BT1

7.3 TIDAL RADII

one expects that repeated (tidal) perturbations of the velocities of stars in a stellar system
⇒ the orbits to diffuse in the phase-space
cause

⇒ to erase all sharp features in the spatial distribution

BUT ... we see sharp edges in globular clusters and cluster galaxies

Why? This is an effect of the large-scale gravitational field of the galaxy or galaxy cluster which host globular cluster and galaxies

host system — satellite system (⟵ inside)

→ Tidal radius r_t (used in King models)

satellite system (m) on a circular orbit assume beyond of the outer edge of the host system (M)

spherical systems — distance between centers is D

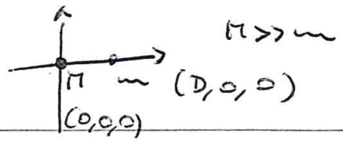
$\Omega^2 r^2 = \frac{Gm}{r}$

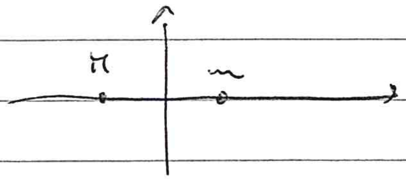
angular speed with which the systems orbit around their common center

$$\Omega^2 r = \frac{v_c^2}{r} = \frac{Gm}{r^2} \qquad \Omega = \sqrt{\frac{Gm}{r^3}}$$

$$\Omega = \sqrt{\frac{G(M+m)}{D^3}} \qquad \boxed{7.78}$$

$$\bar{D} = \bar{x}_m - \bar{x}_M$$





$\bar{x}_m = [D(1+\frac{m}{M})^{-1}, 0, 0]$. $\bar{x}_M = [-D(1+\frac{M}{m})^{-1}, 0, 0]$

orbits on steadily rotating systems ($\Omega \sim$ cons)

Jacobi's integral is a constant $C(\bar{x}_1, \bar{v}_1, t_1) \equiv C(\bar{x}_2, \bar{v}_2, t_2)$

$E_J = \frac{1}{2} v^2 + \phi(\bar{x}) - \frac{1}{2} |\bar{\Omega} \times \bar{x}|^2$

↳ potential energy which cause the centrifugal force

$\bar{\Omega} = (0, 0, \Omega)$

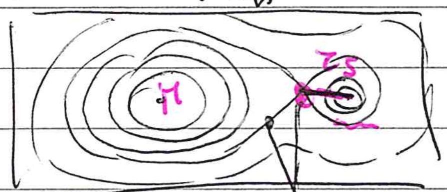
$= \frac{1}{2} v^2 + \phi_{eff}(\bar{x})$ effective

Since $v^2 \geq 0$ a star with E_J never trespass into a region where $\phi_{eff}(\bar{x}) > E_J$ never!

surface $\phi_{eff}(\bar{x}) = E_J$ is called zero velocity surface

fig 7.8

Roche lobe

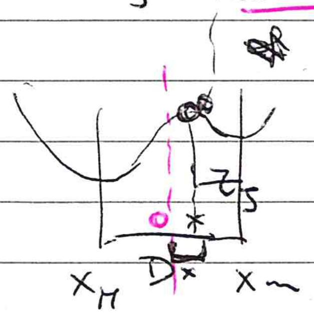


zero velocities surface of ϕ_{eff}
small r, centered on m
large r surface surrounds both bodies

critical value of E_J last surface enclosing only m

Tidal radius $\equiv r_J$ saddle point (punto di sella)

$\bar{D} = \bar{x}_m - \bar{x}_M$



$\left(\frac{\partial \phi_{eff}}{\partial x}\right)_{\bar{x} = \bar{x}_m - \bar{r}_J} = 0$

for 7.78

$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 1 \\ x & 0 & 0 \end{vmatrix} =$

↳ $x_m - r_J$

$\phi_{eff}(\bar{x}) = -G\left[\frac{M}{|\bar{x} - \bar{x}_M|} + \frac{m}{|\bar{x} - \bar{x}_m|}\right] + \frac{1}{2}\frac{M+m}{D^3}|\hat{e}_z \times \bar{x}|^2 =$

$\hat{k}$

$= \hat{\jmath} x$

$$\cancel{\phi} = -G\left[\cancel{\frac{M}{|x_m - r_J - x_M|}}\right]$$

$$x = \frac{x_m - r_J}{D\left(1+\frac{m}{M}\right)^{-1} - r_J}$$

$$= -G\left[\frac{M}{|\bar{x}-\bar{x}_M|} + \frac{m}{|\bar{x}-\bar{x}_m|} + \frac{1}{2}\frac{M+m}{D^3}\cdot x^2\right]$$

$$\left\{ \bar{\nabla}_x \frac{1}{|\bar{x}'-\bar{x}|} = -\frac{1}{2}\frac{1}{[(\bar{x}'-\bar{x})^2]^{3/2}}\cdot 2(\bar{x}'-\bar{x}) = -\frac{\bar{x}'-\bar{x}}{|\bar{x}'-\bar{x}|^3}\right.$$

$$\to 2x \qquad (x_m - r_J)$$

r_J positive

$$\frac{1}{G}\left(\frac{\partial \phi_{eff}}{\partial x}\right)_{x_m - r_J} = +\frac{M}{(\bar{x}_m - r_J - \bar{x}_M)^2} \mp \frac{m}{(\bar{x}_m - r_J - \bar{x}_m)^2} - \frac{1}{2}\cdot 2\frac{M+m}{D^3}\left(\frac{D}{1+\frac{m}{M}} - r_J\right)$$

r_J negative

$= 0$ punto di sella

$$\frac{M}{(D-r_J)^2} \mp \frac{m}{r_J^2} - \frac{M+m}{D^3}\left(\frac{D}{1+\frac{m}{M}} - r_J\right) = 0$$

$$0 = M(D-r_J)^{-2} \mp \frac{m}{r_J^2} - \frac{M+m}{D^3}\cdot\frac{MD}{M+m} + \frac{M+m}{D^3} r_J$$

if $m \ll M$ $\quad r_J \ll D$

$(D-r_J)^{-2}$ can be expanded in powers of $\frac{r_J}{D}$

$$(D-r_J)^{-2} = \frac{1}{D^2}\left(1-\frac{r_J}{D}\right)^{-2} = \frac{1}{D^2}\left(1+2\frac{r_J}{D}+\dots\right)$$

$$0 = \frac{M}{D^2}\left(1+2\frac{r_J}{D}+\dots\right) \mp \frac{m}{r_J^2} - \frac{M}{D^2} + \frac{M+m}{D^3} r_J \qquad \boxed{7.83}$$

at the first order in $\frac{r_J}{D}$

$$0 = \cancel{\frac{M}{D^2}} + \frac{2M r_J}{D^3} \mp \frac{m}{r_J^2} - \cancel{\frac{M}{D^2}} + \frac{M r_J}{D^3} + \frac{m}{D^3} r_J$$

$$\left(\frac{3M+m}{D^3}\right) r_J \mp \frac{m}{r_J^2} = 0$$

$$r_J^3 = \pm \frac{m D^3}{3M+m}$$

$$\boxed{r_J = \pm D \left(\frac{m}{M\left(3+\frac{m}{M}\right)}\right)^{1/3} \sim \pm \left(\frac{m}{3M}\right)^{1/3} D} \qquad (7.84)$$

Jacobi limit of the mass m

→ estimate of r_T tidal radius

only approximation!

i) zero velocity surface is not spherical

ii) a star can be confined also if it zero-velocity surface does not close around m (Sim...)

iii) m is non on circular orbit

iv) satellite orbits involve the body of the host

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x galaxy clusters

a galaxy around the center on circular orbit $r_p \lesssim 0,5 \frac{\sigma_*}{\sigma_{gal}} R_0$ → cluster core radius

$\sigma_{gal} \uparrow$ $r_p \downarrow$

→ legato a masse!

only galaxy at the center are not limited

cD galaxy!

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