Electrons in crystals
Final written test
academic year 2008/2009
January 26, 2009

(*Time: 3 hours*)

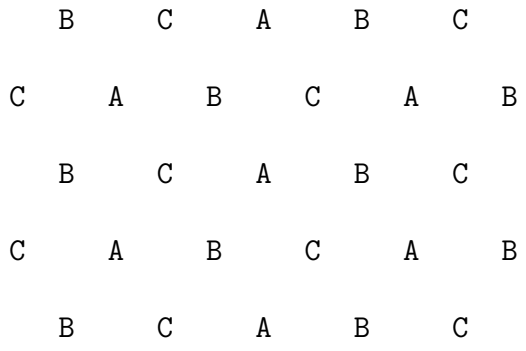
Exercise 1: *Drude and Sommerfeld model of free electron gas*

Copper has a mass density $\rho = 8.95$ g/cm^{-3}, and electrical resistivity $1.56 \cdot 10^{-8}$ ohm·m at room temperature. Assuming that the effective mass of electron in Cu $m^* = m_0$, (m_0 is the free electron mass) calculate:

1. The concentration of the conduction electrons and the mean relaxation time τ
2. The Fermi energy E_F and the Fermi velocity v_F
3. The mean free path ℓ_F at the Fermi level l_F

Exercise 2: *Bravais lattices with basis and structure factors*

Consider the 2D lattice in the figure, constituted by 3 different atomic species A, B and C, occupying sites of a triangular lattice. Let d be the AB distance.



1. Describe and sketch the primitive cell, write the formula unit ($A_nB_mC_\ell$ with m, n, ℓ=...?), a possible choice of the primitive translation vectors $\{\mathbf{a}_i\}$ and the basis vectors $\{\mathbf{d}_i\}$.
2. Write the primitive translation vectors of the reciprocal lattice $\{\mathbf{b}_i\}$.
3. Assuming that the atomic form factors are f_A, f_B and f_C, calculate the geometrical structure factor S($\mathbf{K}$) on the reciprocal lattice vectors, and specify the conditions that need to be satisfied to have interference maxima in case of $f_A = f_B = f_C$.

Exercise 3: *Electrons in 2d: Fermi surfaces, weak potential*

1. Consider a two-dimensional metal with rectangular Bravais lattice with basis vectors $\mathbf{a_1}$=(a,0) and $\mathbf{a_2}$=(0,b), with a=2Å and b=2a. Give the basis vectors $\mathbf{b_1}$ and $\mathbf{b_2}$ of the reciprocal lattice (using cm^{-1}) and sketch the first Brillouin zone.

2. Consider free electrons, give the general expression of the Fermi "sphere" (or better, "circle", in 2d) as a function of the electron density; then, calculate explicitely the numerical value of its radius in the present case, considering one electron per unit cell.

3. Draw such free-electron Fermi "sphere" in the same (k_x, k_y) plot of the first Brillouin zone, showing and specifying whether it is totally within the first Brillouin zone or not. If not, make a sketch concerning *where* and *how* it would be modified by the presence of a weak periodic potential, and a sketch of a possible second band within the first Brillouin zone.

Exercise 4: *Tight-binding model*

Consider s-type electrons in a 1D lattice with lattice spacing a. Use a *tight binding* model with *first and second neighbor hopping* and no *overlap*. With reference to the definition $\gamma(\mathbf{R})$ in the texbook, consider: $\gamma(\mathbf{R}_{NN})$=t where $\mathbf{R}_{NN}$ is between two nearest beighbors, $\gamma(\mathbf{R}_{NNN})$=t' where $\mathbf{R}_{NNN}$ is between two next- nearest beighbors, otherwise $\gamma(\mathbf{R})$=0.

1. Show that the explicit expression for the energy band (using $E_s - \beta \equiv E_0$ (see again the textbook for the definition of β) is:

$$E(k) = E_0 - 2[t\ cos(ka) + t'\ cos(2ka)]$$

2. Consider $t' = -t$ and make a plot in the first Brillouin zone, identifying and writing the maxima and minima points.

3. Calculate the effective mass in those points.

NOTE:

- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

- When required, numerical evaluations should be given exactly with 3 significant figures, if not otherwise indicated.