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Mercoledì 12 ore 15

Aula A Fisica Tecnica

piano terra Ed C5

4 ott. Martedì pomeriggio

Sommatorie. Dati $n \in \mathbb{N}$ numeri

$$a_1, a_2, \dots, a_n$$

$$\sum_{j=1}^{n} a_j := a_1 + \dots + a_n \qquad \sum_{j=1}^{2} a_j = a_1 + a_2$$

Se $n = 1$ $\qquad \sum_{j=1}^{1} a_j = a_1$

$$\sum_{j=1}^{10} j = 1 + 2 + 3 + \dots + 9 + 10$$

Regole Dati $a_1, \dots, a_n \in \mathbb{R}$ e dato $c \in \mathbb{R}$

$$\sum_{j=1}^{n} c\, a_j = c \sum_{j=1}^{n} a_j$$

$a_1, \dots, a_n$, e $b_1, \dots, b_n$

$$\sum_{j=1}^{n} (a_j + b_j) = \sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j$$

$$(a_1 + b_1) + (a_2 + b_2) = a_1 + b_1 + a_2 + b_2 = (a_1 + a_2) + (b_1 + b_2)$$

$$\sum_{j=1}^{n} a_j = \sum_{k=1}^{n} a_k = \sum_{j=1}^{n-1} a_j + a_n$$

$a_j \qquad j \in \{6, 6, 6\}$

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2} \qquad (P_n) \qquad \forall n \in \mathbb{N}$$

Per induzione

$n=1$
$$\left.\frac{n(n+1)}{2}\right|_{n=1} = \frac{1 \cdot 2}{2} = 1$$

$$\sum_{j=1}^{1} j = 1$$

uguali

$n \Rightarrow n+1$ se $\displaystyle\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$ è vera allora

$$\sum_{j=1}^{n+1} j = \frac{(n+1)(n+2)}{2} \qquad P_{n+1}$$

$$\sum_{j=1}^{n+1} j = \sum_{j=1}^{n} j + (n+1) = \frac{n(n+1)}{2} + (n+1)$$

$$= (n+1)\left[\frac{n}{2} + 1\right] = (n+1)\,\frac{n+2}{2}$$

$$\sum_{j=1}^{n} j = 1 + 2 + 3 + 4 + \cdots + (n-3) + (n-2) + (n-1) + n$$

$n+1$

$n+1$

$n+1$

Teorema (Somme geometriche) Se $r \neq 1$ allora

$$\sum_{j=0}^{n} r^j = \frac{1-r^{n+1}}{1-r} \qquad \left(\text{dove } \begin{matrix} r^0 = 1 \\ 0^0 = 1 \end{matrix} \right)$$

Dim. Si tratta di dimostrare il caso $n=0$ e poi l'implicazione $n \Rightarrow n+1$.

1) Caso $n=0$ $\quad \sum_{j=0}^{0} r^j = r^0 = 1$

$$\left. \frac{1-r^{n+1}}{1-r} \right|_{n=0} = \frac{1-r}{1-r} = 1 \qquad \forall r \neq 1$$

2) Dimostriamo che se è vero $\sum_{j=0}^{n} r^j = \frac{1-r^{n+1}}{1-r}$

anche $\sum_{j=0}^{n+1} r^j = \frac{1-r^{n+2}}{1-r}$ è vero

$$\sum_{j=0}^{n+1} r^j = \sum_{j=0}^{n} r^j + r^{n+1} = \frac{1-r^{n+1}}{1-r} + r^{n+1}$$

$$= \frac{1-r^{n+1} + r^{n+1}(1-r)}{1-r} = \frac{1 - \cancel{r^{n+1}} + \cancel{r^{n+1}} - r^{n+2}}{1-r}$$

$$= \frac{1-r^{n+2}}{1-r}$$

$$\sum_{j=0}^{n} r^j = \frac{1-r^{n+1}}{1-r} \qquad \cdot (1-r)$$

$$(1-r) \sum_{j=0}^{n} r^j = 1 - r^{n+1}$$

$$= (1-r)\left(1 + r + r^2 + r^3 + \cdots + r^{n-1} + r^n\right)$$

$$= (1-\cancel{r}) + (\cancel{r} - \cancel{r^2}) + (\cancel{r^2} - r^3) + \cdots + (r^{n-1} - \cancel{r^n}) +$$

$$+ (\cancel{r^n} - r^{n+1}) = 1 - r^{n+1}$$

$$\sum_{j=1}^{n} (a_j - a_{j+1}) = a_1 - a_{n+1}$$

Coefficienti binomiali

$n \in \mathbb{N} \cup \{0\} = \{0, 1, 2, \dots\}$ $\quad n!$ $\quad$ n fattoriale

$0! = 1$, per $n \geq 1$

$n! = 1 \cdot \ldots \cdot n$

$1! = 1$ $\qquad 2! = 1 \cdot 2 = 2$

$3! = 2!\, 3 = 1 \cdot 2 \cdot 3 = 6$

$4! = 3!\, 4 = 24$

Coefficienti binomiali , $0 \leq k \leq n$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{\cancel{(1 \cdots (n-k))}\,(n-k+1) \cdots n}{\cancel{(n-k)!}\; k!}$$

$$= \frac{n \cdots (n-k+1)}{k!}$$

$$\binom{n}{k} = \binom{n}{n-k} = \frac{n!}{(n-k)!\; k!}$$

$$\binom{n}{0} = 1 \qquad \binom{n}{n} = 1$$

$$\frac{n!}{0!\,(n-0)!} = \frac{\cancel{n!}}{0!\,\cancel{n!}} = 1$$

$$\binom{n}{1} = n = \binom{n}{n-1}$$

$$\binom{n}{1} = \frac{n!}{(n-1)!} = \frac{\cancel{(n-1)!}\; n}{\cancel{(n-1)!}} = n$$