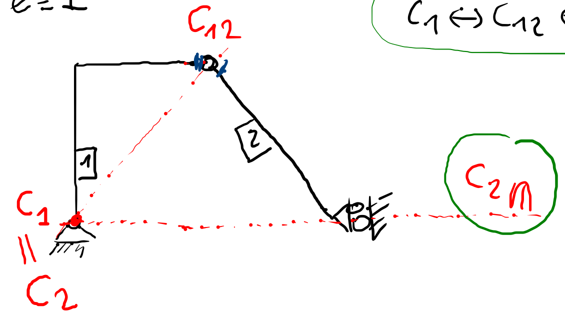
..... ANCORA CATENE CINEMATICHE

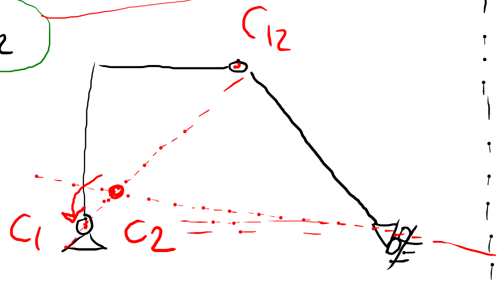
20/12/22

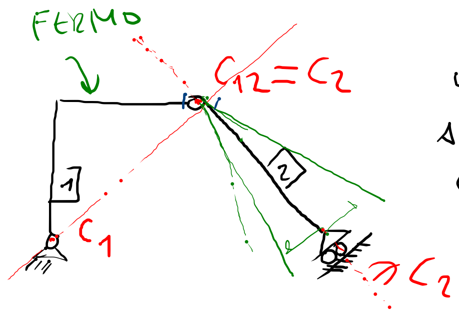
— ALCUNI CASI PARTICOLARI (COINCIDENZA DI CENTRI)

e=1

$C_1 \leftrightarrow C_{12} \leftrightarrow C_2$





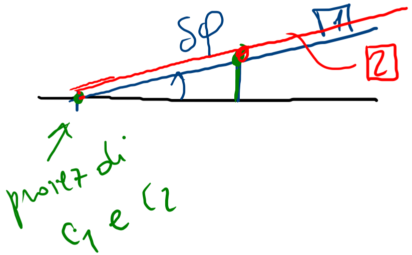


UN CENTRO ASS. E UN CIR RELATIVO COINCID.

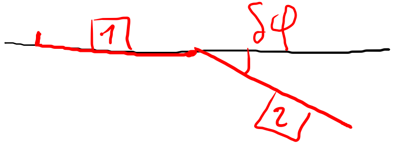
⇓

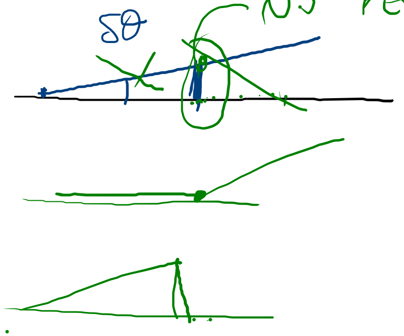
CINEMATISMO PARZIALE

DUE CENTRI ASSOLUTI COINCIDENTI

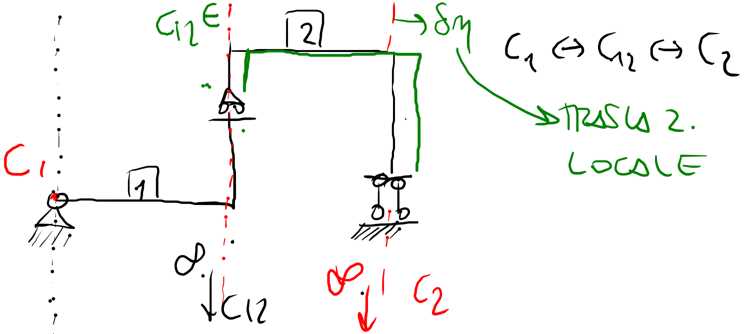


$\boxed{1}$ e $\boxed{2}$ RUOTANO COME SE FOSSERO UN C.R. UNICO.

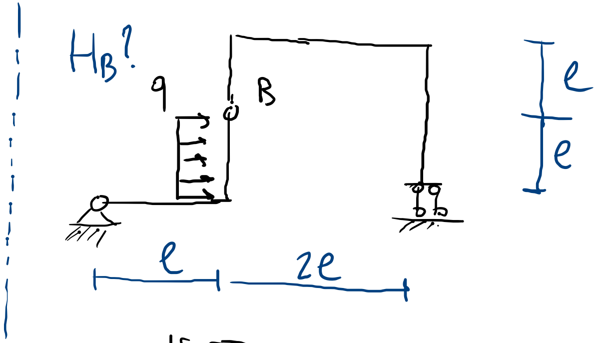


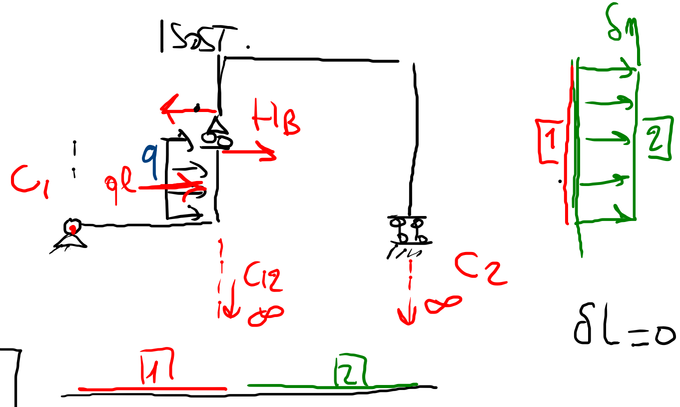


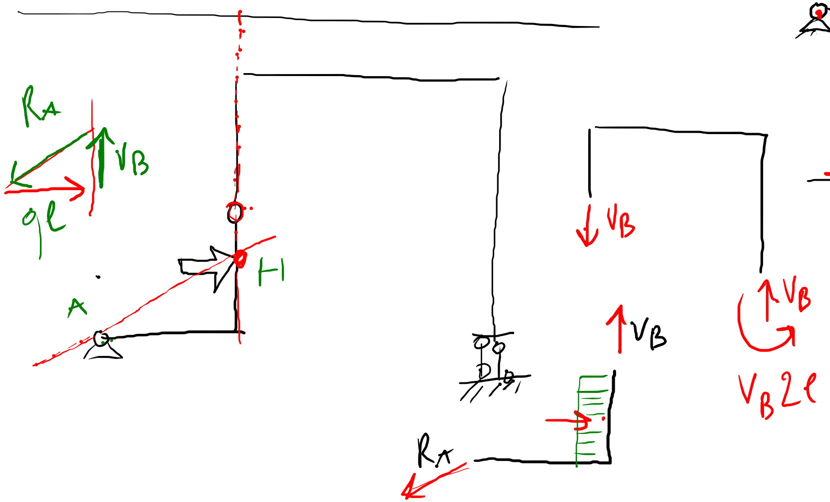
NO PERCHÉ PER $\boxed{3}$ QUESTO PUNTO DEVE AVERE SPOSTAM. NULLO



COINCID.
CINEMATISMO
PARZIALE







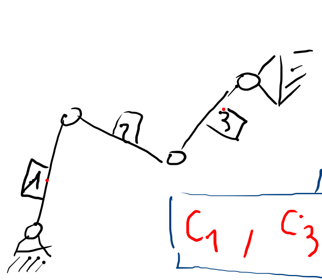
$$\delta L = 0 \quad \forall \delta\eta \Rightarrow H_B$$

$$\underbrace{q\ell \cdot 0 + H_B \cdot 0}_{\boxed{1}} - \underbrace{H_B\,\delta\eta}_{\boxed{2}} = 0 \quad \forall \delta\eta \Rightarrow \boxed{H_B = 0}$$

II TEOREMA DELLE CATENE CINEMATICHE (PIÙ DI 2 C.R.)

C.N. PERCHÉ SI SVILUPPI UN CINEMATISMO IN UNA STR LABILE CON UN GDL FORMATA DA PIÙ DI 2 C.R. È CHE I CENTRI LORO RELATIVI SIANO ALLINEATI A 3 A 3:

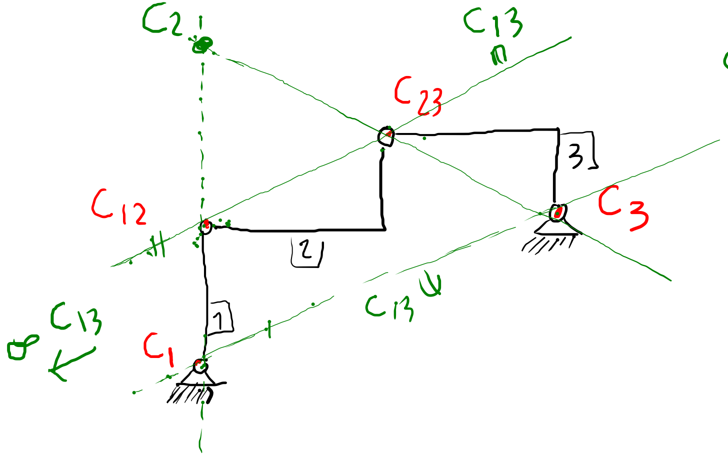
$$\boxed{C_{ij} \leftrightarrow C_{ik} \leftrightarrow C_{jk}}$$



C_1, C_3, C_{12}, C_{23}

C_2 C_{13} ???

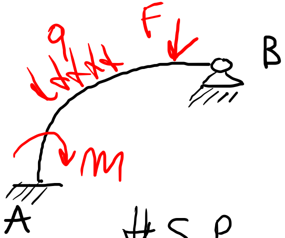
$C_i \leftrightarrow C_{ij} \leftrightarrow C_j$



C_2? $\begin{cases} C_1 \leftrightarrow C_{12} \leftrightarrow C_2 \\ C_2 \leftrightarrow C_{23} \leftrightarrow C_3 \end{cases}$

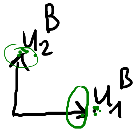
C_{13}? $\begin{cases} C_1 \leftrightarrow C_{13} \leftrightarrow C_3 \\ C_{12} \leftrightarrow C_{13} \leftrightarrow C_{23} \end{cases}$

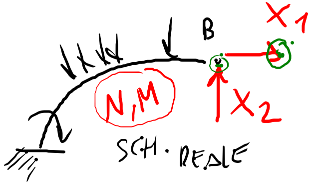
EQ. DI MÜLLER-BRESLAU (1908): FORMALIZZAZIONE DEL METODO DELLE FORZE UTILIZZ. IL T.L.V. (ORA CON VINCOLI FISSI E SENZA DISTORS. TERMICHE)



(2 V. IPERST.)

$\Downarrow$ S.P.





$$\begin{cases} u_1^B = 0 \\ u_2^B = 0 \end{cases}$$

EQ. DI CONGR.

u_1^B SCH. FITTIZ.

$(X_1 = 1)$

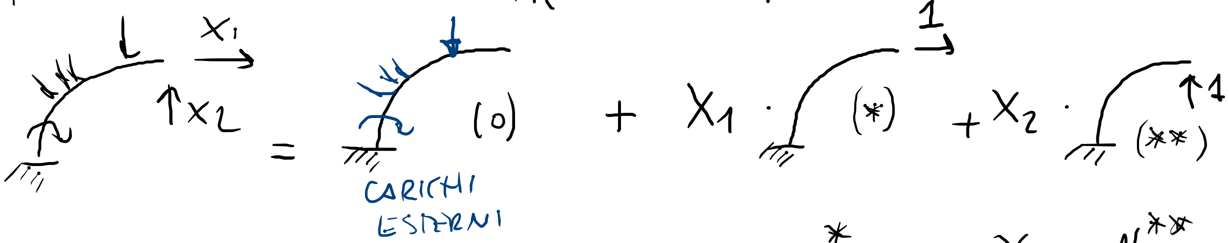
$(L_{ve}^* = L_{vi}^*)$

$$1 \cdot u_1^B = \int_{STR} N^* \overbrace{\frac{N}{EA}}^{\bar{\varepsilon}} + M^* \overbrace{\frac{M}{EJ}}^{\chi} \, dz \longrightarrow$$

RAGIONO SU N, M $\Longrightarrow$

$N = N(q, F, m, X_1, X_2)$

$M = M(\quad\quad\quad)$



$$N = N^0 + X_1 N^* + X_2 N^{**}$$

$$M = M^0 + X_1 M^* + X_2 M^{**}$$

$u_1^B(q, F, m, X_1, X_2)$

$u_2^B(\text{'' '' '' }, X_1, X_2)$

CON IL T.L.V. VOGLIO CALCOLARE FORMALM. u_1^B e u_2^B (SPOST. DI UNA STR. ISOSTATICA)

$$1\cdot u_1^B = \int_{str} N^*\left[\frac{N^0}{EA} + X_1\frac{N^*}{EA} + X_2\frac{N^{**}}{EA}\right] + M^*\left[\frac{M^0}{EJ}+X_1\frac{M^*}{EJ}+X_2\frac{M^{**}}{EJ}\right] dz$$

$$u_1^B = \underbrace{\int_{str} N^*\frac{N^0}{EA}+M^*\frac{M^0}{EJ}\,dz}_{u_{10}} + X_1 \underbrace{\left\{\int \frac{N^{*2}}{EA}+\frac{M^{*2}}{EJ}\,dz\right\}}_{\eta_{11}} + X_2 \underbrace{\left\{\int N^*\frac{N^{**}}{EA}+M^*\frac{M^{**}}{EJ}\,dz\right\}}_{\eta_{12}}$$

$\Downarrow$

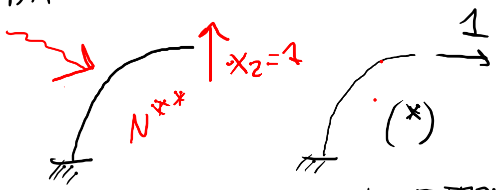
CONGRUENZA

$$0 = u_{10}+X_1\eta_{11}+X_2\eta_{12}$$

u_{10}: SPOST. DELLA STR. REALE NELLA DIREZ. DI X_1 CAUSATO DAI CARICHI ESTERNI

η_{11}: SPOST. NELLA STR. REALE NELLA DIREZ. DI X_1 CAUSATO DA $X_1 = 1$

η_{12}: SPOST. " " " NELLA DIREZ DI X_1 CAUSATO DA $X_2 = 1$



STR. REALE

SCH. FITTIZIO

RIPETO IL PROCEDIMENTO RICERCANDO LO SPOST. u_2^B DELLA STR REALE (PRINCIPALE)

SCH. FITTIZIO

$$L_{ve}^{**} = L_{vi}^{**}$$

$$1\cdot u_2^B = \int_{str} N^{**}\frac{N}{EA}+M^{**}\frac{M}{EJ}\,dz$$

CONGR

$$0 = u_{20}+X_1\eta_{21}+X_2\eta_{22}$$

$$0=\underbrace{\int_{sm} N^{**}\frac{N^0}{EA}+M^{**}\frac{M^0}{EJ}\,dz}_{u_{20}}+X_1\underbrace{\left\{\int_{sm} N^{**}\frac{N^*}{EA}+M^{**}\frac{M^*}{EJ}\,dz\right\}}_{\eta_{21}}+X_2\underbrace{\left\{\int \frac{N^{**2}}{EA}+\frac{M^{**2}}{EJ}\,dz\right\}}_{\eta_{22}}$$

$$\boxed{0=\underbrace{u_{i0}}_{\text{CONTRIB. CARICHI ESTERNI}}+\sum_{j=1}^{m}\underbrace{\eta_{ij}X_j}_{\text{CONTRIB. INCOGNITE IPERST.}}\qquad (i=1,\dots,m)}$$

m INCOGNITE IPERST. $(X_1,\dots,X_m)$

EQ. DI MÜLLER BRESLAU

COEFF. DI INFLUENZA

$$\begin{cases}\eta_{ii}>0\\ \eta_{ij}=\eta_{ji}\quad (i\neq j)\end{cases}$$

$$\left[\eta_{ij}=\int N^{*i}\frac{N^{*j}}{EA}+M^{*i}\frac{M^{*j}}{EJ}\,dz\right]$$

$$[0]=[u_0]+[H]\,[X]$$

$[H]$ → MATRICE QUADRATA $H_{ij}=\eta_{ij}$ $(m\times m)$ MATRICE DI CEDEVOLEZZA o FLESSIBILITÀ

$[X]$ → vettore incognite iperst.

$[u_0]$ → VETTORE SPOST. u_{i0}

→ $[H]$ È DEFINITA POSITIVA

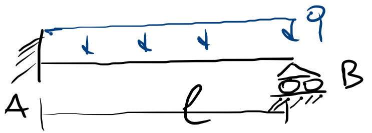
$[H]^{-1}$ È SEMPRE DEFINITA, C'È SEMPRE SOLUZ. AL SIST.

DUE OSSERVAZ.

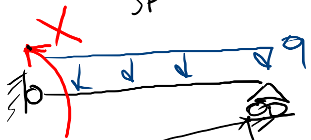
- PER UN PROBL 1 VOLTA IPERST, TUTTI I COEFF DI X HANNO LO STESSO SEGNO SE PORTATI NELLO STESSO MEMBRO DELL' EQ. DI CONGR. $\Rightarrow$ $\underline{\eta_{11} > 0}$

$$0 = \eta_{10} + \eta_{11} X \qquad \eta_{11} > 0$$

- PER UN PROBL 2 VOLTE IPERST POSSO VERIFICARE CHE $\eta_{12} = \eta_{21}$



SP



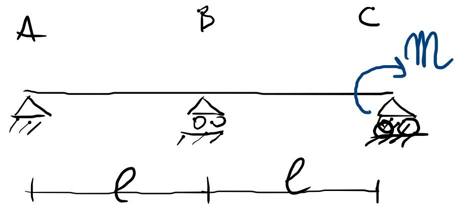
$$\boxed{\varphi_A = 0} \quad +\circlearrowleft$$

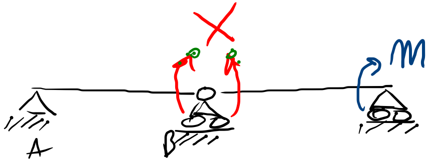
$$\boxed{X = \frac{q\ell^2}{8}}$$

MOM. ALL' INCASTRO

$$\frac{X\ell}{3EJ} - \frac{q\ell^3}{24EJ} = 0 \quad ; \quad 0 = \underbrace{-\frac{q\ell^3}{24EJ}}_{\eta_{10}} + \underbrace{\frac{\ell}{3EJ}}_{\eta_{11}} \cdot X$$

$$\frac{q\ell^3}{24EJ}$$

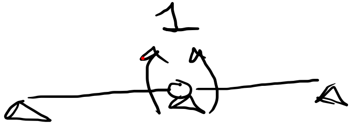




$\varphi_{BA} = \varphi_{BC}$ $\circlearrowright +$

$$\frac{Xl}{3EJ} = -\frac{Xl}{3EJ} - \frac{ml}{6EJ}$$

$$X\left(\frac{l}{3EJ} + \frac{l}{3EJ}\right) = -\frac{ml}{6EJ}$$



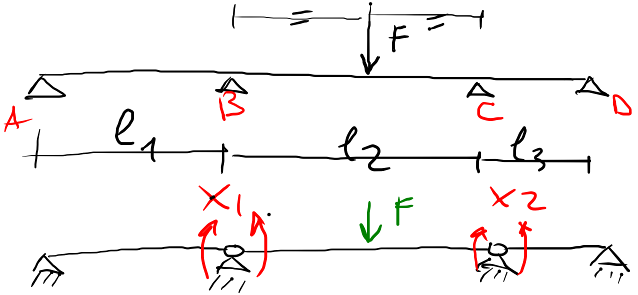
SCH. FITTIZIO

$$\underbrace{\Delta\varphi_B}_{\varphi_{BA}-\varphi_{BC}} = 0 \quad (+)$$

$$\Downarrow$$

$\varphi_{10} = \eta_{10}$

$$X \cdot \underbrace{\frac{2l}{3EJ}}_{\eta_{11}} + \frac{ml}{6EJ} = 0$$



$$\begin{cases} \Delta\varphi_B = 0 & (+) \\ \Delta\varphi_C = 0 & (+) \end{cases}$$

$$\begin{cases} \dfrac{X_1 l_1}{3EJ} + \dfrac{X_1 l_2}{3EJ} - \dfrac{F l^2}{16EJ} + \dfrac{X_2 l_2}{6EJ} = 0 \\[2ex] + \dfrac{X_1 l_2}{6EJ} - \dfrac{F l^2}{16EJ} + \dfrac{X_2 l_2}{3EJ} + \dfrac{X_2 l_3}{3EJ} = 0 \end{cases}$$

$$\begin{cases} X_1 \overbrace{\left(\dfrac{l_1}{3EJ} + \dfrac{l_2}{3EJ} \right)}^{\eta_{11} > 0} + X_2 \overbrace{\dfrac{l_2}{6EJ}}^{\eta_{12}} \overbrace{- \dfrac{F l^2}{16EJ}}^{u_{10}} = 0 \\[2ex] X_1 \cdot \underbrace{\dfrac{l_2}{6EJ}}_{\eta_{21}} + X_2 \underbrace{\left(\dfrac{l_2}{3EJ} + \dfrac{l_3}{3EJ} \right)}_{\eta_{22} > 0} \underbrace{- \dfrac{F l^2}{16EJ}}_{u_{20}} = 0 \end{cases}$$

$$\eta_{12} = \eta_{21}$$