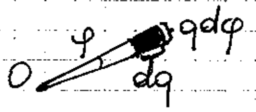
(c) $$g_n(\mathcal{E}) = \int \frac{1}{4\pi^3\sqrt{\alpha_x\alpha_y\alpha_z}}\, d\vec{q}\; \delta\left[\mathcal{E}-\mathcal{E}_0 - q_x^2 + q_y^2 + q_z^2\right]$$

Uso coord. cilindriche nel piano (x,y): $\{q_x, q_y\} \rightarrow \{q, \varphi\}$

$$d\vec{q} = dq_x\, dq_y\, dq_z$$
$$= q\, dq\, d\varphi\, dq_z$$
con $q^2 = q_x^2 + q_y^2$



$$\rightarrow g_n(\mathcal{E}) = \frac{1}{4\pi^3\sqrt{\alpha_x\alpha_y\alpha_z}} \cdot \underbrace{2\pi}_{\int_0^{2\pi} d\varphi} \iint q\, dq\, dq_z\, \delta\left[\mathcal{E}-\mathcal{E}_0 - q^2 + q_z^2\right]$$

$$= \frac{1}{2\pi^2\sqrt{\alpha_x\alpha_y\alpha_z}} \int_0^{\infty} q\, dq \left\{\int_{-\infty}^{+\infty} dq_z\, \delta\left[\mathcal{E}-\mathcal{E}_0 - q^2 + q_z^2\right]\right\}$$

Ved. ex ✱ [formula δ-function]. Si ha:

$$\frac{d}{dq_z}\left[\mathcal{E}-\mathcal{E}_0 - q^2 + q_z^2\right] = 2q_z$$

$$\mathcal{E}-\mathcal{E}_0 - q^2 + q_z^2 = 0 \quad \text{per} \begin{cases} q_z = \pm\sqrt{q^2+\mathcal{E}_0-\mathcal{E}} & \text{se } \mathcal{E}-\mathcal{E}_0 < q^2 \\ \text{NO ZERI} & \text{per } \mathcal{E}-\mathcal{E}_0 > q^2 \end{cases}$$

Allora

viene da $\sum_{\text{zeri}}$, che sono $\pm \ldots$

$$g_n(\mathcal{E}) = \begin{cases} \dfrac{1}{2\pi^2\sqrt{\alpha_x\alpha_y\alpha_z}} \displaystyle\int_0^{\infty} q\, dq\, \frac{\cancel{2}\,\Theta(q^2+\mathcal{E}_0-\mathcal{E})}{\cancel{2}\sqrt{q^2+\mathcal{E}_0-\mathcal{E}}} & \text{per } q^2+\mathcal{E}_0-\mathcal{E} > 0 \\ \emptyset & \text{per } q^2+\mathcal{E}_0-\mathcal{E} < 0 \end{cases}$$

$\rightarrow$ fz. scalino $\Theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$

K piccolo $\leftrightarrow$ q piccolo, allora considero $\int_0^a$, $a \equiv q_{max}$.

Pongo:
$$x \equiv q^2 + \mathcal{E}_0 - \mathcal{E}$$
$$2q\,dq = dx \rightarrow \begin{cases} q = 0 \rightarrow x = \mathcal{E}_0 - \mathcal{E} \\ q = a \rightarrow x = a^2 + \mathcal{E}_0 - \mathcal{E} \end{cases}$$

$$\boxed{g_n(\mathcal{E}) = \frac{1}{4\pi^2\sqrt{\alpha_x\alpha_y\alpha_z}} \int_{\mathcal{E}_0-\mathcal{E}}^{a^2+\mathcal{E}_0-\mathcal{E}} \frac{\Theta(x)\, dx}{\sqrt{x}}}$$

Discutiamo il valore dell' $\int$ a seconda dei limiti di integr.

$\boxed{\mathcal{E}_0 - \mathcal{E} > 0}$ i limiti dell' $\int$ sono $>0 \Rightarrow \Theta(x) \equiv 1$ in quel range $\Rightarrow$

$$g_n(\mathcal{E}) \propto \int_{\mathcal{E}_0-\mathcal{E}}^{a^2+\mathcal{E}_0-\mathcal{E}} \frac{dx}{2\sqrt{x}} = \sqrt{x}\Big|_{\mathcal{E}_0-\mathcal{E}}^{a^2+\mathcal{E}_0-\mathcal{E}}$$

$$\propto \sqrt{a^2+\mathcal{E}_0-\mathcal{E}} - \sqrt{\mathcal{E}_0-\mathcal{E}}$$

$\lim_{\mathcal{E}\to\mathcal{E}_0} g(\mathcal{E}) \propto a$

$$\frac{dg_n(\mathcal{E})}{d\mathcal{E}} \propto \frac{-1}{\sqrt{a^2+\mathcal{E}_0-\mathcal{E}}} + \frac{1}{\sqrt{\mathcal{E}_0-\mathcal{E}}} \underset{\mathcal{E}\approx\mathcal{E}_0}{\sim} -\frac{1}{a} + \frac{1}{\sqrt{\mathcal{E}_0-\mathcal{E}}}$$

trascurabile rispetto all' $\to \infty$

$$\Rightarrow \boxed{\frac{dg_n(\mathcal{E})}{d\mathcal{E}} \sim (\mathcal{E}_0-\mathcal{E})^{-1/2} \quad \text{per } \mathcal{E} < \mathcal{E}_0}$$

$\boxed{\mathcal{E}_0 - \mathcal{E} < 0}$ il limite inf dell' $\int$ è <0, allora l' $\int$ si riduce a:

$$g_n(\mathcal{E}) \propto \int_0^{a^2+\mathcal{E}_0-\mathcal{E}>0} \frac{dx}{2\sqrt{x}} = \sqrt{x}\Big|_0^{a^2+\mathcal{E}_0-\mathcal{E}_0}$$

$$\propto \sqrt{a^2+\mathcal{E}_0-\mathcal{E}}$$

$$\boxed{\frac{dg_n(\mathcal{E})}{d\mathcal{E}} \propto \frac{-1}{\sqrt{a^2+\mathcal{E}_0-\mathcal{E}}} \underset{\mathcal{E}\approx\mathcal{E}_0}{\sim} \frac{1}{a} \quad \text{costante per } \mathcal{E} \to \mathcal{E}_0}$$

$-\sqrt{x_0 - x}$