Condensed Matter Physics I
I test - 12 November 2009
(2 hours 30')

- Solve all the exercises, corresponding to a total maximum score of 36. If the score is between 33 and 36 it is considered equal to 30/30 *cum laude*, if it is between 30 and 32 it is considered equal to 30/30.
- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise: *Free electron gas*

1. The expression of the density of states at the Fermi energy in terms of the Fermi energy E_F and electron density n for a 3D free electron gas (Sommerfeld model) is (Eq. (2.65) of the texbook): $g(E_F) = 3n/(2E_F)$. What is the analogous expression in 1D?
2. And in 2D?
3. Consider from now on the 2D case. Show that the expression of the chemical potential $\mu(n, T)$ as a function of temperature T and electron density n is:
$$\mu = k_B T ln[e^{\beta n/g_0} - 1] \qquad (\beta = (k_B T)^{-1})$$
where $g_0 = g(E_F)$.
4. Calculate from this expression the limit for $T \to 0$ and comment (is it what do you expect?)
5. What's the behaviour for increasing T, and in particular for very high T?
6. Consider now another two dimensional free fermionic gas, made of ^3He atoms instead of electrons. Let's extend to it use again the Sommerfeld model. Given that m = 4.8 10^{-27} kg, $\mu(T = 0) = E_F = 10^{-22}$ J, how many atoms there are within a circle of radius of 10 cm?

Exercise: *Crystalline structures*

1. Determine the atomic density in the crystalline planes (001), (110) and (111) in the BCC structure as a function of the lattice parameter (side of the cubic cell) a_0.
2. Given that Fe has a BCC structure with a_0=2.86 Å, calculate explicitely the atomic density in the (110) plane.
3. Calculate the packing fraction of BCC structure.

Exercise: *Diffraction: Atomic form factors and Structure factors*

1. Calculate the geometrical structure factor for diamond.
2. Show that for a proper choice of the origin, it becomes real.
3. The electronic density of atomic H in the ground states is

$$n(r) = \frac{1}{\pi a_B^3} e^{\frac{-2r}{a_B}}$$

   where a_B is the Bohr radius). Show that its atomic form factor is: $f_H(K) = 16/(4 + K^2 a_B^2)^2$.

Useful notes:

$$\int_0^\infty x e^{-ax} \sin(bx) dx = \frac{2ab}{(a^2 + b^2)^2} \quad (a > 0)$$