# Pedagogical Content Knowledge: Math/Phys interplay

Lecture 3 - 11/10/2021

#### Teachers' PCK assists them in fostering the following goals

#### (Magnusson et al., 1999)

- a) Help students develop the 'science process' skills
- b) Represent a particular body of knowledge
- c) Transmit the facts of science
- d) Facilitate the development of scientific knowledge by confronting students with contexts to explain that challenge their naïve concepts
- e) Have students be active with materials; "hands-on" experiences.
- f) Involve students in investigating solutions to authentic problems
- g) Represent science as inquiry
- h) Constitute a community of learners whose members share responsibility for understanding the physical world, particularly with respect to using tools for science.

## Observation on Physics Teachers' PCK

Is it present a PCK or not?

Which PCK features do you recognize?

https://www.youtube.com/playlist?list=PLAA7AA6B0E433653C PSSC - Physical Science Study Committee

https://www.youtube.com/watch?v=CDEDBXuwYvo Fisica della professoressa Ida

https://www.youtube.com/watch?v=AsNxXS3kYho\_"Te lo avevo detto" - INCIDENTE SENZA CINTURE

https://www.youtube.com/watch?v=282D-YkMxyl PoliMi - Storia del primo principio

https://youtu.be/p0zDfi 8TKo FisicaFast

# https://forms.gle/Ud82zom4X nKovpmx5

## Declining PCK for Math-Phys interplay

(Lehavi et al., 2014; 2017)

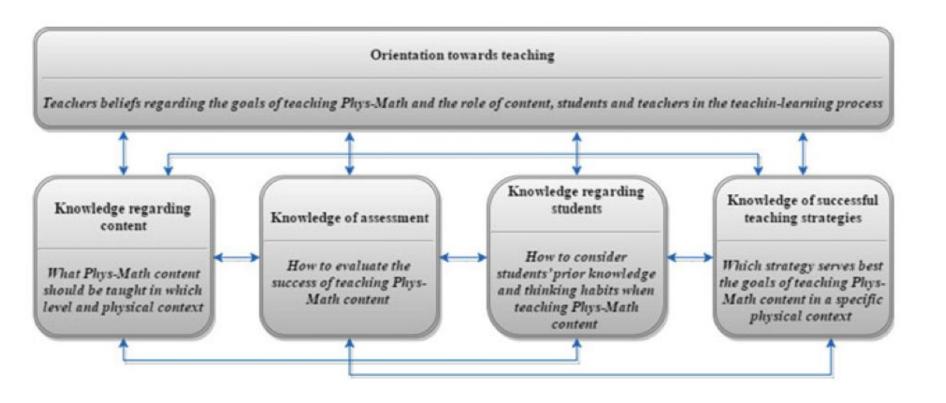


Table 1 Phys-Math patterns, teaching goals and teaching practices (note close relations to goals a-f in the Magnusson PCK model)

Pattern

The teaching goal

The teaching practices

Pattern	The teaching goal	The teaching practices	
A. Exploration	To demonstrate how phys-math is used to explore the behavior of physical systems	Exploring within math ramifications for the physical system: borders (of validity, of approximation), extreme cases, etc.	
B. Construction To demonstrate how phys-math is used in constructing a model for		Constructing and developing (from experiments or from first principles)	

B. Construction	To demonstrate how phys-math is used in constructing a model for physical systems	experiments or from first principles) mathematical tools to describe and analyse physical phenomena
C. Broadening To demonstrate how phys-math		Adopting a bird's-eye view and

		analyse physical phenomena	
C. Broadening	To demonstrate how phys-math can be used in broadening the scope of a physical context	Adopting a bird's-eye view and employing general laws of physics, symmetries, similarities and analogies	
D. Application	To demonstrate how phys-math provides aid in problem solving	- 10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00 (10.00	

The practice of employing different patterns of the Phys-Math interplay can distinguish master teachers from other expert teachers.

In order to perform a deep investigation of physics teachers' Pedagogical Content Knowledge our observations consisted in actions and methodologies according to this sequence:

- 1. Collecting information BEFORE the observations of the lesson activity about teacher's beliefs, methodologies, physics insights and then about class and students' skills, attitudes and everything is concerning learning strategies, emerging and recurrent difficulties, assessments.
- 2. Collecting information DURING the observations of lesson activities, from explanations to evaluation time.
- 3. Collecting information AFTER the observations of lesson activities from the teachers' point of view to the students' self reflection about their performance and learning.

All information has been collected from interviews, meetings and taking notes during class activities.

## Sequence of monitoring

BEFORE DURING AFTER

Teachers' monitoring in their lessons planning.

Teachers' interviewing for collecting information about students and class educational trend.

Observations during class activities in presence and on line (in the first COVID lockdown period) for the extension of a learning module.

In some cases, preparing evaluation tests together with teachers at the end of the learning module, with particular attention to the integration of math and physics languages.

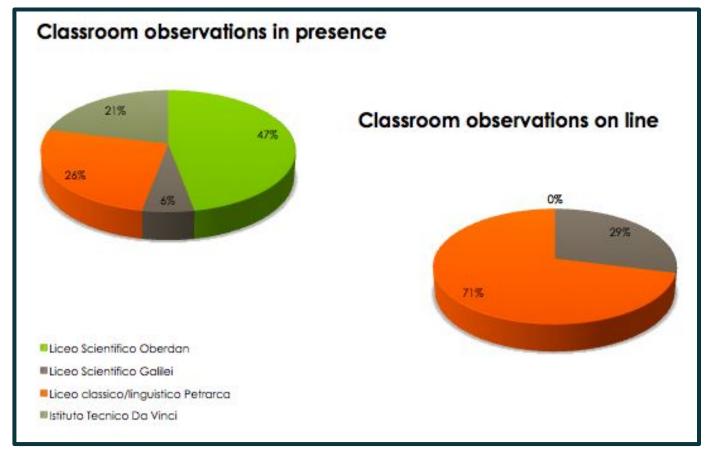
Feedback discussions with teachers on monitoring activities.

Test revisions and corrections trying to identify the most frequent mistakes and to classify them in terms of mastery and knowledge of physics languages.

Collecting students' interviews about their difficulties in that learning module and final evaluation test.

We monitored classes at the first year of the curricular physics studies: this is an important point of our investigation, because we would like to find whether difficulties in learning and studying physics arise from a particular teacher PCK adopted at this stage, i.e. at the beginning of high secondary school, when the students' background is characterized by a basic knowledge of Math.

#### Sample features



	Monitoring		Testing	
	Hours in presence	Hours on line	Hours of case studies	
High school for Scientific Studies "Oberdan"	40	Θ	12 + 18	
High school for Scientific Studies "Galilei"	5	11	0	
High school for Ancients and Modern Languages Studies "Petrarca"	22	27	Θ	
High School for Technical Studies	18	0	0	HOURS TOTAL
	85	38	30	153

The main feature of this sample is the age difference among students. It happens that someone starts studying curricular physics at 14 years old and some of them later (16 years). Of course, this is relevant for the stage of cognitive development, in terms of abstraction functions and metacognitive processes. And it also would be relevant in terms of content building processes for successful learning.

We analysed our observations according to the PCK model suggested by Magnusson et al., adapted by Etkina and used in the framework of the content of the Math-Phys interplay by Lehavi et al. and Pospiech obtaining some interesting results.

Prevalent pattern of Math/Phys Interplay



APPLICATION AND FORMULAS MANIPULATION

Prevalent patterns of Math/Phys Interplay

APPLICATION
AND FORMULAS
AND FORMULAS
MANIPULATION



MODEL BUILDING



EXPLORATION AND BROADENING

At the last curricular year of Physics study in the high secondary school

## Main results from the observation analysis

Where the lack of math competencies is relevant, the teacher adopts strategies converging to the strong use of observative/descriptive language for conceptualization.

If the teacher is aware of students difficulties in Math, or of the absence of Math-Phys interplay, he/she tries to support their learning process focusing on mathematical languages.

This causes a large use of math in the demonstration of physics laws and a great number of math exercices applied to physics phenomena, with the consequence that even the evaluation tests seem to be mathematically rather than physically oriented.

## Main results from the observation analysis

On the other side some teachers try to resolve the students difficulties in Math using improperly the different language structures (formulas, graphs...) making a conceptual reduction of Physics, separating what comes from mathematics results and what corresponds to a physical phenomenological observation.

This kind of approach tends to amplify the distance between the two disciplines instead of favouring their interplay and integration also in a form of interdisciplinarity to be thought and taught.

# Teachers' content knowledge for teaching

E.Etkina et al. (2018) PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH 14, 010127

#### Motivation

Research into teacher learning and practice over the last three decades shows that the teachers of a specific subject need to possess knowledge that is different from the knowledge of other content experts. Yet this specialized version of content knowledge that teachers need to plan instruction, respond to student ideas, and assess student understanding in real time is a theoretically elusive construct. It is crucial for the fields of precollege teacher preparation, teacher professional education, and postsecondary faculty professional development to

- (a) clarify the construct that underlies this specialized content knowledge,
- (b) operationalize it in some domain,
- (c) measure it in both static contexts and as it is enacted in the classroom,
- (d) correlate its presence with "richness" of classroom instruction and its effect on student learning.

#### APPENDIX A: TASKS OF TEACHING

This section provides a list of the tasks of teaching.

Task of teaching	Description	Specific tasks
I. Anticipating student thinking around science ideas	While planning and implementing instruction teachers are able to anticipate particular patterns in student thinking. They understand and recognize challenges students are likely to confront in developing an understanding of key science concepts and mathematical models. Teachers are also familiar with student interests and background knowledge and enact instruction accordingly.	<ul> <li>Teachers:</li> <li>I. a) anticipate specific student challenges related to constructing scientific concepts, conceptual and quantitative reasoning, experimentation, and the application of science processes</li> <li>I. b) anticipate likely partial conceptions and alternate conceptions, including partial quantitative understanding about particular science content and processes</li> <li>I. c) recognize student interest and motivation around particular science content and practices</li> <li>I. d) understand how students' background knowledge both in physics and mathematics can interact with new science content</li> </ul>

II. Designing, selecting, and sequencing learning experiences and activities Classroom learning experiences and activities are designed around learning goals and involve key science ideas, key experiments, and mathematical models relevant to the development of ideas and practices. Learning experiences reflect an awareness of student learning trajectories and support both individual and collective knowledge generation on the part of students.

- II. a) design or select and sequence learning experiences that focus on sense-making around important science concepts and practices, including productive representations, mathematical models, and experiments in science that are connected to students' initial and developing ideas
- II. b) include key practices of science including experimentation, reasoning based on collected evidence, experimental testing of hypotheses, mathematical modeling, representational consistency, and argumentation
- II. c) address projected learning trajectories that include both longterm and short-term goals and are based on evidence of actual student learning trajectories
- II. d) address learners' actual learning trajectories by building on productive elements and addressing problematic ones
- II. e) provide students with evidence to support their understanding of short- and long-term learning goals
- II. f) integrate, synthesize, and use multiple strategies and involve students in making decisions
- II. g) prompt students to collectively generate and validate knowledge with others
- II. h) help students draw on multiple types of knowledge, including declarative, procedural, schematic, and strategic
- II. i) elicit student understanding and help them express their thinking via multiple modes of representation
- II. j) help students consider multiple alternative approaches or solutions, including those that could be considered to be incorrect

III. Monitoring, interpreting, and acting on student thinking Teachers understand and recognize challenges and difficulties students experience in developing an understanding of key science concepts; understanding and applying mathematical models and manipulating equations; designing and conducting experiments, etc. This is evident in classroom work, talk, actions, and interactions throughout the course of instruction so that specific learning needs or patterns are revealed. Teachers also recognize productive developing ideas and problem solutions and know how to leverage these to advance learning.

Teachers engage in an ongoing and multifaceted process of assessment, using a variety of tools and methods. Teachers draw on their understanding of learners and learning trajectories to accurately interpret and productively respond to their students' developing understanding.

- III. a) employ multiple strategies and tools to make student thinking visible
- III. b) interpret productive and problematic aspects of student thinking and mathematical reasoning
- III. c) identify specific cognitive and experiential needs or patterns of needs and build upon them through instruction
- III. d) use interpretations of student thinking to support instructional choices both in lesson design and during the course of classroom instruction
- III. e) provide students with descriptive feedback
- III. f) engage students in metacognition and epistemic cognition
- III. g) devise assessment activities that match their goals of instruction

IV. Scaffolding meaningful engagement in a science learning community Productive classroom learning environments are community-centered. Teachers engage all students as full and active classroom participants. Knowledge is constructed both individually and collectively, with an emphasis on coming to know through the practices of science. The values of the classroom community include evidence-based reasoning, the pursuit of multiple or alternative approaches or solutions, and the respectful challenging of ideas.

- IV. a) engage all students to express their thinking about key science ideas and encourage students to take responsibility for building their understanding, including knowing how they know
- IV. b) develop a climate of respect for scientific inquiry and encourage students' productive deep questions and rich student discourse
- IV. c) establish and maintain a "culture of physics learning" that scaffolds productive and supportive interactions between and among learners
- IV. d) encourage broad participation to ensure that no individual students or groups are marginalized in the classroom
- IV. e) promote negotiation of shared understanding of forms, concepts, mathematical models, experiments, etc., within the class
- IV. f) model and scaffold goal behaviors, values, and practices aligned with those of scientific communities
- IV. g) make explicit distinctions between science practices and those of everyday informal reasoning as well as between scientific expression and everyday language and terms
- IV. h) help students make connections between their collective thinking and that of scientists and science communities
- IV. i) scaffold learner flexibility and the development of independence
- IV. j) create opportunities for students to use science ideas and practices to engage real-world problems in their own contexts

V. Explaining and using examples, models, representations, and arguments to support students' scientific understanding

Teachers explain and use
representations, examples, and models
to help students develop their own
scientific understanding. Teachers also
support and scaffold students' ability
to use models, examples, and
representations to develop
explanations and arguments.
Mathematical models are included as a
key aspect of physics understanding
and are assumed whenever the term
model is used.

- V. a) explain concepts clearly, using accurate and appropriate technical language, consistent multiple representations, and mathematical representations when necessary
- V. b) use representations, examples, and models that are consistent with each other and with the theoretical approach to the concept that they want students to learn
- V. c) help students understand the purpose of a particular representation, example, or model and how to integrate new representations, examples, or models with those they already know
- V. d) encourage students to invent and develop examples, models, and representations that support relevant learning goals
- V. e) encourage students to explain features of representations and models (their own and others') and to identify/evaluate both strengths and limitations
- V. f) encourage students to create, critique, and shift between representations and models with the goal of seeking consistency between and among different representations and models
- V. g) model scientific approaches to explanation, argument, and mathematical derivation and explain how they know what they know. They choose models and analogs that accurately depict and do not distort the true meaning of the physical law and use language that does not confound technical and everyday terms (e.g., heat and energy).
- V. h) provide examples that allow students to analyze situations from different frameworks such as energy, forces, momentum, and fields

VI. Using experiments to construct, test, and apply concepts

Teachers provide timely and meaningful opportunities throughout instruction for students to design and analyze experiments to help students develop, test, and apply particular concepts. Experiments are an integral part of student construction of physics concepts and are used as part of scientific inquiry in contrast with simple verification.

- VI. a) provide opportunities for students to analyze quantitative and qualitative experimental data to identify patterns and construct concepts
- VI. b) provide opportunities for students to design and analyze experiments using particular frameworks such as energy, forces, momentum, field, etc.
- VI. c) provide opportunities for students to test experimentally or apply particular ideas in multiple contexts
- VI. d) provide opportunities for students to pose their own questions and investigate them experimentally
- VI. e) use questioning, discussion, and other methods to draw student attention during experiments to key aspects needed for subsequent learning, including the limitations of the models used to explain a particular experiment
- VI. f) help students draw connections between classroom experiments, their own ideas, and key science ideas
- VI. g) encourage students to draw on experiments as evidence to support explanations and claims and to test explanations and claims by designing experiments to rule them out

#### APPENDIX B: ENERGY-SPECIFIC STUDENT TARGETS (ENERGY-RELATED CONTENT AND PRACTICE IDEAS)

This section provides a list of energy targets for the students.

#### 1. Connections of energy and everyday experiences

The student

- 1) uses energy ideas to interpret or explain everyday phenomena
- 2) recognizes the important role of internal energy in interpreting or explaining everyday phenomena

#### 2. Choice of system

The student

- 1) recognizes that the energy accounting in a phenomenon depends on the choice of system
- 2) explains the relative advantage of a given system choice (i.e., relative ease of analysis)
- 3) recognizes that the choice of system determines whether springs or Earth do work (i.e., if the spring or Earth are in the system they do not do any work on the system, but the system can possess elastic or gravitational potential energy)
- 4) identifies and differentiates between forms of energy and other physics concepts

## 3. Identification of and differentiation between forms of energy and other physics concepts

- 1) recognizes that energy cannot be observed directly and knows how different forms of energy correspond to different measurable physical quantities
- 2) recognizes and maintains a consistency of scale (microscopic or macroscopic) during energy analysis
- 3) differentiates between energy and related ideas (e.g., force, power, stimulus, trigger, activation, speed, distance, temperature)
- 4) distinguishes between forms of energy and energy transfers

# **4.** Transfer of energy (environment → system; system → environment)

- 1) recognizes that the energy of a system is always conserved but might not be constant
- 2) recognizes that work is the way in which energy is transferred mechanically and may result in a change in temperature in some cases
- 3) avoids double counting when analyzing processes involving work and energy
- 4) recognizes when to use compensatory models for tracking energy into and out of a system and when quantitative models are of limited use

#### 5. Use of mathematics

- 1) understands that when considering potential energy, it is important to think about the change. The zero level of potential energy is arbitrary, but the change is not. The energy of attraction is negative if the zero level is set at infinity.
- 2) can account for vector and scalar quantities in energy analysis
- 3) understands that work is a scalar quantity and the positive or negative sign of work does not indicate direction but addition or subtraction
- 4) connects forms of energy and the factors on which they depend through appropriate linear and non-linear mathematical relationships
- 5) applies conservation as a mathematical constraint on the outcomes of possible processes
- 6) recognizes that the mathematical analysis of energyrelated processes depends on the choice of initial and final state and the choice of system

#### **6.** Use of representations

- 1) selects/creates and uses appropriate verbal, mathematical, and graphical/pictorial representations (specific for energy, such as bar charts, energy diagrams, etc.) to describe, analyze, and/or communicate a physical situation or process
- 2) interprets different representations used to describe, analyze, and/or communicate a physical situation or process
- 3) understands the relationships between different representations of the same phenomenon and seeks consistency among different representations
- 4) understands standard technical representations and language used to communicate energy-related ideas

#### 7. Use of science practices

- 1) uses a range of representations to communicate ideas and illustrate or defend explanations
- 2) connects energy ideas to other learning and real-life processes and projects through experimental investigations, energy problem solutions, and engineering designs
  - 3) designs experiments to test competing hypotheses
- 4) makes choices in data collection and analysis that allow for inferring the amounts and transfers of energy even when they cannot be measured directly
- 5) connects experiments and data to the mathematical representations of energy
- 6) evaluates and negotiates choices/options by considering the merits, limitations, and relative advantages of different engineering designs in terms of, for example, different choices of energy models for the same physical process
- 7) provides evidence-based arguments concerning energy processes and engineering designs
- 8) demonstrates consistency and coherence in modelbased and evidence-based reasoning in making predictions and interpreting results