\partial M}\right)_m = -ds$$

$$\boxed{A(m) = \omega \int_0^{\infty} ds\, \phi(M)\, s^2 \nu(s)}$$

FUND. EQ. OF STELLAR STATISTICS

(3.26)

MALMQUIST BIAS (DEM.)

$\langle M \rangle_m$ the average in the mag limited sample

$$\langle M \rangle_m = \frac{\int_{-\infty}^{+\infty} dM\, M \frac{d^2N}{dm\,dM}}{\int_{-\infty}^{+\infty} dM \frac{d^2N}{dm\,dM}} =$$

$\frac{d^2N}{dm\,dM} \longrightarrow$ distribution function

REMIND:

$$\langle X \rangle_{m} = \frac{\sum w_i x_i}{\sum w_i}$$

$\downarrow$ weighted avg.

spatial average

$$\langle X \rangle = \frac{\int X\, \rho(\bar{x})\, d^3\bar{x}}{\int \rho(\bar{x})\, d^3\bar{x}}$$

$$= \frac{\omega \int_0^\infty ds\, M\, \phi(M)\, s^2 \nu(s)}{\omega \int_0^\infty ds\, \phi(M)\, s^2 \nu(s)}$$

$\omega \int_0^\infty ds\, \phi(M)\, s^2 \nu(s) \longrightarrow$ is $A(m)$

$$\frac{dA}{dm} = \omega \int_0^\infty ds \frac{d\phi}{dm} s^2 \nu(s) = \omega \int_0^\infty \frac{d\phi}{dM} s^2 \nu(s)$$

$$\frac{d\phi}{dm} = \frac{d\phi}{dM} \cdot \underbrace{\frac{dM}{dm}}_{=1}$$

From distance modulus

$dm - dM = \cancel{\phi}$

$$\left\langle \frac{1}{\phi} \frac{d\phi}{dM} \right\rangle_m = \frac{\omega \int_0^\infty ds \frac{1}{\cancel{\phi}} \frac{d\phi}{dM} \cancel{\phi(M)}\, s^2 \nu(s)}{\omega \int_0^\infty ds\, \phi(M)\, s^2 \nu(s)} = \frac{1}{A} \cdot \frac{dA}{dm}$$

$$\ldots\ldots \frac{1}{A} \frac{d^2A}{dm^2} = \left\langle \frac{1}{\phi} \frac{d^2\phi}{dM^2} \right\rangle_m$$

To go on

We must adopt a definite functional form for LF

$$\phi(M) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{(M-M_0)^2}{2\sigma^2}\right) \; ; \; \frac{d\phi}{dM} = \phi \cdot \frac{M_0 - M}{\sigma^2}$$

.... set the local stellar density to unity

$$\left\langle \frac{1}{\phi}\frac{d\phi}{dM} \right\rangle_m = \frac{\omega \int_0^\infty ds \; \phi \, \frac{M_0-M}{\sigma^2} \, s^2 \nu(s)}{\omega \int_0^\infty ds \; \phi \; s^2 \nu(s)} = \left\langle \frac{M_0 - M}{\sigma^2} \right\rangle_m$$

$$\frac{1}{A}\frac{dA}{dm} = \left\langle \frac{M_0 - M}{\sigma^2} \right\rangle_m = \frac{M_0 - \langle M \rangle_m}{\sigma^2}$$

$$\langle M \rangle_m = M_0 - \sigma^2 \frac{1}{A}\frac{dA}{dm} = M_0 - \sigma^2 \frac{d\ln A}{dm} \qquad (3.34)$$

$$\boxed{\langle M \rangle_m < M_0}$$

MALMQUIST BIAS !

obs is > 0

obvious case if population is distributed in an uniform way

.....

$$\sigma_m^2 = \sigma^2 + \sigma^4 \frac{d^2 \ln A}{dm^2} \qquad (3.36)$$

CASE OF UNIFORM POPULATION

$\nu(s) = \text{const.}$

$m - M = 5 \lg s - 5 \qquad s = 10^{\frac{m-M+5}{5}} = 10^{0.2m} \cdot 10^{-0.2M} \cdot 10$

For fixed M $\quad s = 10^{0.2m} \cdot \text{const.} \qquad m = 5 \lg s + \text{const}$

$$dm = 5 \frac{d \ln s}{\ln 10} = \frac{5}{\ln 10} \frac{ds}{s}$$

$$\left(\frac{\partial s}{\partial m}\right)_M = 0.2 \ln 10 \cdot s$$

gaussian

$$\text{3.25} \longrightarrow A(m) \propto \int_{-\infty}^{+\infty} dM \, \phi(M) \, s^3 \propto \int_{-\infty}^{+\infty} dM \, e^{-\frac{(M-M_0)^2}{2\sigma^2}} s^3$$

$$\propto 10^{0.6m} \int_{-\infty}^{+\infty} dM \, e^{-\frac{(M-M_0)^2}{2\sigma^2}} \cdot 10^{-0.6M}$$

$$\propto e^{(0.6 \ln 10) m} \int_{-\infty}^{+\infty} dM \, e^{-\frac{M^2 + M_0^2 - 2MM_0 + 2M \, 0.6 \ln 10 \, \sigma^2}{2\sigma^2}}$$

$$\propto e^{(0.6 \ln 10) m} \int_{-\infty}^{+\infty} dM \, e^{-\frac{M^2 - 2M[0.6 \ln 10 \, \sigma^2 - M_0] + M_0^2 + \ldots}{2\sigma^2}} \cdot e^{-0.6 \ln 10 \, M_0}$$

$$\propto e^{0.6 \ln 10 (m - M_0)} \cdot \underbrace{\int_{-\infty}^{+\infty} \ldots}_{\sqrt{\pi} = \text{const}}$$

So che

$$\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$$

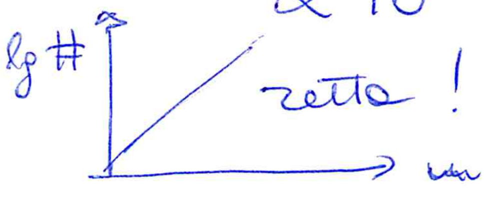
$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = 1$$

normalized gaussian

$$z = \frac{x - \mu}{\sigma}$$

$$A(m) \propto e^{0.6 \ln 10 (m - M_0)} \propto 10^{0.6(m - M_0)} \propto 10^{0.6}$$

$$\boxed{A(m) \propto 10^{0.6m}}$$



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$$A(m) = \text{const} \cdot 10^{0{,}6\, m}$$

$$\lg A = \text{const}' + 0{,}6\, m$$

$$d \lg A = 0{,}6\, dm$$

$$\frac{d \ln A}{\ln 10} = 0{,}6\, dm \qquad \frac{d \ln A}{dm} = \underbrace{0{,}6 \cdot \ln 10}_{\text{const}}$$

in eq. 3.34

diff. between $\langle M \rangle_m$ and M_0 does not depend on m!

$$\frac{d^2 \ln A}{dm^2} = 0 \rightarrow$$ eq. 3.35 diff. between σ_m and σ does not depend on m