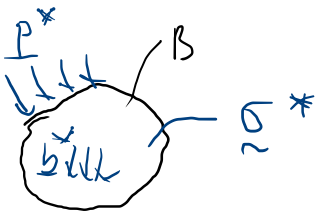
UTILIZZO DEL T.L.V. PER IL CALCOLO DEGLI SPOST. 13/12/22
IN STR ISOSTATICHE $(L_{ve} = L_{vi})$

LEZ DOMANI h 14-16
AULA A - IDRAULICA (C2)

MECC. DEI SOLIDI

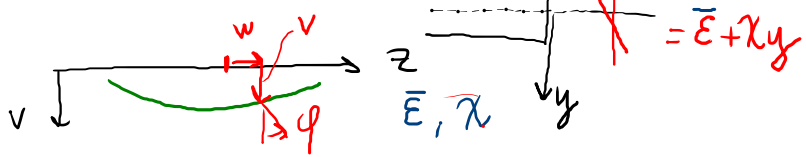


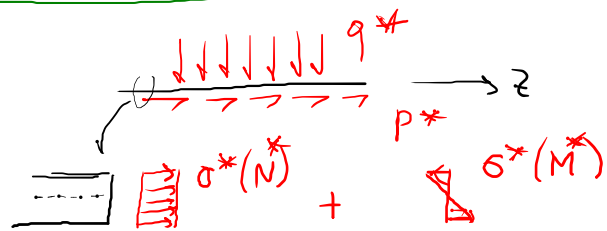
SIST. SPOSTAM - DEFORMAZIONI (CINEMATICAM. AMMISSIBILE)

SIST. FORZE - TENSIONI (*) (STATICAMENTE AMMISSIBILE)

$$L_{ve} = \int_{\partial B} \underline{p}^* \cdot \underline{u}\, dS + \int_B \underline{b}^* \cdot \underline{u}\, dV \quad ; \quad L_{vi} = \int_B \underset{\sim}{\sigma}^* \cdot \underset{\sim}{\varepsilon}\, dV$$

MECC. DELLE STRUTTURE





TRA q^*, p^* e M^*, N^*, T^* valgono le EQUAZ. PUNTUALI DI EQUILIBRIO

$$L_{ve} = \int_{STR} q^* v + p^* w\, dz + \text{LAVORO VIRTUALE DELLE REAZ. VINCOLARI}$$

$$L_{vi} = \int_B \underset{\sim}{\sigma}^* \cdot \underset{\sim}{\varepsilon}\, dV \quad \overset{\text{TRAVE}}{\underset{\text{SNELLA}}{=}} \quad \int_{\text{TRAVE}} \sigma_{zz}^* \varepsilon_{zz}\, dV = \int_{\text{TRAVE}} \sigma_{zz}^* \left(\bar{\varepsilon} + \chi y\right) dV =$$

$$\underset{\sim}{\sigma}^* = \begin{bmatrix} 0 & 0 & \tau_{xz}^* \\ 0 & 0 & \tau_{yz}^* \\ \cdot & \cdot & \sigma_{zz}^* \end{bmatrix}$$

→ IL LORO CONTRIBUTO AL LAVORO SI TRASCURA PERCHÈ GLI SCORRIM. γ_{xz}, γ_{yz} SONO PICCOLI

$$\overline{\varepsilon_{zz}(y)} = \int_{\text{STR.}} \left\{ \int_A \sigma_{zz}^* \left(\bar{\varepsilon} + \chi y\right) dS \right\} dz$$

$$= \int_{\text{STR}} \left\{ \bar{\varepsilon} \overbrace{\int_A \sigma_{zz}^* dS}^{N^*} + \chi \overbrace{\int_A \sigma_{zz}^* y\, dS}^{M^*} \right\} dz$$

DALLE EQ. CDS DEL PROBL. DSV

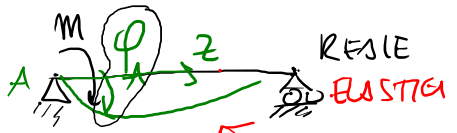
LAVORO VIRTUALE INTERNO PER TRAVI SNELLE

$$\boxed{L_{vi} = \int_{STR} \bar{\varepsilon} N^* + \chi M^*\, dz}$$

CALCOLO DI SPOST. IN STR ISOSTATICHE

ES: CALCOLARE LA ROTAZ. NEL PUNTO A DELLA TRAVE (φ_A?)

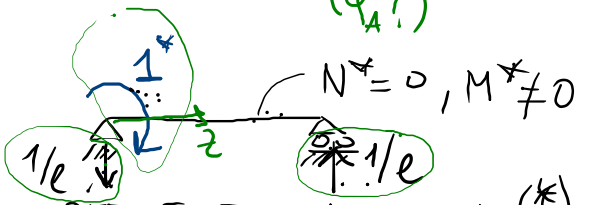
UTILIZZ. $L_{ve} = L_{vi}$



SIST CINEM. AMM. (CONGRUENTE)

$v, \varphi, \chi, \bar{\varepsilon}$

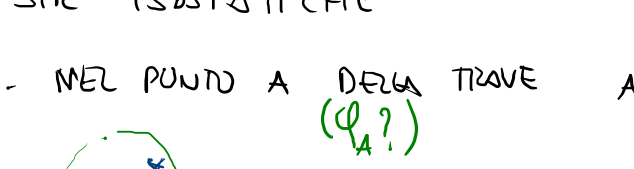
$\chi(z) = \frac{M(z)}{EJ}$



$N^* = 0, M^* \neq 0$

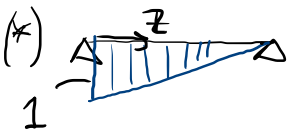
SIST. STAT. AMMISSIBILE (*) (FITTIZIA) (EQUILIBRATO)

F^*, M^*, N^*



STR. "REALE"

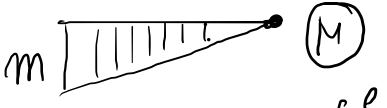
$M^*(z) = 1 - \frac{z}{\ell}$ [-]



$L_{ve} = L_{vi}$

$$1^* \cdot \varphi_A + \frac{1}{\ell} \cdot 0 + \frac{1}{\ell} \cdot 0 = \int_{STR} M^*(z)\, \chi(z)\, dz$$

$M(z) = m - \frac{m}{\ell} z$



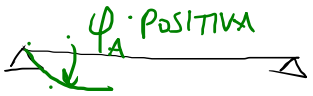
1 EQ. PER L'INCOGNITA φ_A

$$1^* \cdot \varphi_A = \int_0^\ell M^* \frac{M}{EJ}\, dz \Rightarrow \varphi_A = \frac{1}{EJ} \int_0^\ell \left(1 - \frac{z}{\ell}\right) m \left(1 - \frac{z}{\ell}\right) dz$$

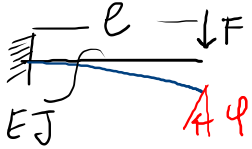
$$\varphi_A = \frac{m}{EJ} \int_0^\ell 1 + \frac{z^2}{\ell^2} - \frac{2z}{\ell}\, dz$$

$$\varphi_A = \frac{m}{EJ} \left[z + \frac{z^3}{3\ell^2} - \frac{2z^2}{2\ell} \right]_0^\ell$$

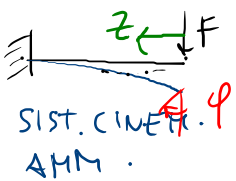
$$\varphi_A = \frac{M}{EJ}\,\frac{\ell}{3} = +\frac{M\ell}{3EJ}$$ (OK)

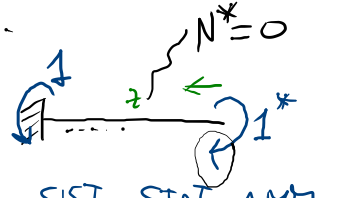


ES.



Lve = Lui

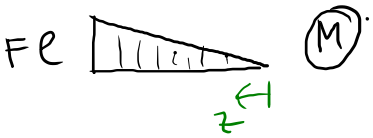




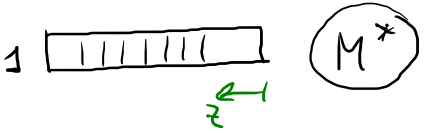
Lve = Lui

$$1^* \cdot \varphi = \int \cancel{N^* \varepsilon} + M^* \chi \, dz$$ (CURVATURA ELASTICA)

$$\varphi = \int_0^\ell M^* \frac{M}{EJ} \, dz$$



$M = -Fz$

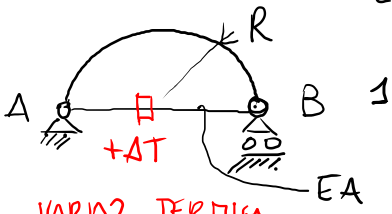


$M^* = -1$

$$\varphi = \frac{1}{EJ}\int_0^\ell (-1)(-Fz)\,dz$$

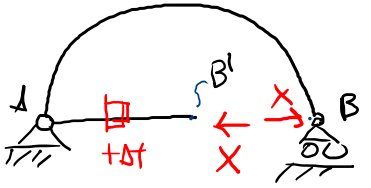
$$= \frac{1}{EJ} F \int_0^\ell z \, dz = +\frac{F\ell^2}{2EJ}$$

ES: RISOLVERE LA STR. IPERSTATICA TRASCURANDO LA DEFORMABILITA' ASSIALE DELL'ARCO E CONSIDERANDO COSTANTE IL COEFF. DI RIGIDEZZA EJ COSTANTE.



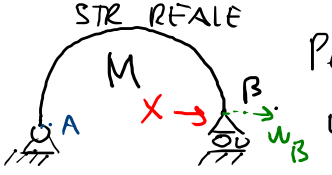
1 V. IPERST. INTERNAMENTE

VARIAZ. TERMICA UNIFORME SULLA SEZIONE SU TUTTA LA BIELLA AB.



$(+\rightarrow)$ $\boxed{w_B = w_{B'}}$ EQ. DI CONGRUENZA

w_B ?

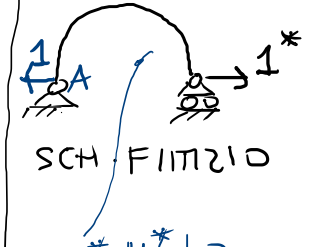


Per calcolare w_B uso T.L.V.

$w_{B'}$ 

$w_{B'} = +\alpha \Delta T \cdot 2R$ — LUNGH. BIELLA $\times \frac{2R}{EA}$

↑ COEFF DI DILAT. LINEARE



SCH. FITTIZIO

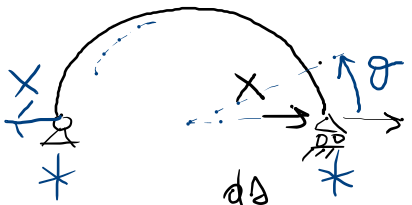
$N^*, M^* \neq 0$

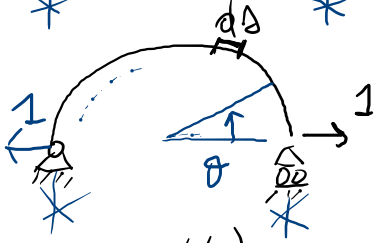
$L_{ve} = L_{vi}$ "PICCOLA"

$1^* \cdot w_B = \int_{STR} N^* \varepsilon + M^* \chi \, dz$

$w_B = \int_{STR} M^* \chi \, dz$ → CURVATURE ELASTICHE

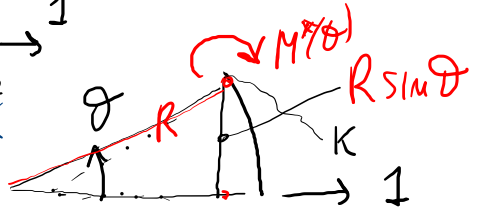
$w_B = \int M^* \frac{M}{EJ} \, dz$





$M^*(\vartheta)$

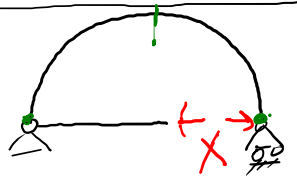
$M(\vartheta) = X M^*(\vartheta)$



$+\circlearrowleft_K: \quad 1 \cdot R\sin\vartheta - M^*(\vartheta) = 0$

$M^*(\vartheta) = R\sin\vartheta \qquad \vartheta \in [0,\pi]$

$\boxed{M(\vartheta) = XR\sin\vartheta}$



dalla pag. preced.

$$W_B = \frac{1}{EJ}\int_0^{\pi} X M^*(\vartheta)^2 \underbrace{R\,d\vartheta}_{ds}$$

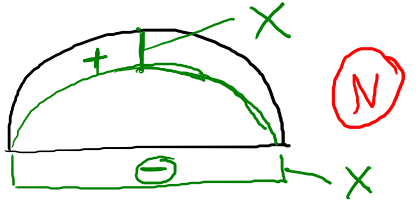
$$= \frac{XR}{EJ}\int_0^{\pi} R^2 \sin^2\vartheta\, d\vartheta = \frac{XR^3}{EJ}\left[\frac{\vartheta}{2} - \frac{\sin 2\vartheta}{4}\right]_0^{\pi}$$

$$= \frac{XR^3}{EJ}\left[\frac{\pi}{2}\right] = + \frac{X\pi R^3}{2EJ}$$

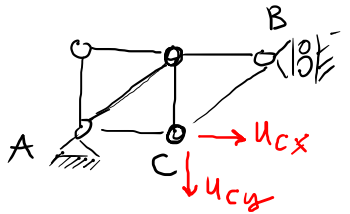
TORNO ALL' IPERST... $\qquad W_B = W_{B'}$

$$\frac{X\pi R^3}{2EJ} = \alpha \Delta T\, 2R - \frac{X\,2R}{EA} \Rightarrow \boxed{X > 0}$$

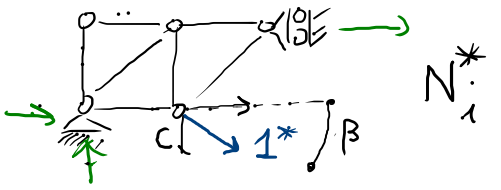
$X(\Delta T)$



CALCOLO SPOST. IN STR. RETICOLARI ISOSTATICHE



SIST. CINEM. AMM.



SIST. STAT. AMM.

$L_{ve} = L_{vi}$

$1^* \cos\beta \cdot u_{cx} + 1^* \sin\beta \cdot u_{cy}$ + LAVORO VIRTUALE ESTERNO DELLE REAZ. VINCOLARI. $= \int N^* \bar{\varepsilon}\, dz = \sum_{i=1}^{m_a} N_i^* \bar{\varepsilon}_i \ell_i$

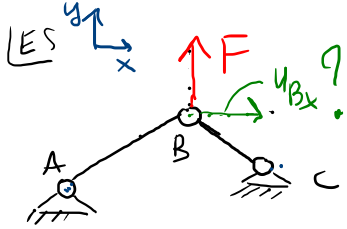
$\bar{\varepsilon}_i \int_{\text{ASTA}_i} dz$

$= \sum_{i=1}^{m_e} N_i^* \bar{\varepsilon}_i \ell_i$ ..... $= \sum_{i=1}^{m_e} N_i^* \left(\frac{N_i}{EA} \pm \alpha \Delta T \right) \ell_i$

SCALDA

RAFFREDDA

DEFORMAZIONE ELASTICA, DEFORMAZ. TERMICA



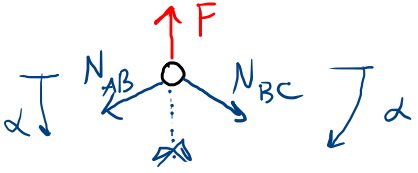
$AB = \ell \quad (\ell_1)$

$BC = \ell/2 \quad (\ell_2)$

EA: cost

$\Delta\alpha$ Δ^{α}

STR REALE (CONGRUENTE)



$+\uparrow: \quad F - 2N_{BC}\sin\alpha = 0$

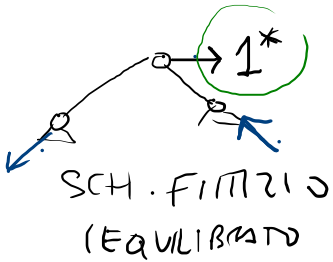
$N_{BC} = \dfrac{F}{2\sin\alpha} = N_{AB}$ (TRAZIONE)

	AB	BC
N	$F/2\sin\alpha$	$F/2\sin\alpha$

$L_{ve} = L_{vi}$

DEF. ELASTICHE

$1^{*} \cdot u_{Bx} = N^{*}_{AB}\dfrac{N_{AB}}{EA}\ell_{AB} + N^{*}_{BC}\dfrac{N_{BC}}{EA}\ell_{BC}$



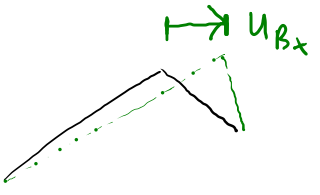
$\sin\alpha = S \qquad \cos\alpha = C$

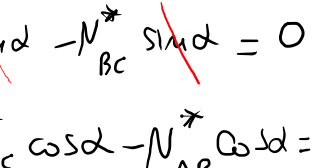
$u_{Bx} = \dfrac{1}{2C}\dfrac{F}{2S}\dfrac{\ell}{EA} + \left(-\dfrac{1}{2C}\right)\left(+\dfrac{F}{2S}\right)\dfrac{\ell}{2EA}$

$N_{AB} = N_{BC}$

$u_{Bx} = \dfrac{F\ell}{EA\,4CS}\left[1 - \dfrac{1}{2}\right] =$

$= +\dfrac{F\ell}{8EA\sin\alpha\cos\alpha}$

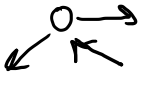




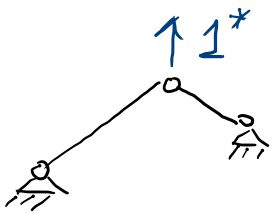
$\begin{cases} +\uparrow: \; -N^{*}_{AB}\sin\alpha - N^{*}_{BC}\sin\alpha = 0 \\ +\rightarrow: \; 1 + N^{*}_{BC}\cos\alpha - N^{*}_{AB}\cos\alpha = 0 \end{cases}$

$N^{*}_{BC} = -\dfrac{1}{2\cos\alpha} \; ; \; N^{*}_{AB} = \dfrac{1}{2\cos\alpha}$

	AB	BC
N^{*}	$+\frac{1}{2\cos\alpha}$	$-\frac{1}{2\cos\alpha}$



u_{By} ? $\Rightarrow$ SCH. FITTIZIO:

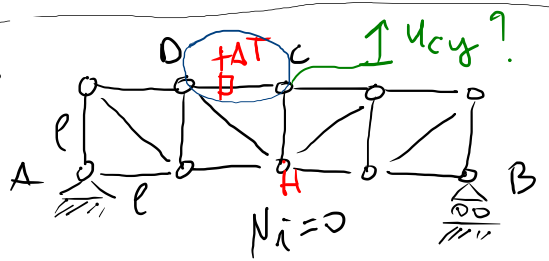


$\Rightarrow$ $N_{BC} = F\,N^*_{BC}$

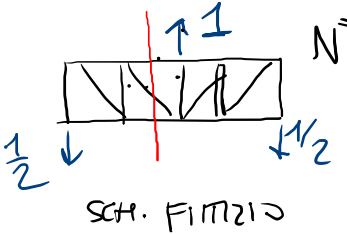
$N_{AB} = F\,N^*_{AB}$

$1 \cdot u_{By} = \dots\dots$

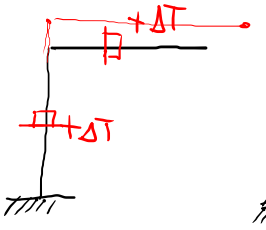
LES



ISOST.

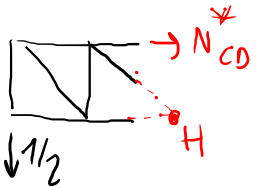


SCH. FITTIZIO



VARIAZ. TERMICHE IN STR. ISOST: SPOSTAMENTI MA NON REAZIONI VINCOLARI $\Rightarrow M, T, N = 0$

ASTA CD TENDE AD ALLUNGARSI PER EFFETTO DELLA ΔT.



$\curvearrowright^+_H : N^*_{CD}\,\ell - \frac{1}{2}\,2\ell = 0$

$N^*_{CD} = +1$

$L_{ve} = L_{vi}$

$1 \cdot u_{cy} = \sum_{i=1}^{Ma} N^*_i \left(\frac{N_i}{EA} \pm \alpha \Delta T_i \right) \ell_i$ (0 sopra N_i; SOLO SU CD sopra $\pm\alpha\Delta T_i$)

$u_{cy} = N^*_{CD}\,(\alpha \Delta T)\,\ell_{CD} \Rightarrow$

$u_{cy} = +1 \cdot \alpha \Delta T \ell = +\alpha \Delta T \ell$