

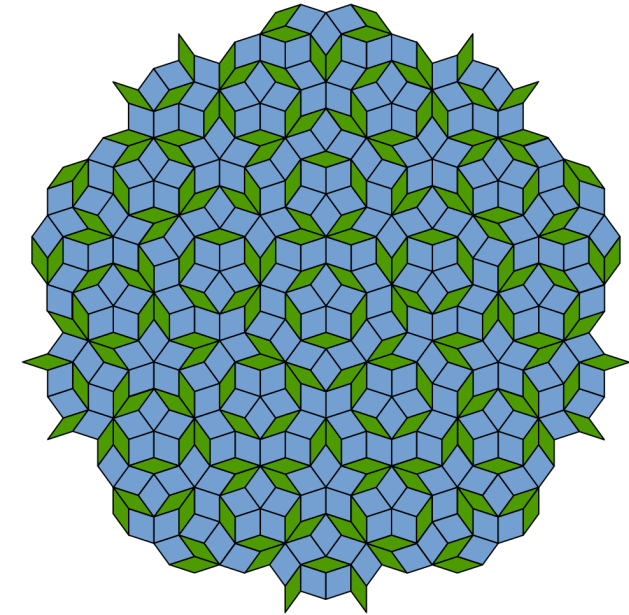
Cristallo : ordine a lungo raggio

- particelle → periodico
 ↘ quasi periodico

⇓
 spettri di diffrazione
 picchi discreti



Zd



tassellazione di Penrose

✗ sistema disordinato:
 no ordine posizionale a lungo raggio

- Spins

Disordine nell'ordine

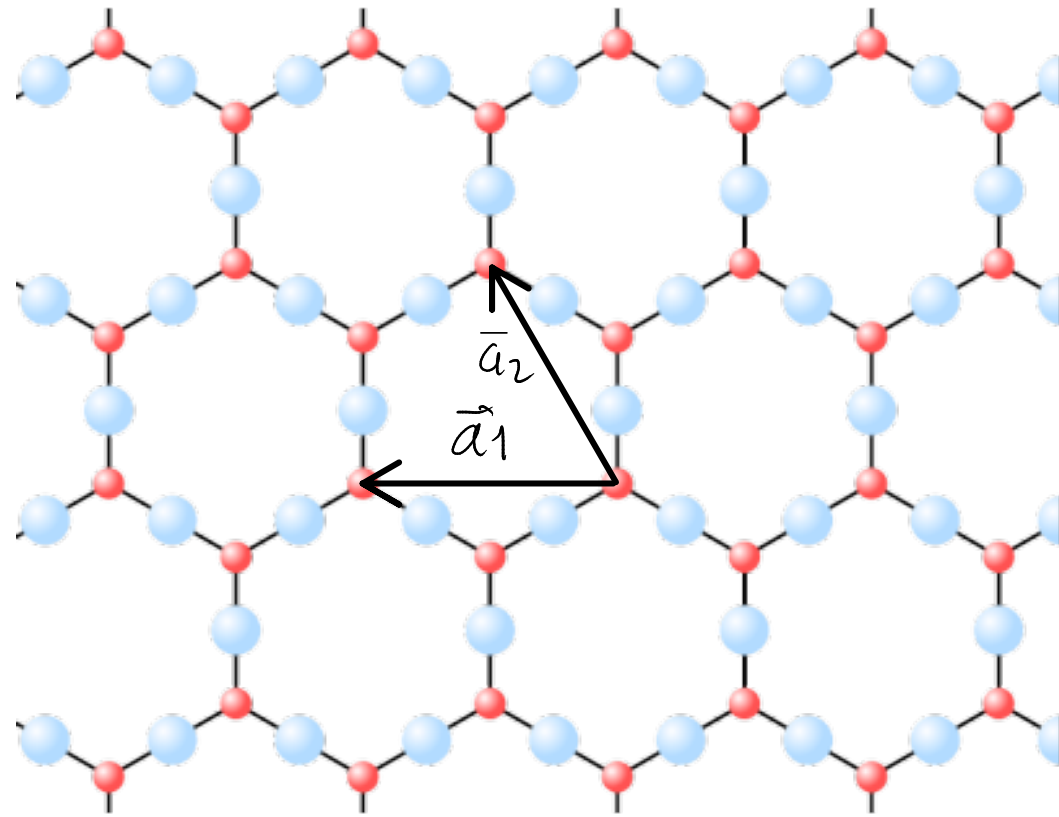
Ordine nel disordine

⚠ casualità ↔ sistema disordinato

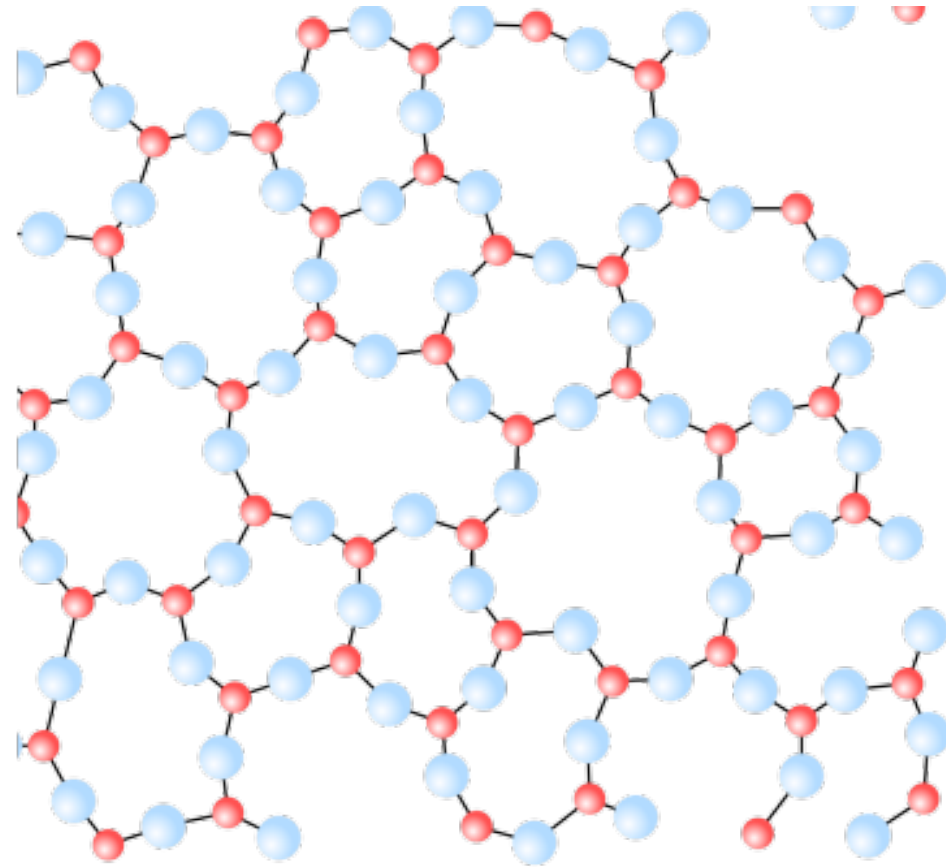
FASI DELLA MATERIA E SIMMETRIE

Simmetria per traslazione discreta : $\vec{r} \rightarrow \vec{r} + \underbrace{l_1 \vec{a}_1 + l_2 \vec{a}_2}_{\vec{R}_l}$ $l_1, l_2 \in \mathbb{Z}$

$$F(\vec{r}) = F(\vec{r} + \vec{R}_l)$$

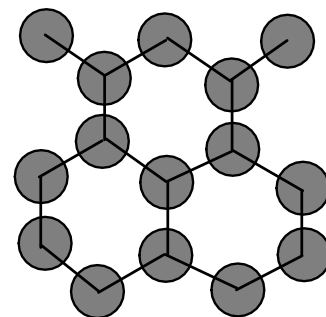
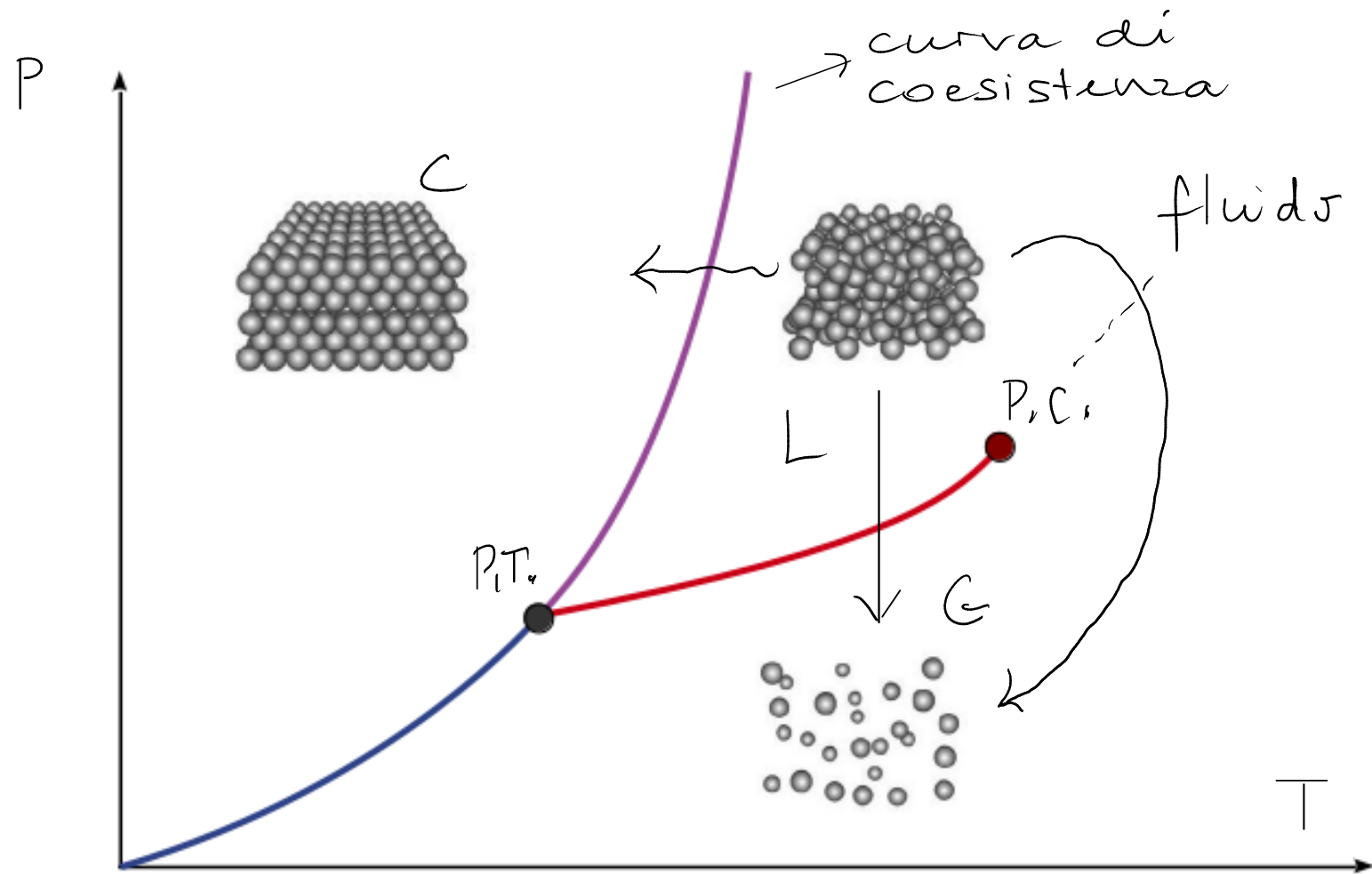


ordinato

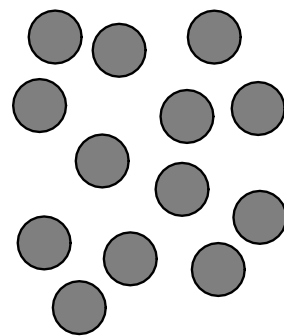


disordinato

Diagramma di fase : Ar



ghiaccio



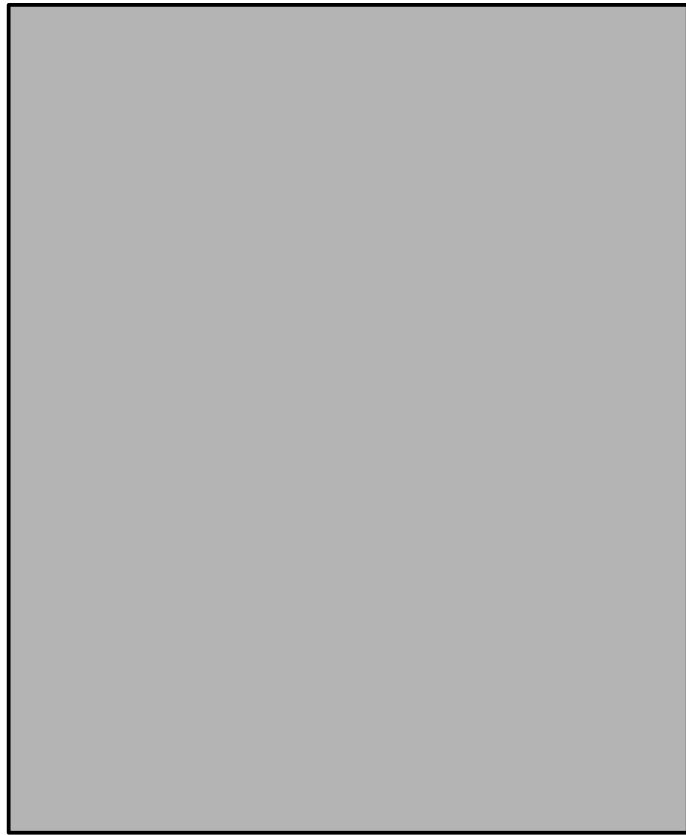
acqua liquida

Media termica:

$$\langle F(\vec{r}) \rangle = \frac{\text{Tr} [e^{-\beta H} F(\vec{r})]}{\text{Tr} [e^{-\beta H}]}$$

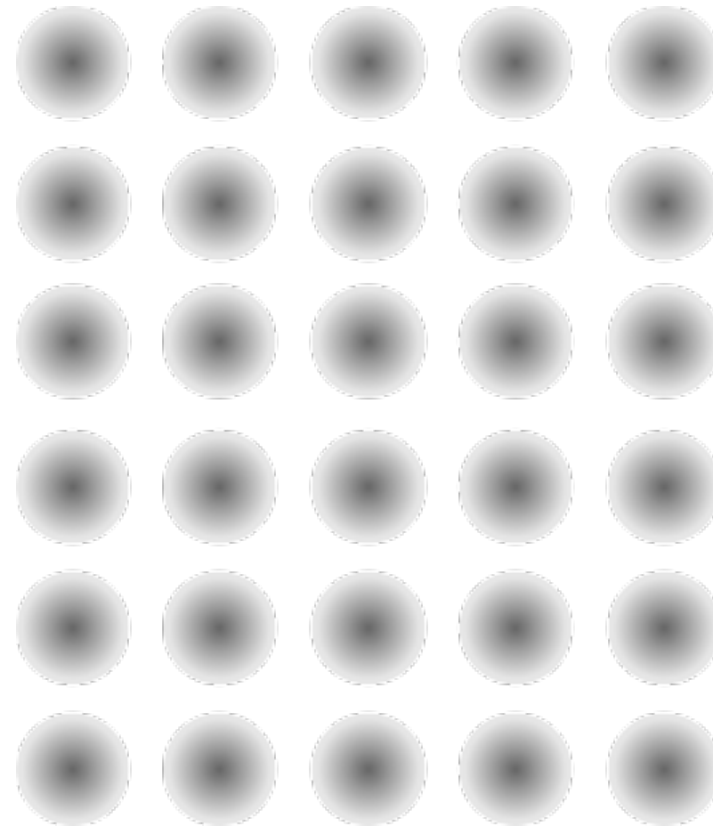
Simmetria traslazione continua :

$$\langle F(\vec{r}) \rangle = \langle F(\vec{r} + \vec{R}) \rangle \quad \forall \vec{R}$$



liquido, gas

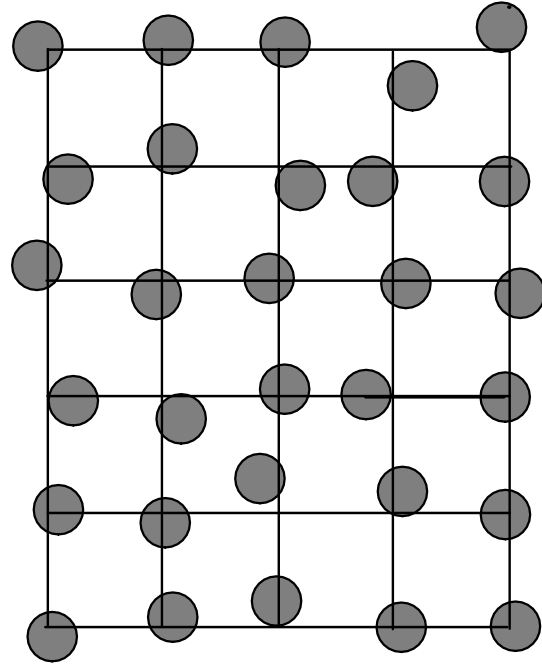
$$\langle F(\vec{r}) \rangle = \langle F(\vec{r} + \vec{R}_e) \rangle$$



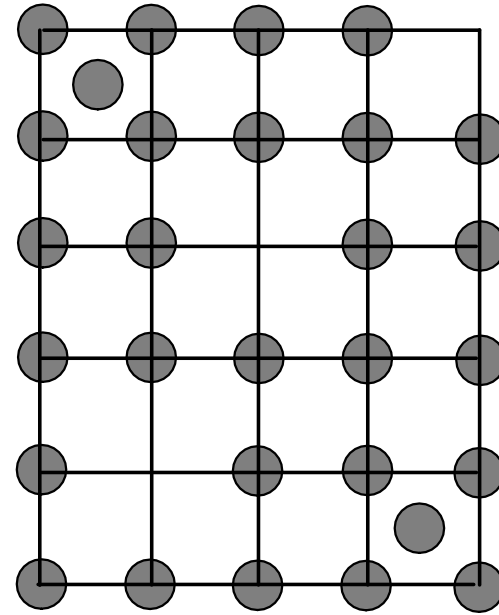
cristallo

TIPOLOGIE DI DISORDINE

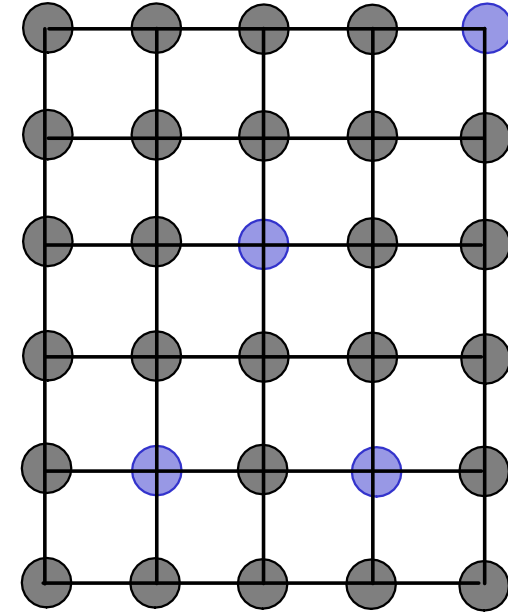
Disordine sostituzionale



crystallo caldo

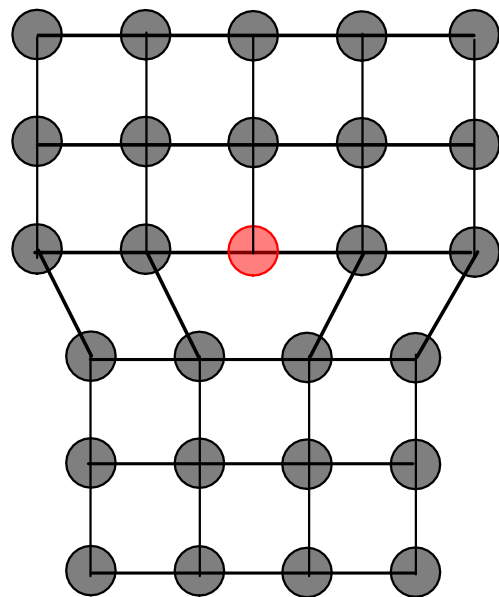


difetti interstiziali
vacanze



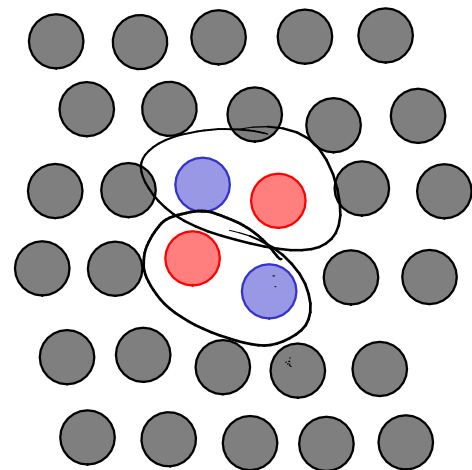
disordine
"chimica"

Disordine topologica

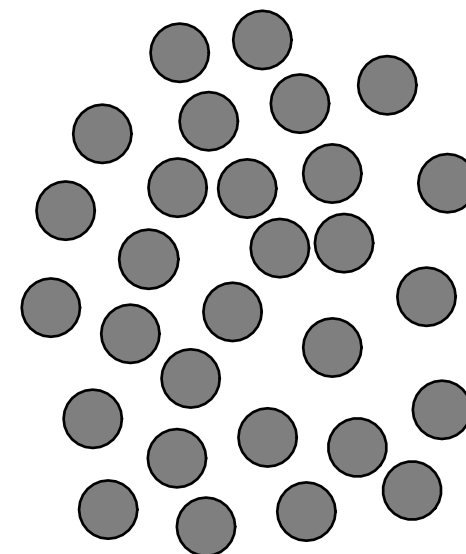


dislocazione

2d

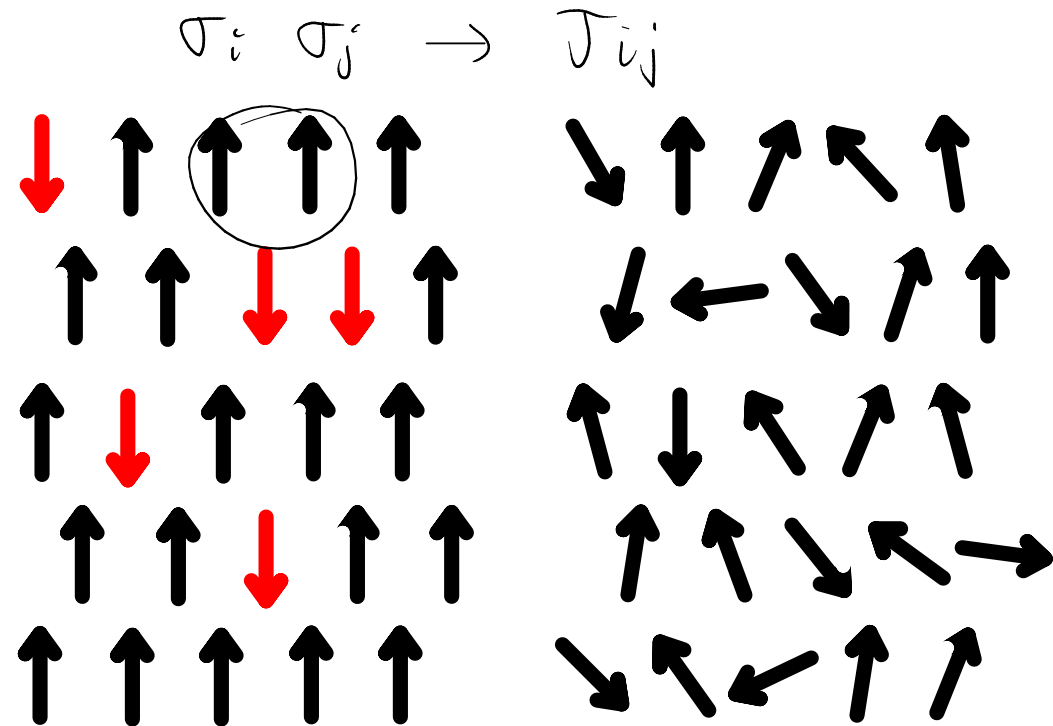


coppia di dislocazioni



gas, liquido, vetro

Disordine gelato "quenched"



vetri di spin

$$H = - \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j$$

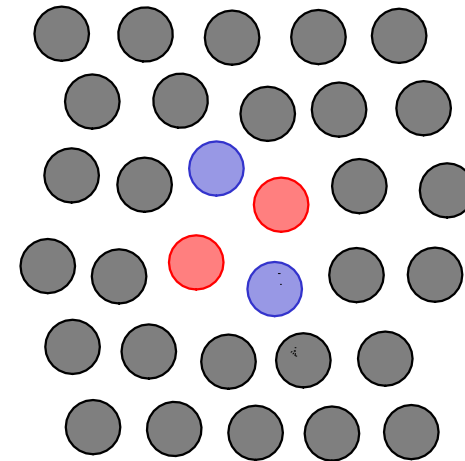
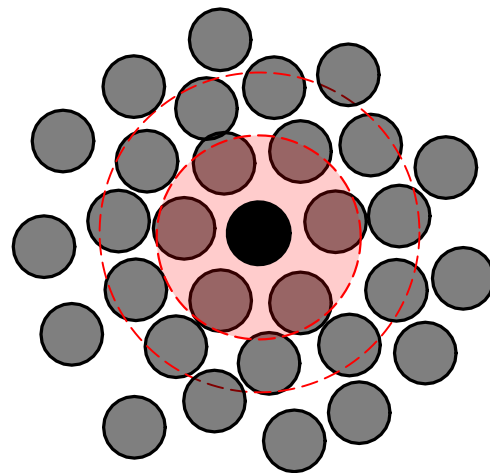
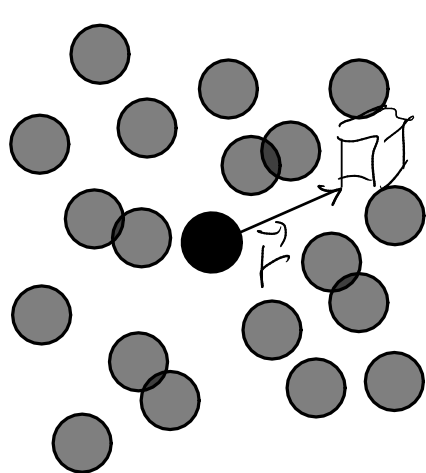
$P(J)$ gaussiana

$\langle \dots \rangle$



fluido in una matrice
porosa

ORDINE E CORRELAZIONI



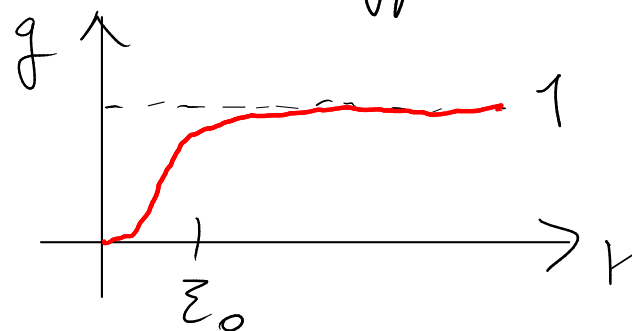
Funzione di distribuzione

$$g(\bar{r}) = \frac{\langle N(\bar{r}) \rangle}{N_{id}}$$

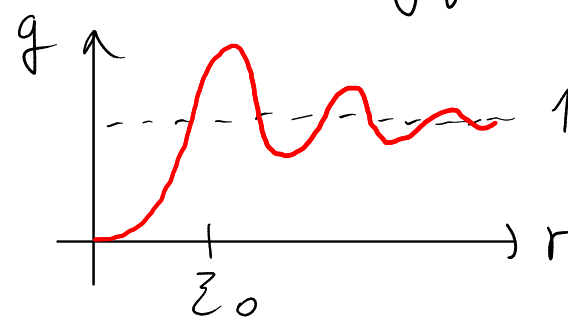
Funzione di correlazione

$$G(\bar{r})$$

Corto raggio



Medio raggio

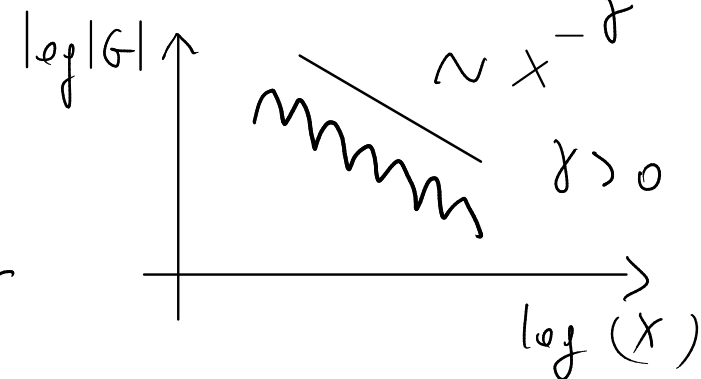
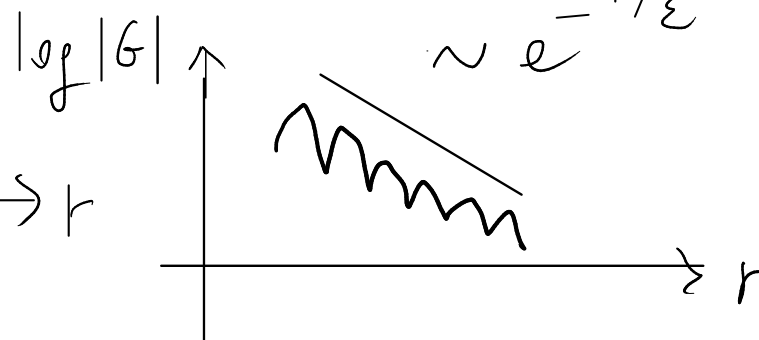
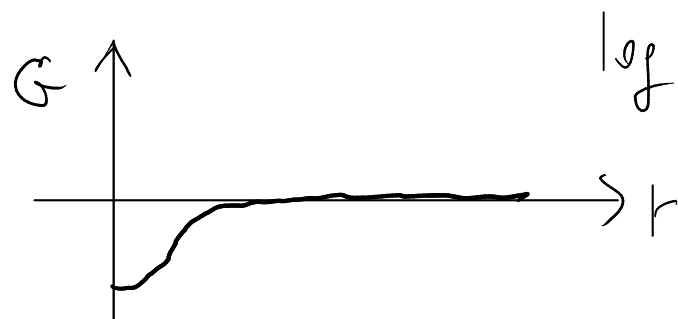


Quasi-lungo

lunghezza di correlazione

$T \rightarrow \infty \quad \xi \rightarrow \xi_0$

$T \rightarrow T_c \quad \xi \rightarrow \infty$

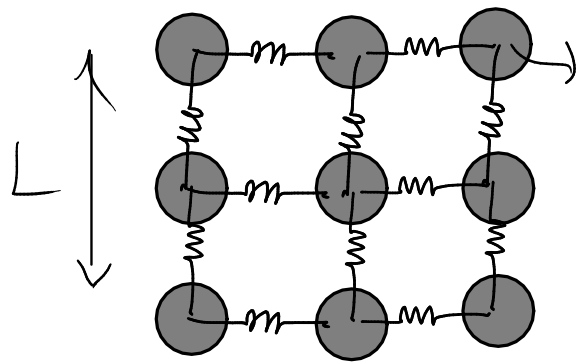


ORDINE E DIMENSIONALITÀ

130-140 : Peierls, Landau

160 : Teor. Mermin - Wagner : Sotto ipotesi abbastanza generali sulla natura delle interazioni, no ordine spontaneo a lungo raggio in 1d e 2d (particelle, spins)

1) **Vibrazioni armoniche** 2d



$\Delta \vec{F}_i$ Spostamento quadratico medio:

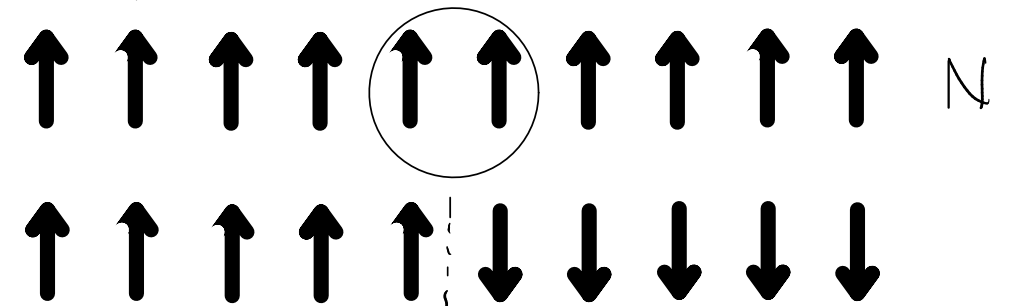
$$\langle |\Delta \vec{F}|^2 \rangle = \frac{1}{N} \sum_{i=1}^N \langle |\Delta \vec{F}_i|^2 \rangle$$

Approx armonica:

$$\langle |\Delta \vec{F}|^2 \rangle \sim K_B T \int_{\frac{2\pi}{L}}^{\infty} \frac{d^d k}{\omega_k^2} \sim \int_{\frac{2\pi}{L}}^{k_c} k^{d-3} dk$$

$$\left. \begin{array}{l} d \geq 3 \rightarrow \text{cost} \\ d = 2 \rightarrow \log L \\ d = 1 \rightarrow L \end{array} \right\} \rightarrow \infty \quad L \rightarrow \infty$$

2) **Spins** 1d $J > 0$



$$\Delta E = J$$

$$\Delta S = K_B \ln N$$

$$\Delta F = \Delta E - T \Delta S$$

$$= J - K_B T \ln N$$



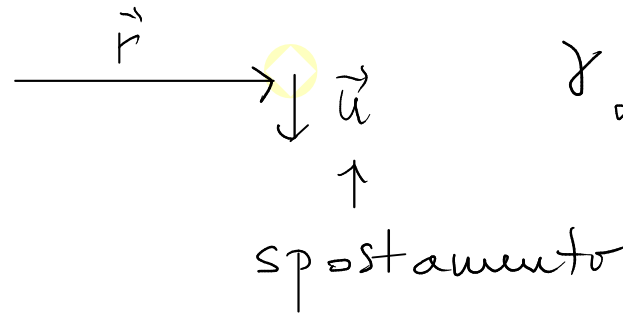
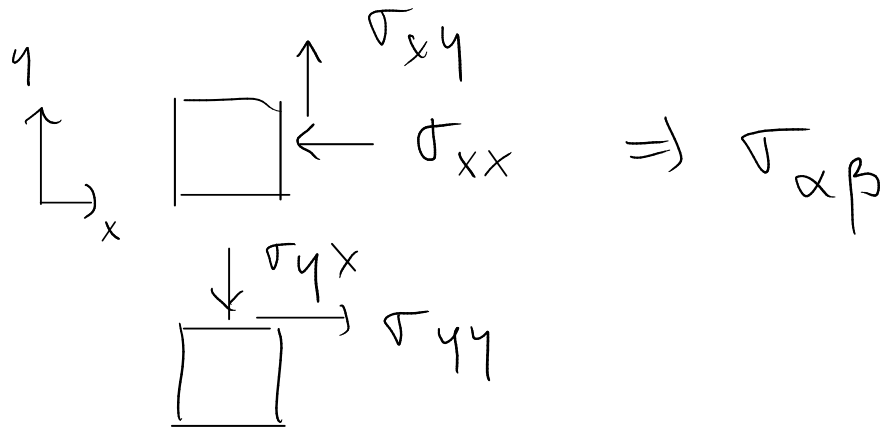
ELASTICITÀ

Sforzo = modulo x deformazione

$$F = k \Delta L \quad (\text{Hooke})$$

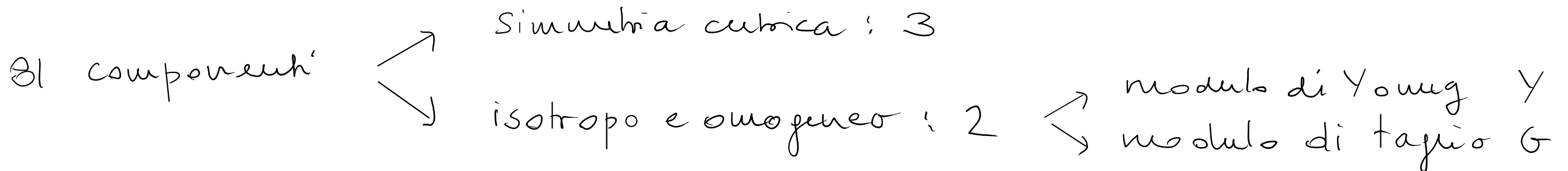
$$\frac{F}{A} = \sigma \quad \downarrow \text{Pa} \quad \text{relativa}$$

\Rightarrow solido Hookiano

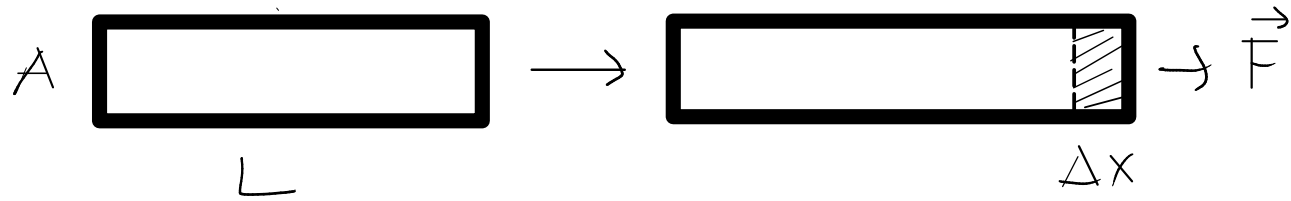


$$\gamma_{\alpha\beta} = \frac{\partial u_\alpha}{\partial r_\beta} \rightarrow \frac{1}{2} \left(\frac{\partial u_\beta}{\partial r_\alpha} + \frac{\partial u_\alpha}{\partial r_\beta} \right)$$

Modulo elastico $C_{\alpha\beta\gamma\theta} \Rightarrow \sigma_{\alpha\beta} = \sum_{\gamma\theta} C_{\alpha\beta\gamma\theta} \gamma_{\gamma\theta}$



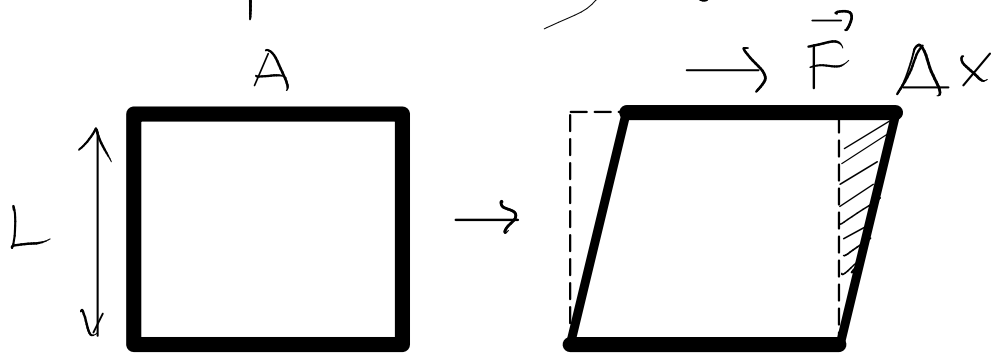
1) Sforzo di trazione



$$\sigma = \frac{F}{A} \quad \gamma = \frac{\Delta x}{L} \quad \sigma = Y \gamma$$

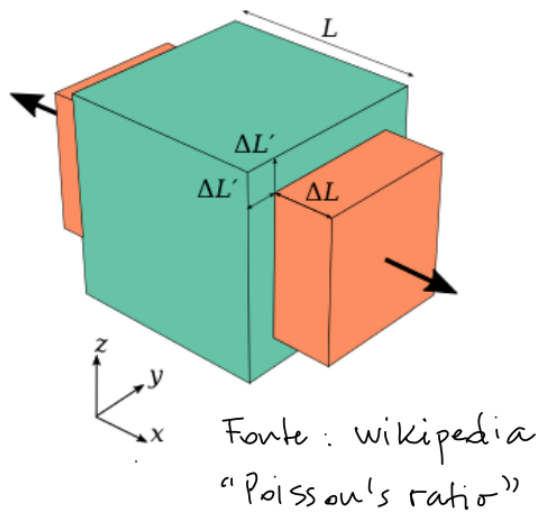
ES. gomma $Y \sim 0,1 - 0,01 \text{ GPa}$
 diamante $\sim 10^3 \text{ GPa}$

2) Sforzo di taglio



$$\tau = \frac{F}{A} \quad \gamma = \frac{\Delta x}{L} \quad \tau = G \gamma$$

ES. gomma $G \sim 6 \times 10^{-4} \text{ GPa}$
 diamante $\sim 500 \text{ GPa}$



Coefficiente di Poisson : $\nu = - \frac{\Delta L'}{\Delta L}$

$$0 \leq \nu < 0,5$$

↑
 sughero

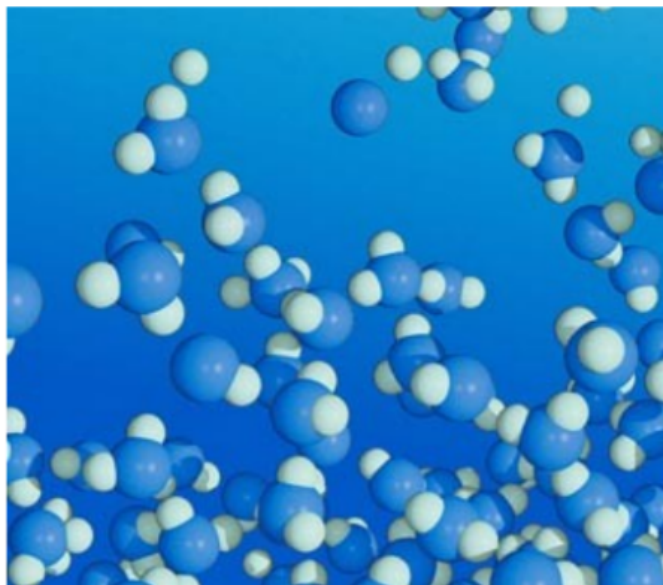
↑
 incompressibile

$$Y = 2G(\nu + 1)$$

MATERIA CONDENSATA DURA E SOFFICE

Scala atomica

10^{-10} - 10^{-9}

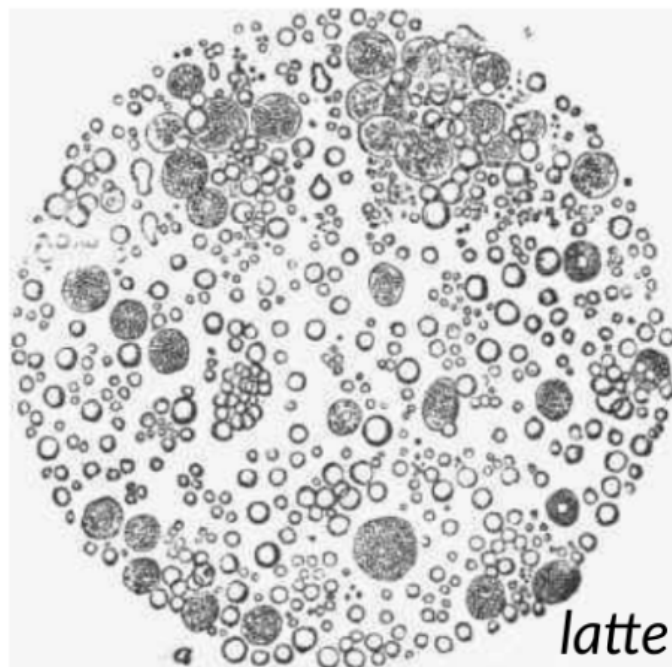


atomi, molecole

DURA

Scala mesoscopica

10^{-7} - 10^{-5}



latte

macromolecole

SOFFICE

Scala macroscopica

10^{-2} - 10^0

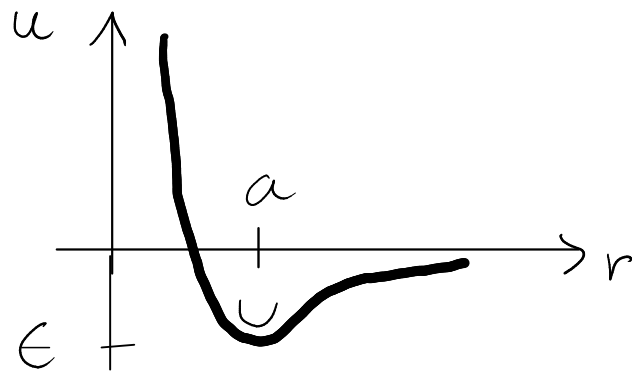
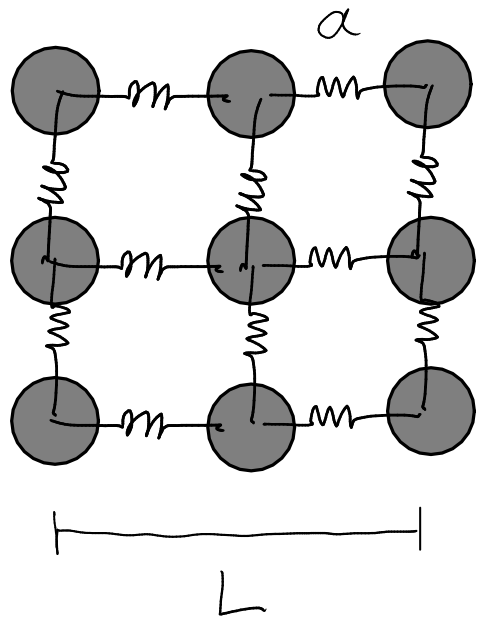


Lunghezza
[m]



Modello micro

Modulo di Young : $\gamma \sim \frac{E}{a^3}$



$$[G, \gamma] = \frac{F}{A} = \frac{E}{V} \sim \frac{E}{a^3}$$

Dura

$$E \sim 100 k_B T$$

$$a \sim 10^{-10} \text{ m}$$

Softice

$$E \sim 10 k_B T$$

$$a \sim 10^{-6} \text{ m}$$

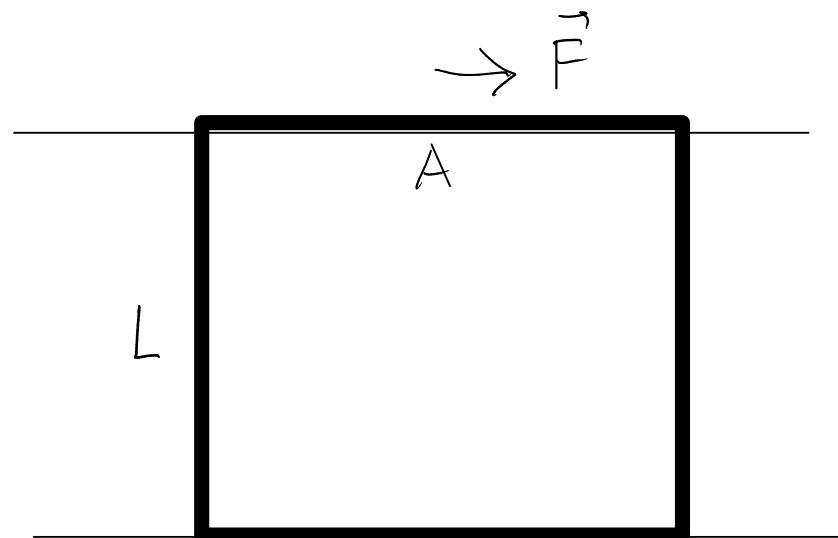
$$\frac{\gamma_{dura}}{\gamma_{softice}} \sim \frac{E_{dura}}{E_{softice}} \left(\frac{a_{softice}}{a_{dura}} \right)^3$$

$$\sim 10 \cdot \left(\frac{10^{-6}}{10^{-10}} \right)^3 \sim \underline{\underline{10^{13}}}$$

Cf. "Effective interactions in soft condensed matter physics"
C.N. Likos 2001 p. 274

VISCOSITÀ

Sforzo \sim deformazione
 « Hookiano »



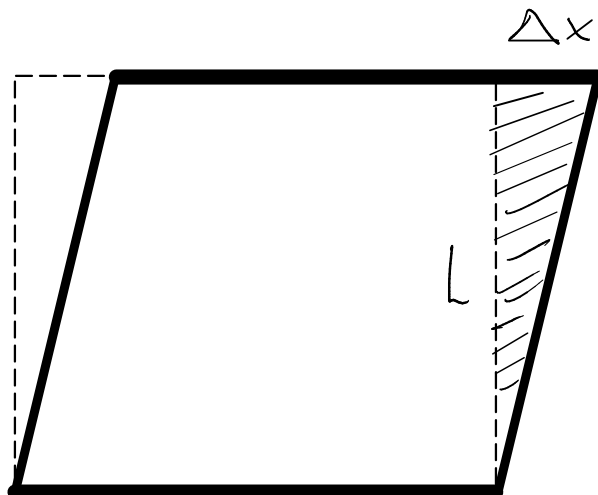
$$\sigma = \frac{F}{A}$$

$$\sigma = \eta \dot{\gamma}$$

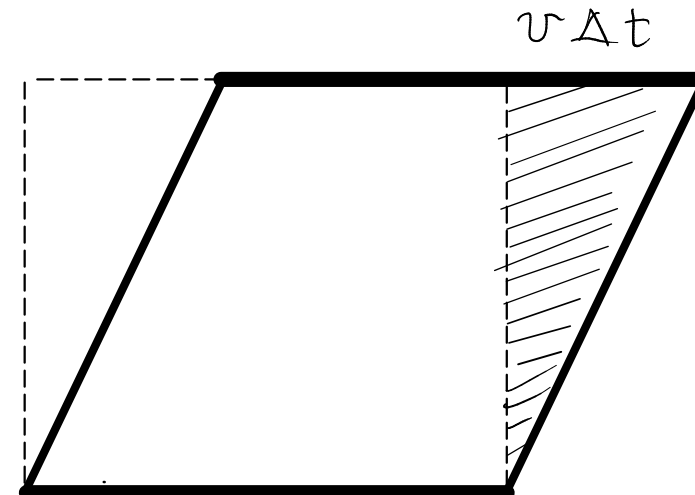
$\eta = \text{viscosità}$

SI: Pa·s

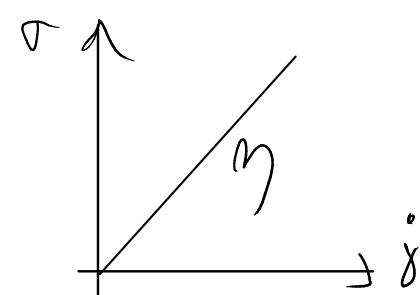
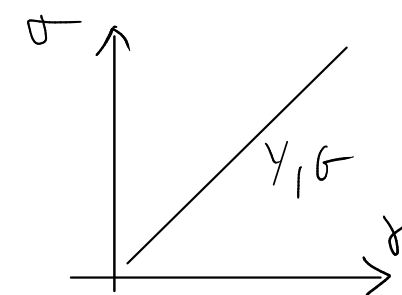
Sforzo \sim tasso di deformazione
 « Newtoniano »



$$\gamma = \frac{v \Delta t}{L}$$



$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{v}{L}$$



ES.: H₂O @ T_{amb}

$\eta \sim 10^{-3} \text{ Pa}\cdot\text{s}$

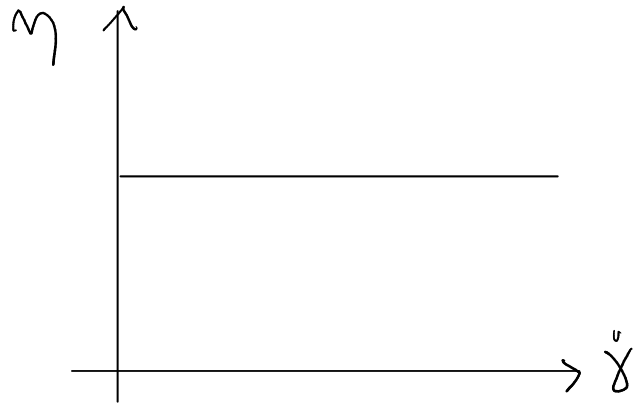
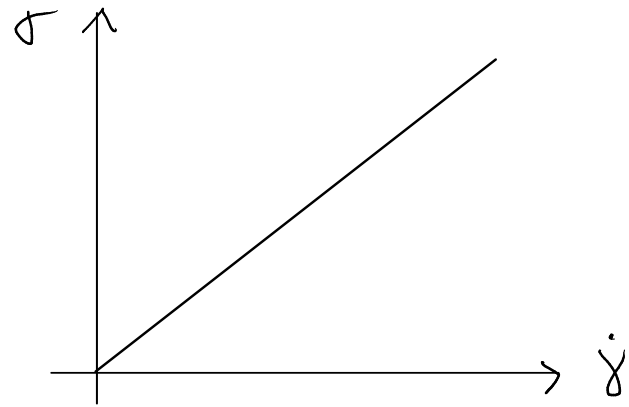
T \approx 90°C

$\eta \sim 10^{-4} \text{ Pa}\cdot\text{s}$

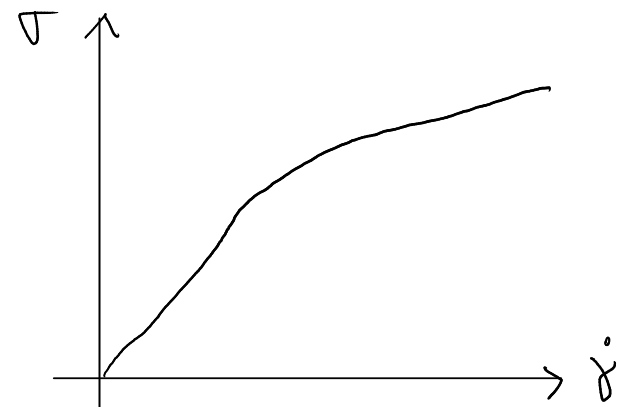
Miele

$\eta \sim 1 \text{ Pa}\cdot\text{s}$

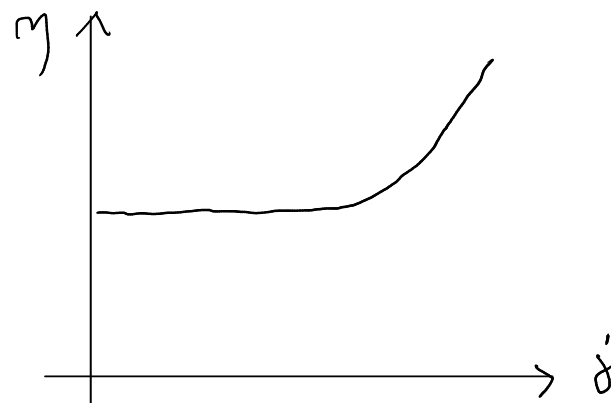
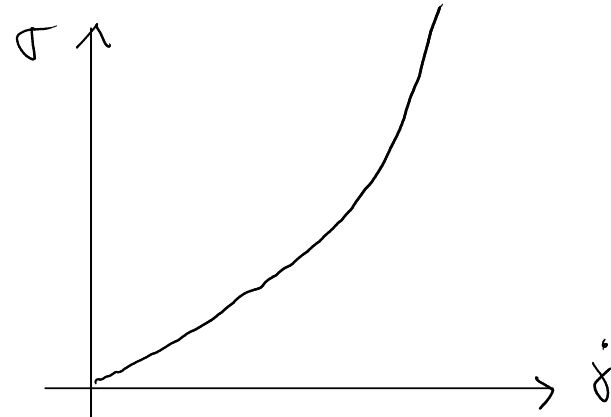
Newtoniano



Non-newtoniano



assottigliamento
al taglio



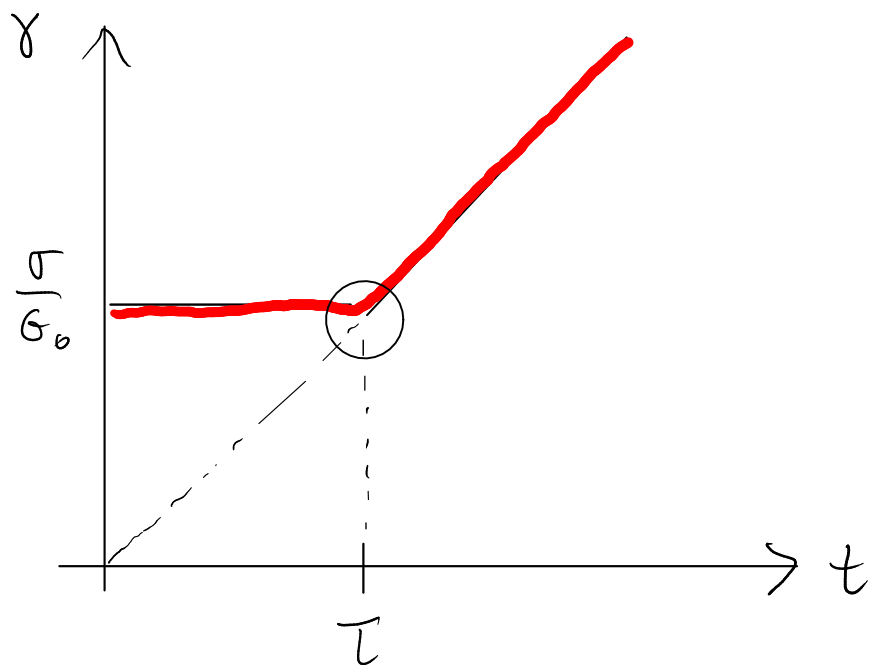
ispessimento
al taglio

$\rightarrow \sigma = \eta(\dot{\gamma}) \dot{\gamma}$

\rightarrow acqua e
maizena

VISCO - ELASTICITA'

Modello di Maxwell



tempo di rilassamento

$$1) \sigma = G_0 \gamma \quad \Rightarrow \quad \gamma = \frac{\sigma}{G_0} \quad \text{elastico (Hooke)}$$

$$2) \sigma = \eta \dot{\gamma} \quad \text{viscoso (Newton)}$$

$$\frac{\sigma}{G_0} = \dot{\gamma} \tau = \frac{\sigma}{\eta} \tau$$

Relazione di Maxwell: $\eta = G_0 \tau$

$$\text{Es.: } \tau \sim 10^{-12} \text{ s} \quad G_0 \sim 10^9 \text{ Pa} \quad \eta \sim 10^{-3} \text{ Pa s}$$

