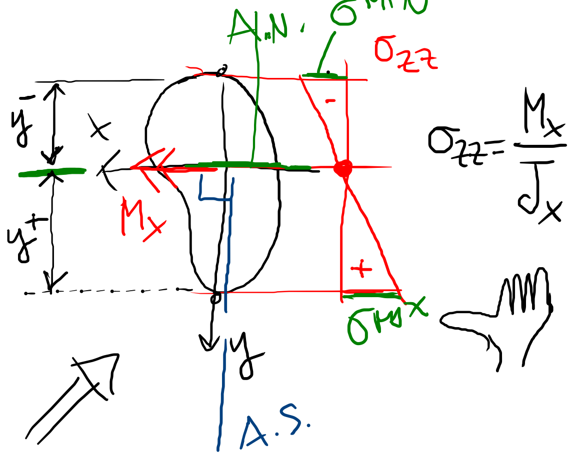
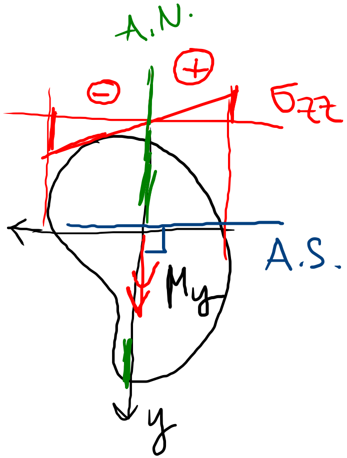
... FLESSIONE RETTA

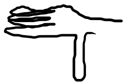
23/11/22



$$\sigma_{zz} = \frac{M_x}{J_x}\, y$$



$$\sigma_{zz} = -\frac{M_y}{J_y}\, x$$



A.N. : ASSE NEUTRO

$$\boxed{\sigma_{zz} = 0}$$

A.S : ASSE DI SOLLECITAZIONE

A.S. $\perp$ M

σ_{zz}^{MAX}, σ_{zz}^{MIN} ?

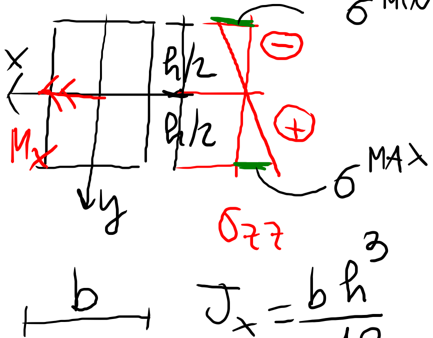
QUOTE

$$\sigma_{zz}^{MAX} = \frac{M_x}{J_x}\, y^+ = \frac{M_x}{W_x^+}$$

$$|\sigma_{zz}^{MIN}| = \frac{M_x}{J_x}\, y^- = \frac{M_x}{W_x^-}$$

W_x^+, W_x^- : MODULI DI RESISTENZA DELLA SEZ. RISPETTO A x

$[W] = [L^3]$



$$J_x = \frac{b h^3}{12}$$

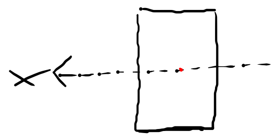
$$|\sigma^{MIN}| = \sigma^{MAX} = \frac{M_x}{J_x}\frac{h}{2} = \frac{M_x}{\frac{bh^3}{12}}\frac{h}{2} = \frac{M}{W_x}$$

$$W_x^+ = W_x^- = W_x$$

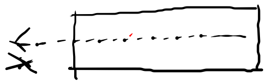
$$W_x = \frac{b h^3}{6\cancel{12}}\,\frac{\cancel{2}}{h} = \frac{b h^2}{6}$$

LE MASSIME TENSIONI NORMALI DIPENDONO DA $W_x^{\pm} = \frac{J_x}{y^{\pm}_{max}}$

M_x



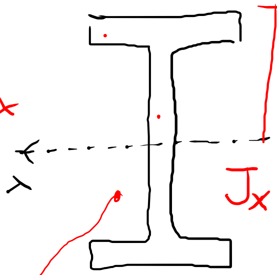
W_x MEDIO



y_{MAX}

W_x MINORE

J_x MOLTO MINORE



y_{MAX}

J_x MOLTO MAGGIORE

W_x MAGGIORE

$\Uparrow$

σ^{MAX} MINORE

$W_x^+ = W_x^- = W_x$

Area costante

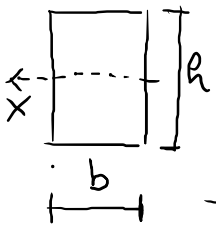
$$\sigma^{MAX} = \frac{M_x}{W_x} \rightsquigarrow \sigma^{MAX} \propto \frac{1}{W_x}$$

IN QUALE SEZ. CI SONO LE σ^{MAX} MINORI

$\rightarrow$ IPE, HE

MASSIMIZZANO J_x e ANCHE W_x

PROGETTO DI SEZ. RETTANGOLARE



b: NOTO

σ_0: MAX TENSIONE DI PROGETTO

M_x: NOTO

h?

CRITERIO:

$\sigma_{zz} \leq \sigma_0$

$\sigma_{zz}^{MAX} = \sigma_0$

$\frac{M_x}{W_x} = \sigma_0$

$$W_x = \frac{M_x}{\sigma_0}$$

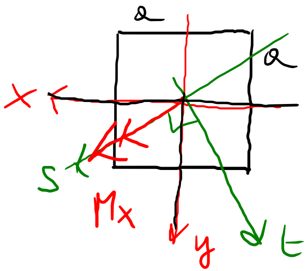
$$\frac{bh^2}{6} = \frac{M_x}{\sigma_0} \Rightarrow \bar{h} = \sqrt{\frac{6 M_x}{\sigma_0 b}} \; ??$$

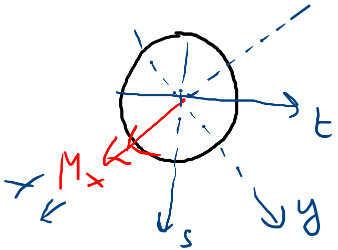
$$\boxed{h \geq \bar{h}}$$

$\bar{h}$: ALTEZZA CALCOLATA

$\bar{h} = [L]$???

OSSERVAZ. (QUADRATO – CERCHIO)





$J_x = J_y = \frac{\pi R^4}{4} = J_s = J_t$

TUTTI GLI ASSI BARICENTRICI SONO PRINCIPALI

$J_x = J_y = \frac{a^4}{12} = J_s = J_t$

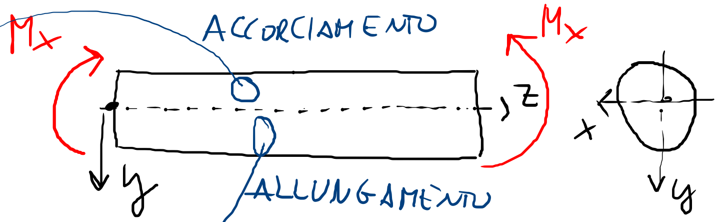
TUTTI I PROBLEMI DI FLESSIONE NEL QUADRATO E NEL CERCHIO SONO "FLESSIONE RETTA" PERCHÉ TUTTI GLI ASSI BARICENTRICI SONO PRINCIPALI.

DEFORMATA DELLA LINEA D'ASSE DEL CILINDRO SOLLECITATO A FLESSIONE RETTA

$$\underset{\sim}{\sigma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{M_x}{J_x} y \end{bmatrix}$$

IN TUTTO IL CILINDRO

$$\underline{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$



$$\underset{\sim}{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

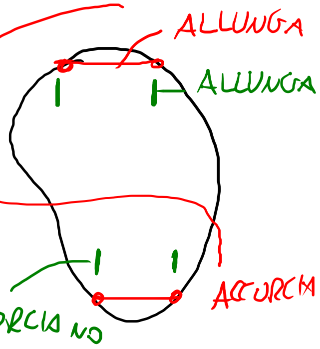
$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} = \frac{M_x}{J_x E} y$$

$$\varepsilon_{xx} = \varepsilon_{yy} = -\frac{\nu}{E}\sigma_{zz} = -\nu \frac{M_x}{E J_x} y$$

$\varepsilon_{xx} > 0,\ y < 0$

$\varepsilon_{yy} > 0,\ y < 0$

POCO IMPORTANTE



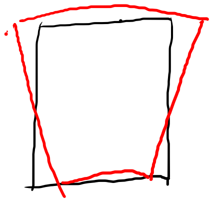
$\nu > 0$

$$u_{z,z} = \varepsilon_{zz} = \frac{M_x y}{E J_x}$$

$$u_{x,x} = u_{y,y} = -\nu \frac{M_x}{E J_x} y$$

PROBLEMA DIFFERENZIALE CHE POSSO INTEGRARE PER DETERMINARE IL CAMPO DI SPOSTAMENTI. $\underline{u}(x,y,z)$

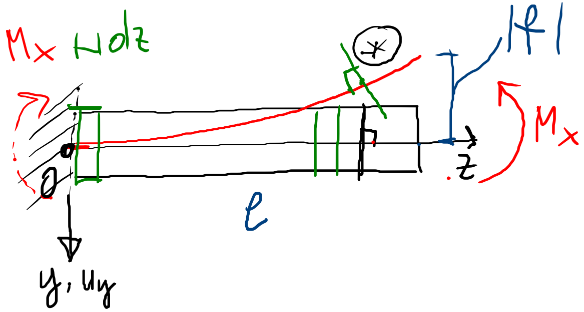
EFFETTO DELLE ε_{xx}



$\boxed{\text{SPOSTAMENTO ASSE: } u_y(0,0,z)}$

$\boxed{\text{M. CAPURSO}}$

INTEGRAZIONE



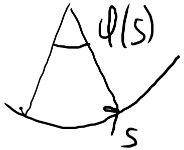
$$u_y(0,0,z) = -\frac{M_x}{2EJ_x} z^2$$

EQ. LINEA D'ASSE DEFORMATA

$$f = u_y(0,0,\ell) = \frac{M_x \ell^2}{2EJ_x} << \ell$$

PICCOLI SPOSTAMENTI

$$\chi = \frac{d\varphi}{ds} \simeq \frac{d\varphi}{dz} = \frac{M_x}{EJ_x}$$



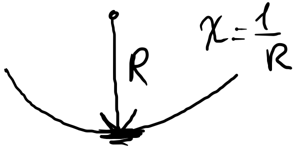
piccoli spostamenti

$$\frac{d u_y(\quad)}{dz} = -\frac{M_x}{EJ_x} z = tg\,\alpha \simeq \alpha$$

COME SI DEFINISCE LA CURVATURA DI UNA FUNZIONE

$$\left[\chi = \frac{y''(x)}{(1+y'^2(x))^{3/2}} \quad \text{per una funzione } y(x) \right]$$

se $y(x)$ è molto vicino all'asse x $\Rightarrow$ $\boxed{\chi \simeq y''(x)}$



$$\chi \simeq \frac{d^2 u_y(\quad)}{dz^2} = -\frac{M_x}{EJ_x}$$

: NEL PROBLEMA DI FLESSIONE RETTA LA CURVATURA È COSTANTE E VALE

$$\Downarrow$$

$$\frac{1}{R} \Rightarrow [\chi] = \left[\frac{1}{L}\right]$$

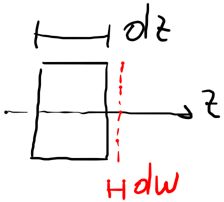
$$\left[\ |\chi| = \frac{M_x}{EJ_x}\ \right]$$

CURVATURA FLESSIONALE

EJ_x : COEFFICIENTE DI RIGIDEZZA FLESSIONALE

OSSERVAZIONE

• NEL PROBL. DI FORZA NORMALE :



$$\frac{dw}{dz} = \frac{N}{EA}$$

COEFFICIENTE DI RIGIDEZZA ASSIALE

TEORIA DELLE TRAVI

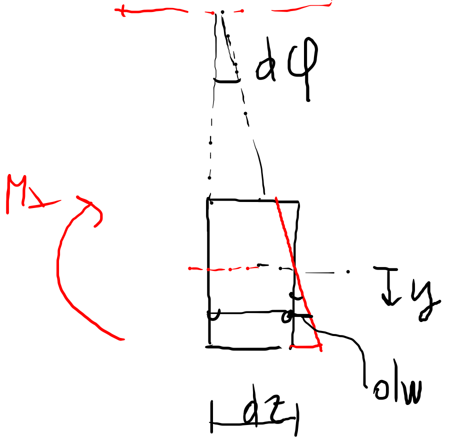
• . . . . FLESSIONE RETTA

$$\chi = \boxed{\frac{M_x}{EJ_x} = \frac{d\varphi}{dz}}$$

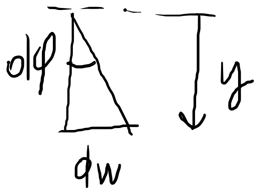
$$[\chi] = \left[\frac{FL}{FL^{-2}L^{4}}\right] = \left[\frac{1}{L}\right]$$

$$\frac{d\varphi}{dz} = \frac{M_x}{EJ_x}$$

φ : ADIMENSIONALE



$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} = \frac{M_x}{EJ_x}\, y \quad : \text{lineare in } y$$



$$d\varphi = \frac{dw}{y} = \frac{\varepsilon_{zz}\, dz}{y} = \frac{M_x}{EJ_x}\cancel{y}\, dz \cdot \frac{1}{\cancel{y}} = \frac{M_x}{EJ_x}\, dz$$

$$\boxed{\frac{d\varphi}{dz} = \chi = \frac{M_x}{EJ_x}}$$