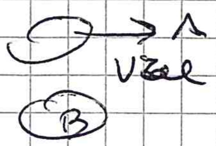
7 COLLISIONS / ENCOUNTERS OF STELLAR SYSTEMS

Galaxy and M31 gas and galaxies → non collisions but effect of ϕ → ordered → random motion, merger?



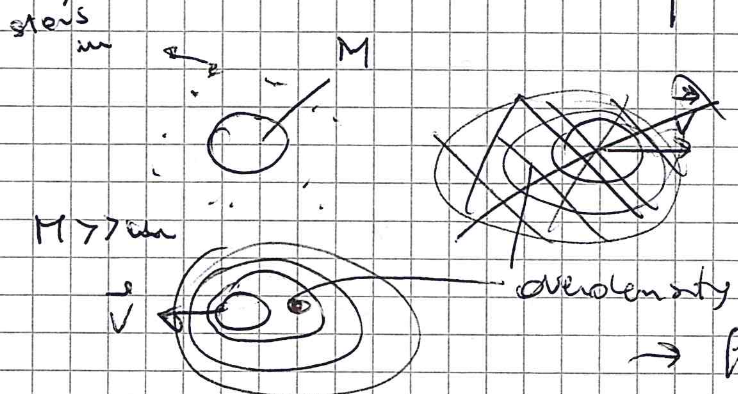
$$\Delta \bar{v} = \int dt\, \bar{g}(t)$$

→ velocity impulse

→ large if V_{rel} is small

sum ←

DYNAMICAL FRICTION



distribution of stars is modified → overdensity of small particles → friction

BT 1

1 star

Effect on M of $\boxed{\text{an encounter}}$ with a $\boxed{\text{single m}}$

$M\ (\bar{x}_M, \bar{v}_M)$, $m\ (\bar{x}_m, \bar{v}_m)$

$\bar{r} = \bar{x}_m - \bar{x}_M$

$\bar{V} = \dot{\bar{r}}$

eq. of motion of the reduced particle in the Keplerian potential of a fixed body of mass $(m+M)$

$$\frac{mM}{m+M}\ddot{\bar{r}} = -\frac{GMm}{r^2}\hat{e}_r$$

→ μ reduced mass

see Appendix

$\bar{F}_{12}$, F_{21}, m_2, m_1

$\bar{F}_{12} = -\bar{F}_{21}$

$\bar{F}_{12} = m_1 \frac{d\bar{x}_1}{dt} = -m_2 \frac{d\bar{x}_2}{dt}$

$\bar{r} = \bar{x}_2 - \bar{x}_1$

$$\frac{d^2 r}{dt^2} = \frac{d^2(\bar{x}_2 - \bar{x}_1)}{dt^2} = \frac{d^2 x_2}{dt^2} - \frac{d^2 x_1}{dt^2} = \frac{-\bar{F}_{12}}{m_1} - \frac{\bar{F}_{12}}{m_2} =$$

$$-\bar{F}_{12}\left(\frac{m_1+m_2}{m_1 m_2}\right) = -\frac{\bar{F}_{12}}{\mu} =$$

$= +\frac{1}{\mu}\left(-\ldots\right)$

$$\Delta \bar V_m - \Delta \bar V_M = \Delta \bar V \qquad 7.2a$$

$$-\;\; m\Delta \bar V_m - m\Delta \bar V_M = m\Delta \bar V$$

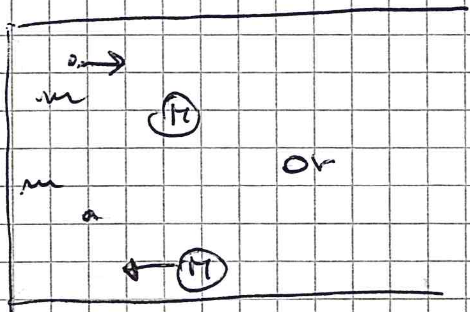
$$+\;\; m\Delta \bar V_m + M\Delta \bar V_M = 0 \qquad 7.2b \;\rightarrow\; \Delta \bar V_m = -\frac{M}{m}\Delta \bar V_M$$

$$\cancel{\Delta \bar V_M = -\frac{m}{M}\Delta \bar V_m = -\frac{m}{M}}$$

$$(M+m)\Delta \bar V_M = -m\Delta \bar V$$

$$\Delta \bar V_M = -\frac{m}{M+m}\Delta \bar V \qquad 7.3$$

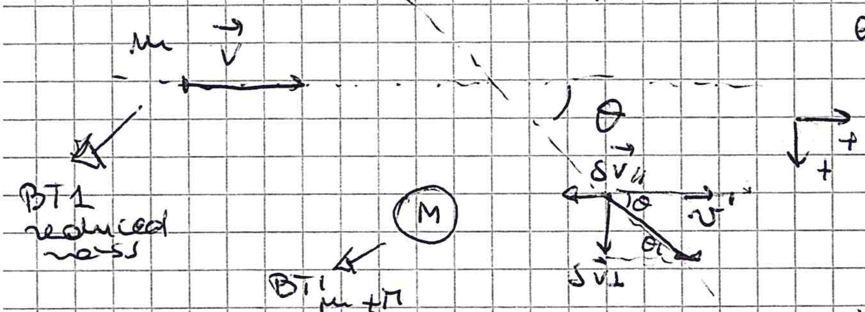
$$\Delta \bar V_m = +\frac{M}{M+m}\Delta \bar V$$



FROM MAMON (easiest → not precise)

$M \gg m$ V big $|\delta \bar v| \ll |\bar v|$ θ small

energy cons. $|\bar v'| \approx |\bar v|$



$$\delta v_\perp \sim v\,\mathrm{sen}\,\theta$$

$$\delta v_\parallel \sim -(v - v\cos\theta)$$

$$\sim -v(1-\cos\theta)$$

$$\left(\frac{\delta v_\perp}{v}\right)^2 = \mathrm{sen}^2\theta = 1-\cos^2\theta = (1-\cos\theta)\underbrace{(1+\cos\theta)}_{\sim 2}$$

$$\sim 2(1-\cos\theta) \qquad (\theta \text{ is small!})$$

$$\frac{\delta v_\parallel}{v} \sim -(1-\cos\theta)$$

$$\frac{\delta v_\parallel}{v} \sim -\frac{1}{2}\left(\frac{\delta v_\perp}{v}\right)^2 \qquad (\overline{VII}-1)$$

From the relaxation time (eq. 4.3 BT1) → with total mass M+m in place of m

$$|\delta \bar v_\perp| \sim \frac{2G(M+m)}{bv}^{*}$$

$$|\delta v_\parallel| \sim \frac{1}{2}\frac{4G^2(M+m)^2}{b^2v^3}$$

→ is $\Delta V_\parallel$

$$\Delta \bar{V}_{M_{\parallel}} = -\left(\frac{m}{m+M}\right) \Delta \bar{V}_{\parallel}$$

$$|\Delta \bar{V}_{M_{\parallel}}| \sim +2 \frac{G^2 m (M+m)}{b^2 v^3}$$ $\boxed{VII}-2$

More precise computation BT1

$$|\Delta \bar{V}_{M_{\parallel}}| = \frac{\frac{2m}{M+m}}{1 + b^2 v^4 / [G^2 (M+m)^2]} \, v$$

if V very big $\rightarrow$ $\boxed{VII}-2$ or b very big

Keplerian orbit

V_0 relative velocity

$\boxed{BT1}$

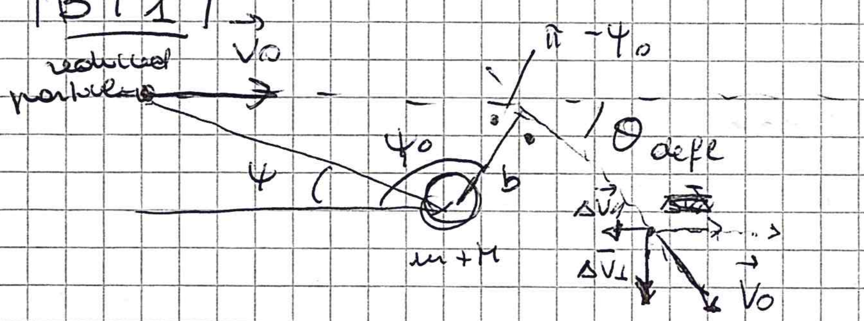


Fig 7.2

motion of reduced particle

$\bar{V}_0 = \bar{V}(t=-\infty)$

b = impact parameter

$$\Theta = \Theta_{defl} = \pi - 2(\pi - \Psi_0) = 2\Psi_0 - \pi$$

$|\vec{V}|_{after} \sim |\vec{V}|_{before}$ conservation of energy ~~conservation of the relative velocity~~

conservation of angular momentum $L = bV_0 = r \cdot r\dot{\Psi} = r^2 \dot{\Psi}$ in the orbit $\boxed{7.4}$

in the time

Eq. $\boxed{3.21}$ of Keplerian orbits $\rightarrow$ hyperbolic

$$\frac{1}{r} = C \cos(\Psi - \Psi_0) + \frac{G(M+m)}{b^2 V_0^2}$$ $\boxed{7.5}$

C and Ψ_0 from initial conditions

$\frac{d}{dt}$ 7.5

$$-\frac{1}{r^2}\frac{dr}{dt} = C\left[-\sin(\Psi - \Psi_0)\right] \cdot \dot{\Psi}$$

$$\frac{dr}{dt} = C r^2 \dot{\Psi} \sin(\Psi - \Psi_0) = C b V_0 \sin(\Psi - \Psi_0)$$ $\boxed{7.6}$

$\boxed{7.4}$

$\psi = 0$ at $t \to -\infty$

$$7.6 \underset{t \to -\infty}{\rightarrow} \quad -V_0 = C\, b\, V_0 \operatorname{sen}(-\psi_0) \qquad 7\text{-}7e$$

$$7.5 \underset{t \to -\infty}{\rightarrow} \quad 0 = C \cos\psi_0 + \frac{G(M+m)}{b^2 V_0^2} \qquad 7.7b$$

$$\operatorname{sen}\psi_0 = \frac{\cancel{V_0}}{C\, b\, \cancel{V_0}} \qquad \cos\psi_0 = -\frac{G(M+m)}{b^2 V_0^2 \cdot C}$$

$$\operatorname{tg}\psi_0 = -\frac{b\, V_0^2}{G(M+m)} \qquad 7.8$$

$$|\Delta \vec{V}_\perp| = V_0 \operatorname{sen}\theta = V_0 \operatorname{sen}(2\psi_0 \pm \pi) = V_0 |\operatorname{sen}(2\psi_0)| =$$

$$= \frac{2 V_0 |\operatorname{tg}\psi_0|}{1 + \operatorname{tg}^2\psi_0} \underset{\text{ep. 7.8}}{=} \frac{2\, b V_0^3}{G(M+m)} \left[1 + \frac{b^2 V_0^4}{G^2(M+m)} \right]^{-1} \qquad 7.9^{e}_{b}$$

$$|\Delta \vec{V}_{\parallel}| = V_0 - V_0 \cos\theta = V_0(1 - \cos\theta) = V_0(1 + \cos(2\psi_0)) =$$

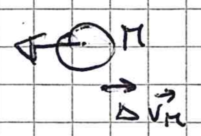
$$= \frac{2 V_0}{1 + \operatorname{tg}^2\psi_0} = 2V_0 \left[1 + \frac{b^2 V_0^4}{G^2(M+m)^2} \right]^{-1}$$

$\Delta \vec{V}_{\parallel}$ points in the direction opposite to $\vec{V}_0$

see Fig.!

$$\Delta \vec{V}_M = -\left(\frac{m}{M+m}\right) \Delta \vec{V} \quad \xleftarrow{7.3}$$

$\Delta \vec{V}_{M\parallel}$ points in the same direction of $\vec{V}_0$



M is slowed down

notice

$$\operatorname{sen} 2\alpha = 2 \operatorname{sen}\alpha \cos\alpha = 2 \operatorname{tg}\alpha \cos^2\alpha = \frac{2 \operatorname{tg}\alpha \cos^2\alpha}{\operatorname{sen}^2\alpha + \cos^2\alpha} = \frac{2\operatorname{tg}\alpha}{\frac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\cos^2\alpha}} =$$

$$= \frac{2\operatorname{tg}\alpha}{1 + \operatorname{tg}^2\alpha}$$

$$1 + \cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha + 1 =$$

$$= 2\cos^2\alpha = \frac{2}{\dots}$$

7,8 a and b →

$$|\Delta \bar{V}_{M\perp}| = \frac{2 m b V_0^3}{G(M+m)^2}\left[1 + \frac{b^2 V_0^4}{G^2(M+m)^2}\right]^{-1} \qquad 7.10_a$$

$$\boxed{|\Delta \bar{V}_{M\parallel}| = \frac{2 m V_0}{M+m}\left[1 + \frac{b^2 V_0^4}{G^2(M+m)^2}\right]^{-1}} \qquad 7.10_b$$

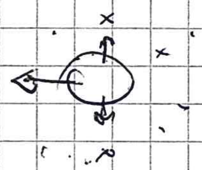
more precise
Than Moumom eq.

M is travelling throug an infinite
homogeneus sea of stars

Jeans swindle → we neglect the gravitat. potent.
of these stars

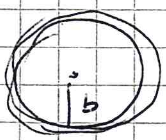
→ motion of each star
is determined only by the gravit. force of M

$$\sum \Delta \vec{V}_{M\perp} = \underset{\text{zero}}{\phi}$$



$$\sum \Delta \vec{V}_{M\parallel} \neq \phi$$

M is decelerated ! dynamical friction



stars phase space number density
$f(\bar{v})$ independent of $\bar{x}$ since homogeneus

the rate of encounters of M with stars with
$\vec{V}$ in $d^3\bar{V}_m$ is

$$2\pi b\, db \cdot V_0 \cdot f(\bar{V}_m)\, d^3\bar{V}_m \qquad 7.11$$

stars with $\bar{V}_m$ and b

stars with $\bar{v}_m$

change of $\bar{V}_M$ due to these encounters over all b

$$\left.\frac{d\bar{V}_M}{dt}\right|_{v_m} = \bar{V}_0 f(\bar{v}_m)\, d^3\bar{v}_m \int_0^{b_{max}} |\Delta \bar{V}_{M\parallel}| \cdot 2\pi b\, db =$$

$$= \bar{V}_0 f(\bar{v}_m)\, d^3\bar{v}_m \int_0^{b_{max}} \frac{2m V_0}{M+m}\left[1+\frac{b^2 V_0^4}{G^2(M+m)^2}\right]^{-1} 2\pi b\, db$$ 7.12

new variable

$$x = \frac{b V_0^2}{G(M+m)} \qquad \lambda = \frac{b_{max} V_0^2}{G(M+m)} \quad \boxed{7.13b}$$

$$\left.\frac{d\bar{V}_M}{dt}\right|_{\bar{v}_m} = f(\bar{v}_m)\, d^3\bar{v}_m\, 2\pi G^2 m(M+m) \frac{\bar{V}_0}{V_0^3} \int_0^{\lambda} \frac{2V_0^4\, b\, db}{(M+m)^2 G^2} (1+x^2)^{-1} =$$

$$= 2\pi G^2 m(M+m) f(\bar{v}_m)\, d^3\bar{v}_m \underbrace{\left(\int_0^{\lambda} \frac{2x\, dx}{(1+x^2)}\right)}_{\ln(1+\lambda^2)} \frac{(\bar{v}_m - \bar{V}_M)}{|\bar{v}_m - \bar{V}_M|^3}$$

$$= 2\pi \overbrace{\underbrace{\ln(1+\lambda^2)}_{\sim 2 \ln\lambda}}^{4\pi \ln \lambda} G^2 m(M+m) f(\bar{v}_m)\, d^3\bar{v}_m \frac{(\bar{v}_m - \bar{V}_M)}{|\bar{v}_m - \bar{V}_M|^3}$$

$\sim 2 \ln\lambda$ — Coulomb logarithm since λ is very large $\boxed{7.13e}$

EX, motion of a globular cluster
$M \sim 10^6 M_\odot$ with $|\bar{V}_0| \sim 100$ km/s
through a galaxy of effective radius b_{max} 2Kpc
with stars $m \sim M_\odot$

$\rightarrow \lambda \sim 4.6 \cdot 10^3$

For $\ln\lambda \sim$ const 7.13e says:

stars with $\bar{v}_m$ exert a force on M
which is parallel to $\bar{v}_m - \bar{V}_M$ and is
inversely proportional to the $|\bar{v}_m - \bar{V}_M|^2$

all stars!

We can obtain $\boxed{\frac{d\bar{V}_M}{dt} = \int \frac{d\bar{V}_M}{dt}\Big|_{v_m} d^3 v_m}$ using (space of velocities)

the analogy with the gravitational field (space of positions)

→ force field generated by $\rho(\bar{v}_m)$

→ gravitational field generated by $\rho(x)$

REMEMBER

$$\bar{F}(\bar{x}) = G \int \frac{\bar{x}' - \bar{x}}{|\bar{x}' - \bar{x}|^3} \rho(\bar{x}')\, d^3\bar{x}'$$

$$= \int \delta\bar{F}(\bar{x}) \qquad \delta\bar{F}(\bar{x}) = \boxed{G}\frac{(\bar{x}' - \bar{x})}{|\bar{x}' - \bar{x}|^3} \rho(\bar{x}')\, \delta^3\bar{x}'$$

in the spherical case

$$\bar{F}(\bar{r}) = -\frac{GM(r)}{r^2}\hat{r} = -\frac{G}{r^2}\frac{\bar{r}}{r}\underbrace{\int_0^r \boxed{4\pi}\rho\, r'^2\, dr'}_{M(r)}$$

$$\delta\bar{F}(r) = +\rho(r)\boxed{G}\frac{\bar{r}}{r^2\, r}$$

$$\frac{d\bar{V}_M}{dt}\Big|_{\bar{v}_m}$$

$$7.13_e \rightarrow \frac{d\bar{V}_M}{dt}\Big|_{\bar{v}_m} = \underbrace{4\pi \ln\Lambda\, G m^{(M+m)} \rho(\bar{v}_m)}\boxed{G}\frac{(\bar{v}_m - \bar{V}_M)}{|\bar{v}_m - \bar{V}_M|^3} d^3\bar{v}_M$$

$|\bar{v}_m|$

$$\boxed{\frac{d\bar{V}_M}{dt}} = -16\pi^2 \ln\Lambda\, G^2 m(M+m) \frac{\int_0^{V_M} \rho(v_m)\, v_m^2\, dv_m}{v_M^3}\bar{V}_M \qquad \boxed{7.14}$$

$(4\pi \cdot \boxed{4\pi})$

That is only stars with $v_m < V_M$ contribute to the force

→ Chandrasekhar dynamical friction

7

if v_M is small $\quad f(v_m)$ in $\int \longrightarrow f(v_m) = f(0)$

and $\int \_\_\_ = f(0) \dfrac{v_M^3}{3}$

$$\frac{d\bar{v}_M}{dt} \sim -\frac{16\pi^2}{3} \ln\Lambda\, G^2 f(0)\, m(M+m)\bar{v}_M \qquad \boxed{7.15}$$

force $\propto v_M$ "force ritondante" $\propto v$

if v_M enough large $\quad \int \sim \rightarrow$ definite limit

force $\propto v_M^{-2}$

Particular case $\quad f(v_m)$ is Maxwellian $\quad f = \dfrac{n_0}{(2\pi\sigma^2)^{3/2}} e^{-\frac{v^2}{2\sigma^2}}$

7.14 $\rightarrow$ 7.17 see note 8 bis

$$\frac{d\bar{v}_M}{dt} = -\frac{4\pi \ln\Lambda\, G^2 (M+m) n_0 m}{v_M^3}\left[\operatorname{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2}\right]\bar{v}_M \qquad \boxed{7.17}$$

$X = \dfrac{v_M}{\sqrt{2\sigma^2}}$

Limit $M \gg m \quad$ 1) force $\propto$ mass density $n_0 m = \rho$ (independent of m)

$M \cdot n_0 m$

2) force $\propto M \cdot M \propto M^2 \longrightarrow$ OBS! velocity segregation in [gals] clusters ! only for big galaxies

Points to be discussed

① if M is extended ?

→ formula is a (small) overestimation

see of stars is not homogeneous and infinite (seems swindle)

1) choice of b_{max} is arbitrary)

2) No self-gravity of stars

3) Keplerian motion (hyperbolae) $\quad$ sims

$$\tau_{df} = \left\langle -\frac{1}{v}\frac{dv_M}{dt}\right\rangle^{-1} \text{ time of dynamical friction}$$

FROM EQ. 7.14 → 7.17

$$\frac{d\bar{V}_M}{dt} = -16\pi^2 \ln\Lambda\, G^2 m(M+m)\left[\int_0^{V_M} \frac{m_0}{(2\pi\sigma^2)^{3/2}} e^{-\frac{V^2}{2\sigma^2}} V_m^2\, dV_m\right] \frac{\bar{V}_M}{V_M^3} =$$

$$= -16\pi^2 \ln\Lambda\, G^2 m(M+m)\, m_0 \left[\int \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{V^2}{2\sigma^2}} \frac{V_m^2}{2\sigma^2} \cdot 2\sigma^2 \cdot d\left(\frac{V_m}{\sqrt{2}\sigma}\right)\sqrt{2}\sigma\right] \cdot \frac{\bar{V}_M}{V_M^3} =$$

$$x = \frac{V_M}{\sqrt{2}\sigma}$$

$$= -16\pi^2 \ln\Lambda\, G^2 m(M+m)\, m_0 \left[\underbrace{\sqrt{\frac{4\cdot 2\cdot \cancel{\sigma^6}}{8\pi^3 \cancel{\sigma^6}}}}_{\frac{1}{\pi}\cdot\frac{1}{\sqrt{\pi}}} \int_0^x e^{-x'^2} x'^2\, dx'\right]$$

$$\frac{1}{\pi}\cdot\frac{1}{\sqrt{\pi}} \left(-\frac{1}{2}\right)\int_0^x \underbrace{e^{-x'^2}(-2x')}_{f'} \underbrace{x'}_{g}\, dx'$$

$$\int f'g = fg - \int fg'$$

$$-\frac{1}{4}\cdot 2 \cdot \left(\frac{1}{\sqrt{\pi}}\left| e^{-x'^2}(-2x')\right|_0^x - \frac{1}{\sqrt{\pi}}\int_0^x e^{-x'^2}\cdot 1\, dx'\right) =$$

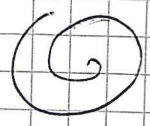
$$= 4\pi \ln\Lambda\, G^2 m_0\, m(M+m) \cdot \frac{\bar{V}_M}{V_M^3}\left[\underbrace{\frac{2}{\sqrt{\pi}}\int_0^x e^{-x'^2} dx}_{\mathrm{erf}(x)} - \frac{2x}{\sqrt{\pi}} e^{-x^2}\right]$$

error function

eq. 7.17

APPLICATIONS OF DYNAMICAL FRICTION

(a) Decay of globular cluster



lose energy → spiral motion (more tangential orbits) → toward galaxy center

$\tau_{OD} \sim 2\,\tau_{DF}$

orbital decay

$r_i = r(0)$

$$\tau_{DF} \sim \frac{2.6 \cdot 10^{11}}{\ln \Lambda} \left(\frac{r_i}{2\,\mathrm{kpc}}\right)^2 \left(\frac{v_c}{250\,\mathrm{km/s}}\right) \left(\frac{10^6 M_\odot}{M}\right) \mathrm{yr}$$

→ typical effective radius

$\ln \Lambda \sim 10$

M31: the most luminous GC ... date

if $r_i \sim 3\,\mathrm{kpc}$ it should be already have spiraled into the center

⟶ M31 nucleus formed of debris of GCs

(b) same fate for Magellanic clouds

(c) galactic cannibalism

→ formation of cD in galaxy clusters

in fact 1) multiplied nuclei in cD

2) sometime dumbbell galaxies 

3) deficit of gals close to cD?

Δm_{12} in fossil group?

From Richstone

gals in clusters

background density

$$\tau_{df} \sim 0{,}3 \cdot 10^{10}\ \text{yr} \left(\frac{v}{10^3\ \text{km/s}}\right)^3 \left(\frac{M}{10^{2} M_\odot}\right)^{-1} \left(\frac{\rho_b}{10^{-3} M_\odot/\text{pc}^3}\right)^{-1}$$

(qui $H_0 = 100$)

in the core

if $\tau_{df} < \tau_{universe} \sim 1{,}3 \cdot 10^{10}$ yr

df is important, is more important in the core for $M > 10^{12} M_\odot$

other

core radius

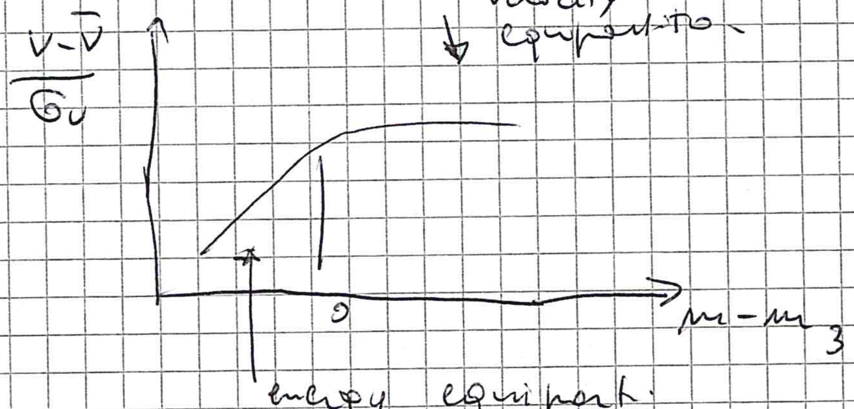
$$\tau_{df} \sim 6 \cdot 10^{9} \left(\frac{\sigma_v}{10^3\ \text{km/s}}\right)\left(\frac{r_0}{0{,}125\, h^{-1}\ \text{Mpc}}\right)\left(\frac{L}{L^*}\right)^{-1}\left(\frac{M/L}{10\ \frac{M_\odot}{L_\odot}}\right)^{-1}$$

$$r_0^2 = \frac{9\sigma_v^2}{4\pi G \rho_0}$$

OBS.

τ_{df} important for very massive galaxies

See **slides**



dynamical friction