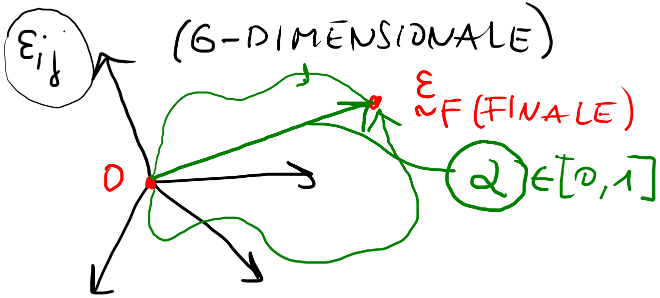
COMPLEMENTI SUGLI ASPETTI ENERGETICI DELL'ELASTICITÀ

4/11/22



<u>IPOTESI</u>: PROBL. CONSERVATIVO; $\exists$ ENERGIA (FUNZIONE DI STATO) φ CHE PERMETTE DI AFFERMARE CHE ESSA <u>NON DIPENDA DAL</u> PERCORSO SEGUITO PER ANDARE DA $\varepsilon_{ij}=0$ A $\underset{\sim}{\varepsilon}_F$

$$\varphi = \int \underset{\sim}{\sigma} \cdot d\underset{\sim}{\varepsilon} \Rightarrow \varphi = \frac{1}{2} \underset{\sim}{\sigma}_F \cdot \underset{\sim}{\varepsilon}_F \quad (\varphi : \text{POTENZIALE ELASTICO})$$

$$\varphi = \frac{1}{2} \sigma_{xx} \varepsilon_{xx}$$

$$\int_{\text{PERCORSO}} \underset{\sim}{\sigma} \cdot d\underset{\sim}{\varepsilon} \overset{\exists \varphi}{=} \varphi(\underset{\sim}{\varepsilon}_F) - \cancel{\varphi(\underset{\sim}{0})}^{\,0} = \varphi(\underset{\sim}{\varepsilon}_F) \Rightarrow \varphi(\underset{\sim}{\varepsilon}) = \int \underset{\sim}{\sigma} \cdot d\underset{\sim}{\varepsilon}$$

$$\underbrace{\int \underset{\sim}{\sigma} \cdot d\underset{\sim}{\varepsilon}}_{d\varphi} = \int_0^{\underset{\sim}{\varepsilon}_F} d\varphi = \varphi(\underset{\sim}{\varepsilon}_F) - \varphi(\underset{\sim}{0})$$

$d\varphi$: DIFFERENZ. ESATTO

SE CONOSCO LA FUNZ. $\varphi(\underset{\sim}{\varepsilon})$ ALLORA

$$d\varphi(\underset{\sim}{\varepsilon}) = \underset{\sim}{\sigma} \cdot d\underset{\sim}{\varepsilon} \quad (\text{DIFF. ESATTO})$$

$$\sigma_{xx} d\varepsilon_{xx} + \sigma_{yy} d\varepsilon_{yy} + \dots + \tau_{yz} d\varepsilon_{yz}$$

$$\frac{\partial \varphi}{\partial \varepsilon_{xx}} d\varepsilon_{xx} + \frac{\partial \varphi}{\partial \varepsilon_{yy}} d\varepsilon_{yy} + \dots + \frac{\partial \varphi}{\partial \varepsilon_{yz}} d\varepsilon_{yz}$$

$$\varphi(\underset{\sim}{\varepsilon}) \longrightarrow \boxed{\sigma_{ij} = \frac{\partial \varphi}{\partial \varepsilon_{ij}}} \quad (*)$$

EL. LIN. ISOTROPO

ELAST. LIN.

$\varphi(\underset{\sim}{\varepsilon}) = \frac{1}{2} \underbrace{\mathbb{C}\,\underset{\sim}{\varepsilon}}_{\underset{\sim}{\sigma}} \cdot \underset{\sim}{\varepsilon}$; VISTO CHE IN TERMINI DI $\underset{\sim}{\varepsilon}$: $\underset{\sim}{\sigma} = 2G\,\underset{\sim}{\varepsilon} + \lambda\,(\mathrm{tr}\,\underset{\sim}{\varepsilon})\,\underset{\sim}{I}$

$$\Rightarrow \boxed{\varphi(\underset{\sim}{\varepsilon}) = G\,\underset{\sim}{\varepsilon} \cdot \underset{\sim}{\varepsilon} + \frac{\lambda}{2}(\mathrm{tr}\,\underset{\sim}{\varepsilon})^2} \Rightarrow \underset{\sim}{\sigma} = \frac{\partial \varphi}{\partial \underset{\sim}{\varepsilon}} = 2G\,\underset{\sim}{\varepsilon} + \lambda\,(\mathrm{tr}\,\underset{\sim}{\varepsilon})\,\underset{\sim}{I}$$

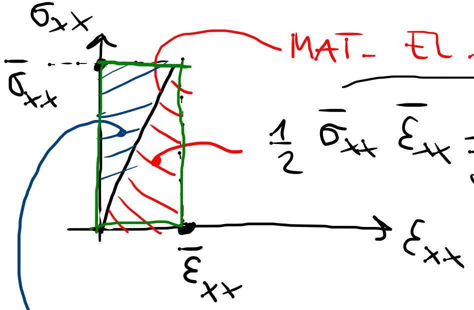
OSS: 1) L'EQ. (*) VALE PER TUTTI I MATERIALI PER CUI φ È UNA FUNZ. DI STATO. IL CASO DI ELASTICITÀ LINEARE ISOTROPA È SOLO UN CASO PARTICOLARE.

2) I MATERIALI PER CUI $\exists\,\varphi$ SI CHIAMANO IPERELASTICI o ELASTICI DI GREEN

3) SE $\varphi \nexists$ (PROBL. NON CONSERVATIVO) $\Rightarrow$ MATERIALI ELASTICI DI CAUCHY.

POTENZ. ELASTICO COMPLEMENTARE (φ^c)

$\sigma_{xx} \leftarrow \square \rightarrow \sigma_{xx}$ (MONOASSIALE)



MAT. EL. LIN.

$\sigma_{xx} = E\,\varepsilon_{xx}$

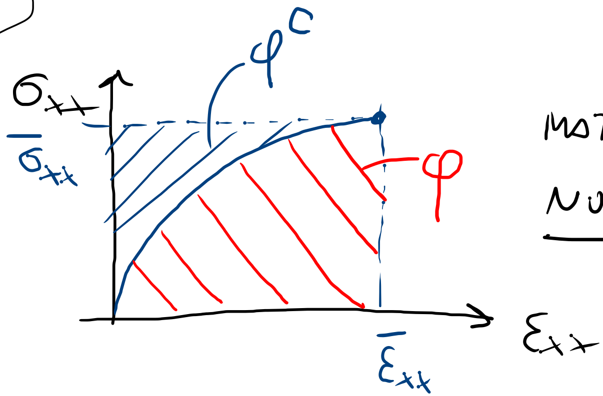
$$\frac{1}{2}\bar{\sigma}_{xx}\bar{\varepsilon}_{xx} = \varphi \; ; \; \varphi(\varepsilon_{xx}) = \frac{1}{2} E\,\bar{\varepsilon}_{xx}^2$$

$$\varphi^c = \frac{1}{2}\bar{\sigma}_{xx}\bar{\varepsilon}_{xx}$$

VALGONO SOLO PER MATERIALI ELASTICI LINEARI

$$\boxed{\varphi + \varphi^c = \bar{\sigma}_{xx}\bar{\varepsilon}_{xx}}$$

PROPR. CHE VALE PER TUTTI I MAT. ELASTICI.



MAT. ELASTICO NON LINEARE

$$\boxed{\varphi \neq \varphi^c}$$

PROPRIETÀ DI φ^c NEL CASO DI MAT. ELASTICO LINEARE

$\rightarrow \varphi + \varphi^c = \underset{\sim}{\sigma} \cdot \underset{\sim}{\varepsilon}$

$\rightarrow$ Se $\varphi(\underset{\sim}{\varepsilon}) \Rightarrow \varphi^c(\underset{\sim}{\sigma}) \Rightarrow \boxed{\varepsilon_{ij} = \dfrac{\partial \varphi^c}{\partial \sigma_{ij}}}$

$\rightarrow \varphi^c(\underset{\sim}{\sigma}) = \frac{1}{2} \underset{\sim}{\sigma} \cdot \underbrace{\mathbb{C}^{-1} \underset{\sim}{\sigma}}_{\underset{\sim}{\varepsilon}}$: FUNZ. QUADRATICA IN σ_{ij}

NEL CASO ISOTROPO:

$$\varphi^c(\underset{\sim}{\sigma}) = \frac{1}{2}\left[\frac{1}{E}(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2) - \frac{2\nu}{E}(\sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz}) + \frac{\cancel{2}(1+\nu)}{E}(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)\right]$$

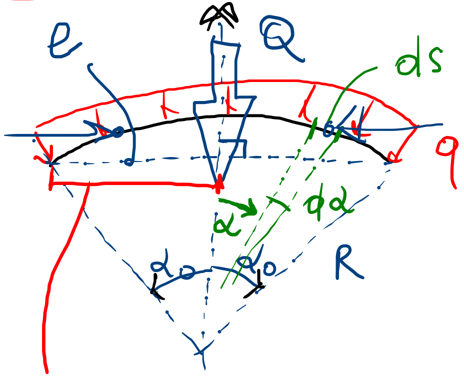
$$\varepsilon_{xx} = \frac{\partial \varphi^c}{\partial \sigma_{xx}} = \frac{1}{\cancel{2}}\left[\frac{1}{E}\cancel{2}\sigma_{xx} - \frac{\cancel{2}\nu}{E}(\sigma_{yy} + \sigma_{zz})\right] = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right]$$

GIÀ VISTO!

OK!

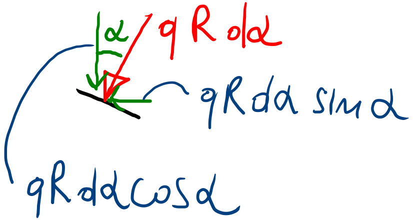
$$\varepsilon_{xy} = \frac{\partial \varphi^c}{\partial \tau_{xy}} = \frac{1}{2}\left[\frac{\cancel{2}(1+\nu)}{E} 2\tau_{xy}\right] = \frac{\cancel{2}(1+\nu)}{E}\tau_{xy} = \frac{\tau_{xy}}{2G}$$

COMPLEMENTI
CARICHI DISTRIBUITI



$Q = q\ell$

$ds = R\,d\alpha$



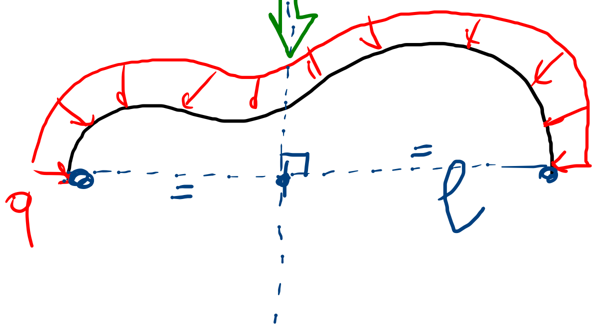
$$Q = \int_{-\alpha_0}^{\alpha_0} qR\,d\alpha\cos\alpha = qR\int_{-\alpha_0}^{\alpha_0}\cos\alpha\,d\alpha$$

$$= qR\left[\sin\alpha\right]_{-\alpha_0}^{\alpha_0} = qR\left[\sin\alpha_0 \underbrace{- \sin(-\alpha_0)}_{+\sin\alpha_0}\right]$$

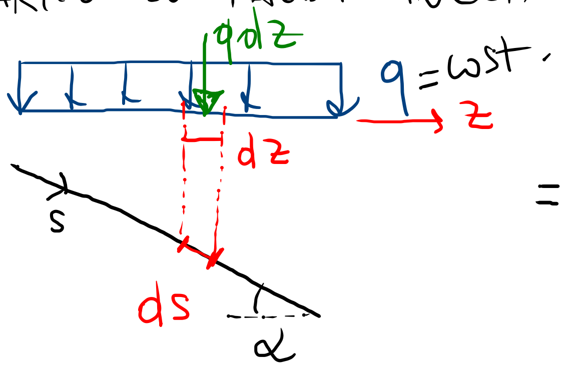
$$Q = \overbrace{2qR\sin\alpha_0}^{\ell} = q\ell$$

ℓ: LUNGH. CORDA

$Q = q\ell$ (SENZA DIMOSTRAZ.)



CARICO SU FALDA INCLINATA



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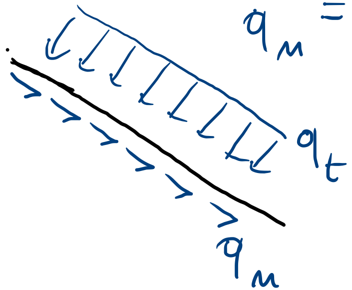
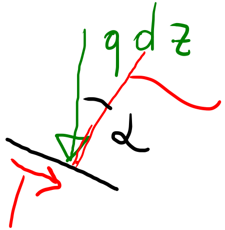


$$q_t,\ q_n = f(q, \alpha)\ ?$$

q: NON SI PROIETTA

q dz: FORZA ⇒ SI PROIETTA (O SCOMPONE)

$$dz = ds\cos\alpha$$

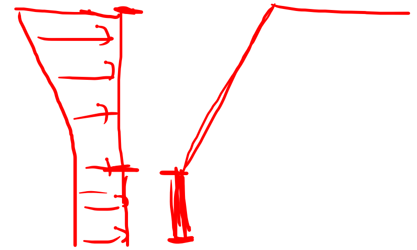


$$q\,dz\cos\alpha = q_t\,ds \longrightarrow q\,ds\cos^2\alpha = q_t\,ds$$

$$q\,dz\sin\alpha = q_n\,ds \longrightarrow q\,ds\cos\alpha\sin\alpha = q_n\,ds$$

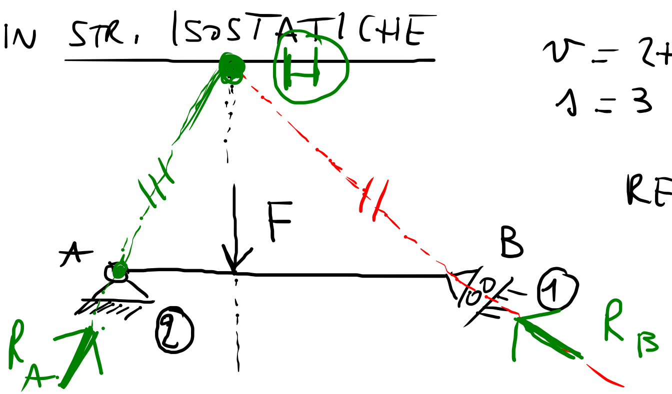
$$\boxed{q_t = q\cos^2\alpha \qquad q_n = q\cos\alpha\sin\alpha}$$

VENTO



SOLUZIONE GRAFICA DELLE STR. ISOSTATICA

METODO ALTERNATIVO A QUELLO "ANALITICO" PER CALCOLARE LE REAZ. VINCOLARI IN STR. ISOSTATICHE

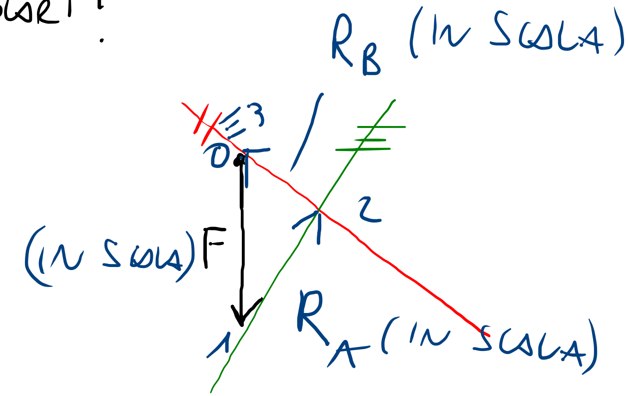


$v = 2+1 = 3$

$l = 3$, $g = 3$

ISOST.

REAZ. VINCOLARI?



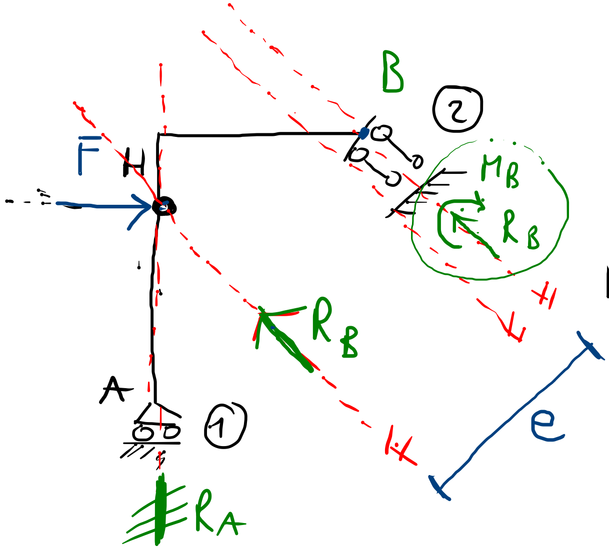
PROBL. RISOLTO

HO 3 FORZE IN GIOCO

$\underbrace{R_A, R_B}_{\text{INCOGNITE}}, F$

PER L'EQUIL. DI 3 FORZE $\boxed{\text{NON PARALLELE}}$ OCCORRE

- R_A, R_B, F SI "INCONTRINO" IN UN PUNTO (H)
- " " " CHIUDANO IL TRIANGOLO DELLE FORZE



$v = 3$

$g = 3$

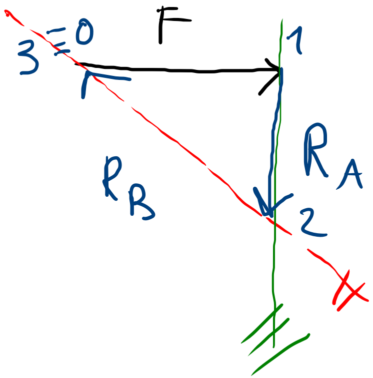
$s = 3$

STR. ISOST.

REAZ. VINCOLARI ? OK

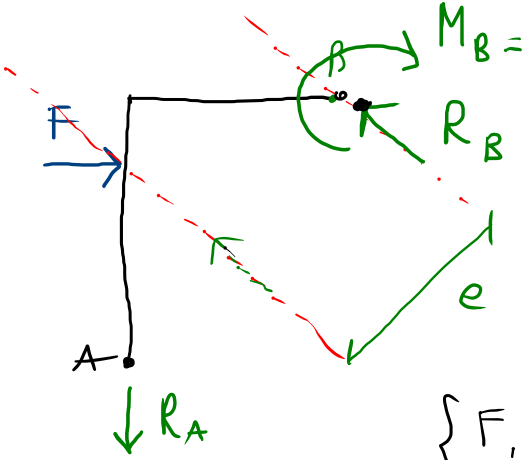
GRAFIC.

$\{F, R_A, R_B\}$: SIST. NULLO



TRIANGOLO CHIUSO

$M_B = R_B \cdot e$



$\{F, R_A, R_B, M_B\}$

SIST. NULLO