Condensed Matter Physics I
final written test
academic year 2012/2013
January 28, 2013

(*Time: 3 hours*)

NOTE: Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: *Crystalline structures*

Consider a 2D solid with primitive lattice vectors $\vec{a}_1 = a(1,0)$ and $\vec{a}_2 = a\left(\frac{1}{2}, 1\right)$.

1. Calculate the area of the primitive cell.
2. Find the primitive vectors of the reciprocal lattice $\vec{b}_1$ and $\vec{b}_2$.
3. Make a *precise* drawing in the reciprocal space of the first and second Brillouin zones, and calculate their areas.
4. Consider now to have a Bravais lattice with the primitive lattice vectors as above and a basis of two atoms within the primitive cell, with $\vec{d}_1 = (0,0)$ and $\vec{d}_2 = \left(\frac{a}{2}, 0\right)$ and atomic form factors f_1 and f_2 respectively. Calculate explicitly the structure factor $S(\vec{K})$.
5. In case of $f_1 = f_2$, on which reciprocal lattice vectors $\vec{K}$ the structure factor is not vanishing? (Give the expression of such $\vec{K}$ vectors)
6. To which structure in the direct space do they correspond? Comment the result.

Exercise 2: *Band structure of a 1D solid*

Consider a 1D band whose energy is given by $E(k) = E_0 - t\cos(ak)$.

1. Calculate explicitly the density of states $g(E)$ and make a plot. Check whether (and, in case, where) $g(E)$ has the expected van Hove singularities.

2. Calculate explicitly the group velocity $v(k)$. Calculate the effective mass $m^*(k)$ as a function of k in the above model and in particular find the values of m^* at the extrema of the band and discuss the character of the corresponding charge carriers.

3. Suppose the band is 1/3 occupied. What is v_F, the group velocity at the Fermi level?

4. Calculate the Fermi energy for 0.5, 1, and 2 electrons per unit cell.

5. For one electron per unit cell, calculate the low-temperature specific heat (per cell)!

6. Consider always a 1D solid, but well described by a free electron picture. Calculate the Fermi energy in case of 1 and 2 electrons per unit cell. Comment his behaviour in terms of conductivity, justifying your answer.