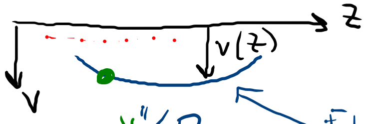
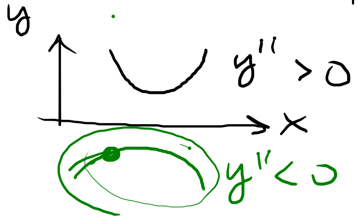
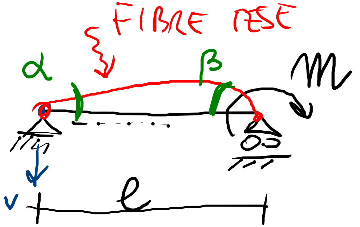
ES. DI INTEGRAZIONE DELLA LINEA ELASTICA (TRAVI SNELLE)

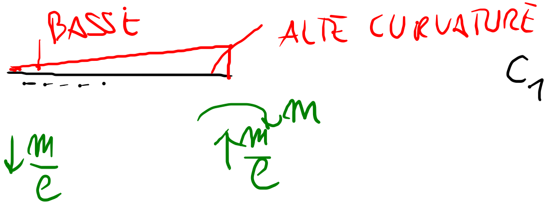
6/12/22

$$v''(z) = -\frac{M(z)}{EJ}$$









$v(z)$?

$$\begin{cases} v''(z) = -\dfrac{M(z)}{EJ} \\ v(0) = 0 \\ v(\ell) = 0 \end{cases}$$

$$M(z) = -\frac{m}{\ell} z$$

$$\longrightarrow v'' = \frac{m}{\ell EJ} z$$

$$v' = \frac{m}{\ell EJ}\frac{z^2}{2} + C_1$$

$$v = \frac{m}{\ell EJ}\frac{z^3}{6} + C_1 z + C_2$$

C_1, C_2 ? C. AI LIMITI

$$v(0) = \boxed{C_2 = 0}$$

$$v(\ell) = \frac{m \ell^{\cancel{3}\,2}}{6 \cancel{\ell} EJ} + C_1 \cancel{\ell} = 0$$

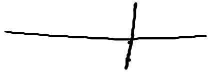
$$C_1 = -\frac{m\ell}{6EJ}$$

$$\boxed{v(z) = \frac{m}{6\ell EJ} z^3 - \frac{m\ell}{6EJ} z}$$

$v \le 0;\ z \in [0, \ell]$

$$\boxed{v'(z) = \frac{m}{2\ell EJ} z^2 - \frac{m\ell}{6EJ}} \Longrightarrow \alpha, \beta\,?$$

Rotazioni α, β :



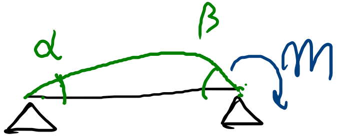
$$\varphi(z) = -v'(z)$$

$\alpha = \varphi(0) > 0$; $\beta = \varphi(l) < 0$

$$\varphi(z) = -\frac{m}{2lEJ} z^2 + \frac{ml}{6EJ}$$

$$\varphi(0) = \alpha = +\frac{ml}{6EJ} > 0$$

$$\varphi(l) = \beta = -\frac{ml}{2EJ} + \frac{ml}{6EJ} = -\frac{ml}{3EJ} < 0$$



$$[\alpha] = \left[\frac{ml}{EJ}\right] = \frac{FL \cdot L}{\frac{F}{L^2} L^4} = \frac{L^4}{L^4} \text{ IN MODULO} = [-] \text{ ADIMENS.}$$

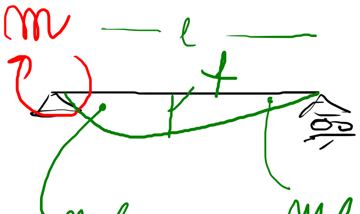
$$\begin{cases} \alpha = \dfrac{ml}{6EJ} \\ \beta = \dfrac{ml}{3EJ} \end{cases} \qquad \boxed{\beta = 2\alpha}$$

α, β = RADIANTI

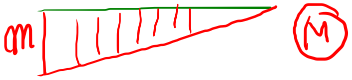
ESEMPIO: $m = 10\ kNm$, $l = 3\ m$

(N, mm) $E = 210\ GPa$, $J = 2000\ cm^4$

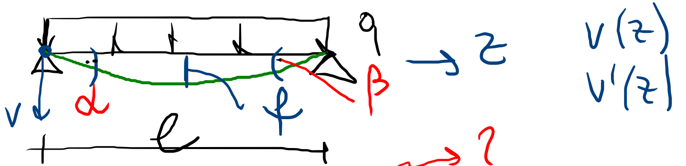
$$\alpha = \frac{10\,\cancel{000\,000} \cdot 3\cancel{000}}{6 \cdot 210\,\cancel{000} \cdot 20\cancel{00\,0000}} = \frac{3}{6 \cdot 210 \cdot 20} = \quad \text{RAD.}$$



$\frac{ml}{3EJ}$ $\qquad$ $\frac{ml}{6EJ}$

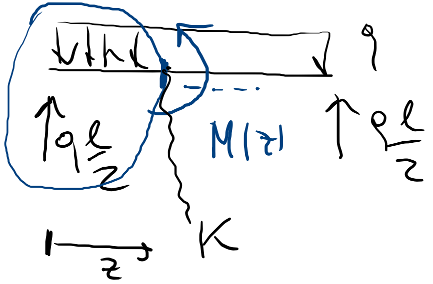


$$f = v\left(\frac{l}{2}\right) = \frac{3}{48}\frac{ml^2}{EJ} = \frac{1}{16}\frac{ml^2}{EJ}$$



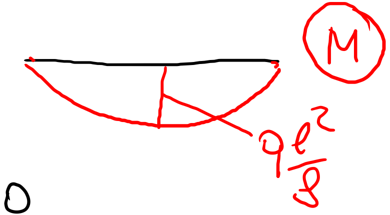
?

$$\begin{cases} v''(z) = -\dfrac{M(z)}{EJ} \\ v(0) = 0 \\ v(l) = 0 \end{cases}$$



$$\circlearrowleft K : \; -q\frac{l}{2}z + q\frac{z^2}{2} + M(z) = 0$$

$$M(z) = \frac{ql}{2}z - q\frac{z^2}{2}$$



$$v'' = \frac{q}{2EJ}\left[z^2 - lz\right] \quad ; \quad v' = \frac{q}{2EJ}\left[\frac{z^3}{3} - l\frac{z^2}{2}\right] + C_1$$

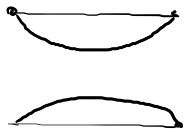
$$v = \frac{q}{2EJ}\left[\frac{z^4}{12} - l\frac{z^3}{6}\right] + C_1 z + C_2$$

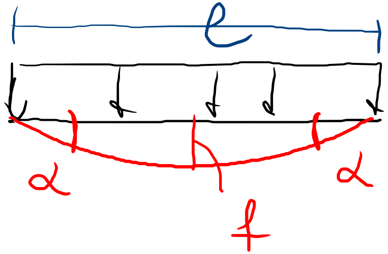
$$v(0) = 0 \implies C_2 = 0$$

$$v(l) = 0 \implies \frac{q}{2EJ}\left[\frac{l^4}{12} - \frac{l^4}{6}\right] + C_1 l = 0 \implies C_1 = \frac{ql^3}{24EJ}$$

$$v'(z) = \frac{q}{2EJ}\left[\frac{z^3}{3} - \frac{\ell z^2}{2}\right] + \frac{q\ell^3}{24EJ}$$

$$f = v\left(\frac{\ell}{2}\right) = \frac{5}{384}\,\frac{q\ell^4}{EJ}$$





$$\alpha = v'(0) = \boxed{\frac{q\ell^3}{24EJ}}$$

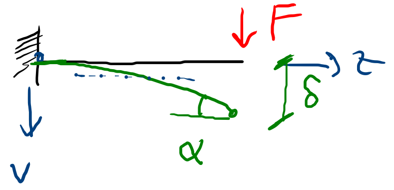
$$\beta = v'(\ell) = \frac{q\ell^3}{2EJ}\left[\frac{1}{3} - \frac{1}{2}\right] + \frac{q\ell^3}{24EJ} =$$

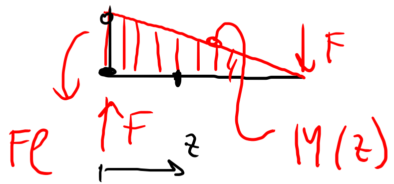
$$= \left(-\frac{1}{12} + \frac{1}{24}\right)\frac{q\ell^3}{EJ}$$

$$= \boxed{-\frac{1}{24}\,\frac{q\ell^3}{EJ}}$$

$$\begin{cases} \alpha = \dfrac{q\ell^3}{24EJ} \\ f = \dfrac{5}{384}\,\dfrac{q\ell^4}{EJ} \end{cases}$$

ES: MENSOLA CON FORZA CONCENTRATA





$$\boxed{\delta = v(\ell) = -\frac{F}{EJ}\left(\frac{\ell^3}{6} - \frac{\ell^3}{2}\right) = \frac{F\ell^3}{3EJ}}$$

$$\boxed{\alpha = v'(\ell) = -\frac{F}{EJ}\left(\frac{\ell^2}{2} - \ell^2\right) = \frac{F\ell^2}{2EJ}}$$

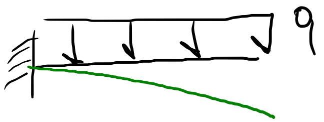
$$M(z) = -F\ell + Fz = F(z-\ell)$$

$$\begin{cases} v'' = -\dfrac{M(z)}{EJ} \\ v(0) = 0 \\ v'(0) = 0 \end{cases} \Longrightarrow \quad C_2 = 0, \quad C_1 = 0$$

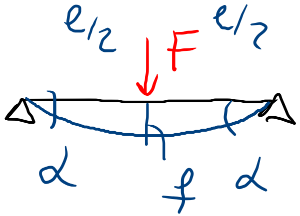
$$v'' = -\frac{F(z-\ell)}{EJ}$$

$$v' = -\frac{F}{EJ}\left(\frac{z^2}{2} - \ell z\right) + C_1$$

$$v = -\frac{F}{EJ}\left(\frac{z^3}{6} - \ell\frac{z^2}{2}\right) + C_1 z + C_2$$



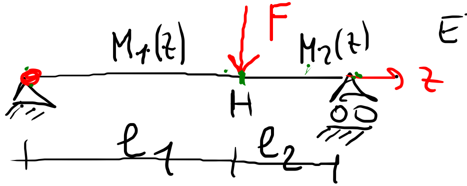
$$\frac{q\ell^4}{8EJ}$$



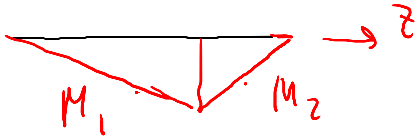
$$\alpha = \frac{F\ell^2}{16EJ}$$

$$f = \frac{F\ell^3}{48EJ}$$

INTEGRAZ. LINEA ELASTICA CON PIÙ CAMPI DI INTEGRAZ.

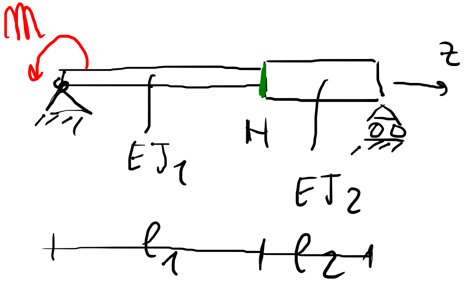


EJ: cost



DISCONT. NEL MOMENTO

$$\begin{cases} v''(z) = -\dfrac{M_1(z)}{EJ} & z \in [0, \ell_1] \\ v''(z) = -\dfrac{M_2(z)}{EJ} & z \in [\ell_1, \ell_1+\ell_2] \end{cases}$$



DISCONT. GEOMETRICA (CAMBIO DI RIGIDEZZA)

M(z): STESSA FUNZIONE PER I DUE DOMINI DI INTEGR

$$\begin{cases} v''(z) = -\dfrac{M(z)}{EJ_1} & z \in [0, \ell_1] \\ v''(z) = -\dfrac{M(z)}{EJ_2} & z \in [\ell_1, \ell_1+\ell_2] \end{cases}$$

IN ENTRAMBI HO 4 COSTANTI DI INTEGRAZIONE:

QUALI SONO LE CONDIZIONI AL CONTORNO?

$$\begin{cases} v(0) = 0 \\ v(\ell_1+\ell_2) = 0 \end{cases}$$

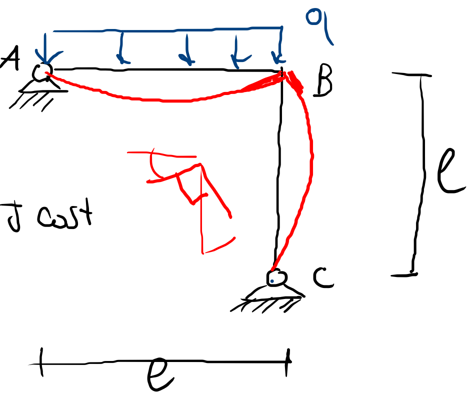
\+ 2 CONDIZ. DI RACCORDO

$$\begin{cases} v(\ell_1^-) = v(\ell_1^+) \\ v'(\ell_1^-) = v'(\ell_1^+) \end{cases}$$

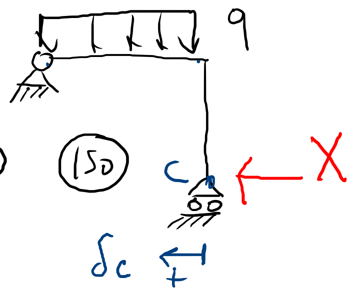
IN H STESSO ABBASSAMENTO (SPOSTAM.)

E STESSA ROTAZIONE

METODO DELLE FORZE PER STRUTTURE INFLESSE



EJ cost



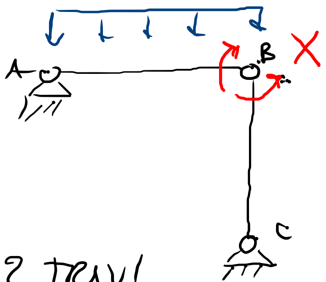
SP1

ISO

EQ. CONGRUENZA

$$\delta_c(q,X)=0$$

SP2



$$\boxed{\varphi_{BA}=\varphi_{BC}}$$

EQ. CONGRUENZA

$$\left(\varphi_{BA}-\varphi_{BC}=0 \qquad \Delta\varphi_B=0\right)$$

1 v. IPERSTATICA

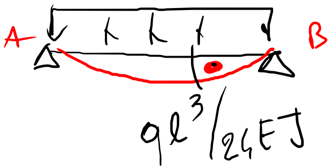
ISO

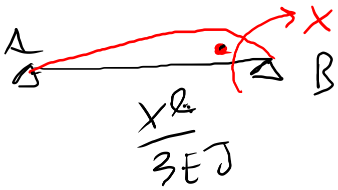
INCOGNITE STATICHE:

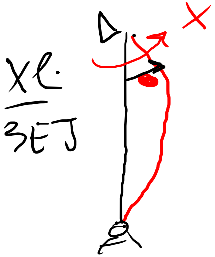
V_A, H_A, V_C, H_C

AB, BC: 2 TRAVI APPOGGIATE!

$$\varphi_{BA}(q,X)=\varphi_{BC}(X)$$

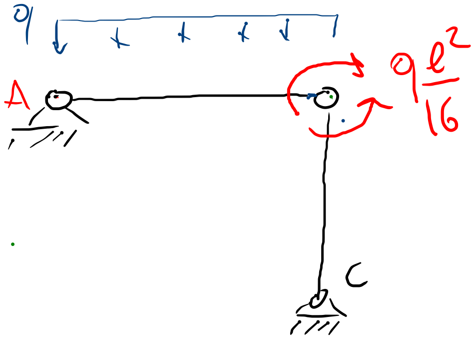






$$-q\frac{l^3}{24EJ}+\frac{Xl}{3EJ}=-\frac{Xl}{3EJ}$$

$$\Rightarrow \boxed{X=+ql^2/16}$$



SP EQUILIBRATA E

CONGRUENTE NELLE

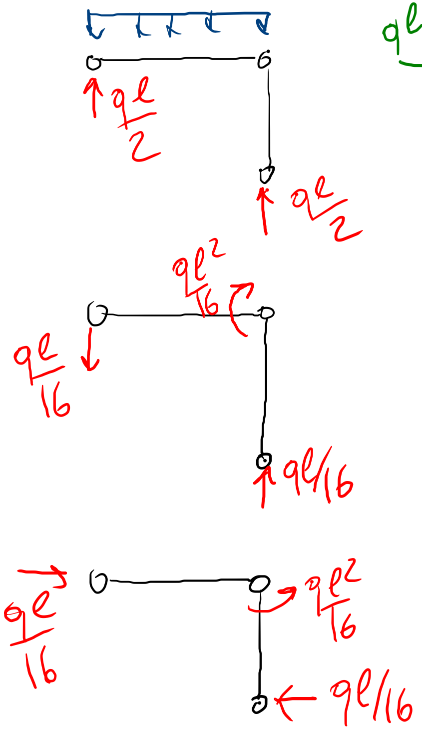
ROTAZIONI IN B

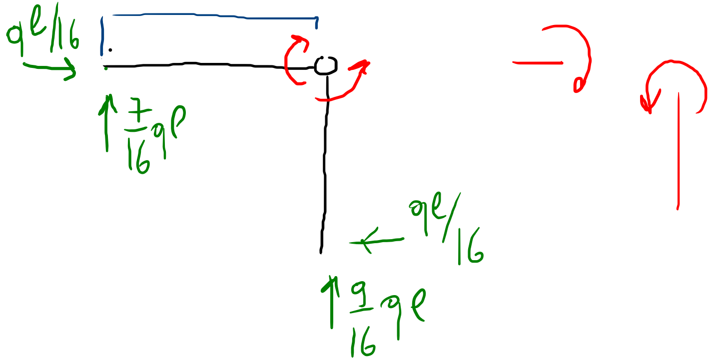
$$\varphi_{BA} = \varphi_{BC}$$

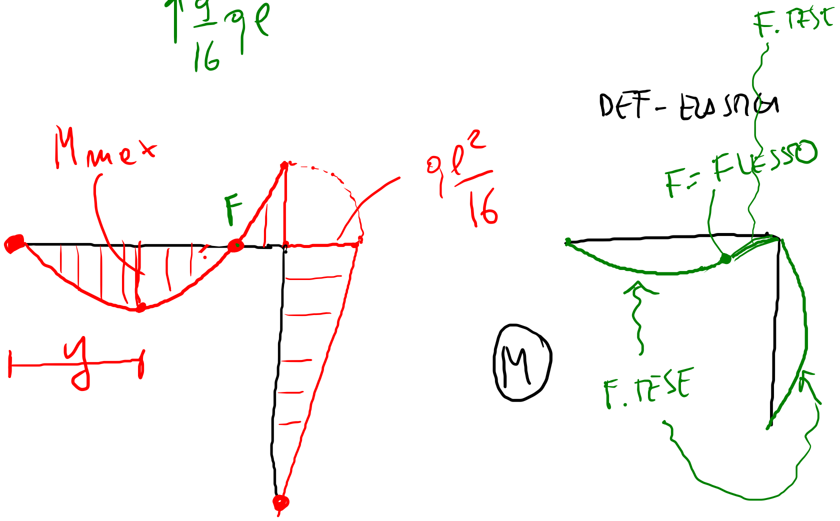
$$\Downarrow$$

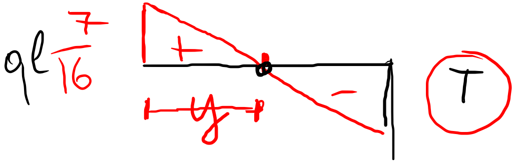
EQUIVALENTE ALLA

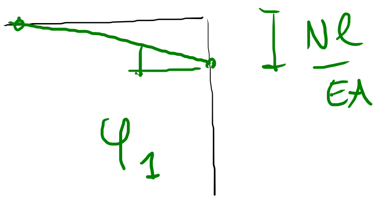
IPERST. ASSEGNATA.





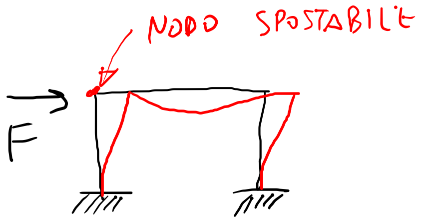






$\frac{Nl}{EA} \longrightarrow 0 \qquad \left(\frac{EA}{l} \to \infty \right)$

Nell'esempio appena risolto si dice che il NODO B è <u>FISSO</u>



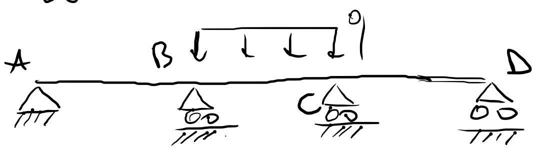
φ_1: ROTAZIONE CAUSATA DALL'ABBASSAM. DEL PUNTO B (ACCORCIAMENTO DEL PILASTRO)

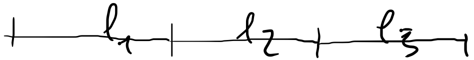
$\varphi_1 \ll \varphi_{BA}(q, X)$
DOVUTA ALLA FLESSIONE

φ_1 SI TRASCURA $\Rightarrow$ IPOTESI DI TRASCURABILITÀ DELLA <u>DEFORMABILITÀ ASSIALE</u>

TRAVE CONTINUA (SU PIÙ DI 2 APPOGGI)

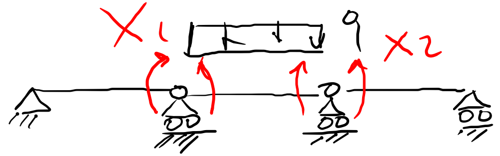
EJ cost

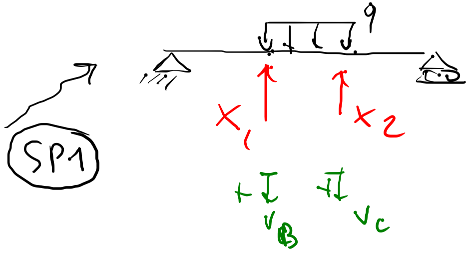




2 V. IPERST.

!!!!





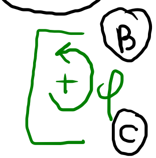
$$\begin{cases} V_B = 0 \\ V_C = 0 \end{cases}$$

SIST. DELLE EQ. DI CONGRUENZA

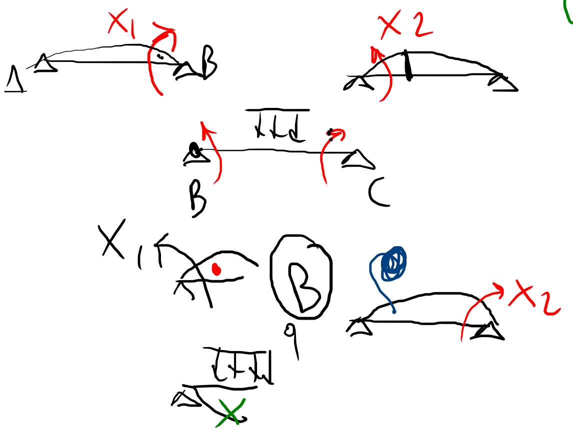
$V_B(q, X_1, X_2)$??

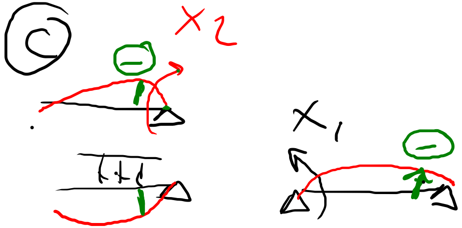
$V_C(q, X_1, X_2)$

SP2



$$\begin{cases} \varphi_{BA} = \varphi_{BC} \\ \varphi_{CB} = \varphi_{CD} \end{cases}$$





$$\begin{cases} -\frac{X_1 l_1}{3EJ} = +\frac{X_1 l_2}{3EJ} - \frac{q l_2^3}{24EJ} + \frac{X_2 l_2}{6EJ} \\ -\frac{X_2 l_2}{3EJ} + \frac{q l_2^3}{24EJ} - \frac{X_1 l_2}{6EJ} = \frac{X_2 l_3}{3EJ} \end{cases}$$

↳ X_1, X_2