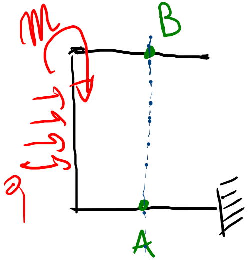
... EQ. DI MÜLLER-BRESLAU

21/12/22

OSSERVAZIONE: COME CALCOLO UNO SPOSTAMENTO RELATIVO (O ROTAZ. RELATIVA) TRA DUE PUNTI DI UNA STR. ISOSTATICA

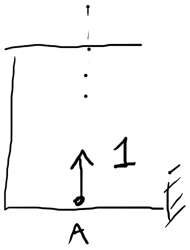


↓ v_B

δ_{AB}: SPOST. RELATIVO TRA A E B $\Rightarrow$ $v_A + v_B = \delta_{AB}$ positivo se i punti si AVVICINANO

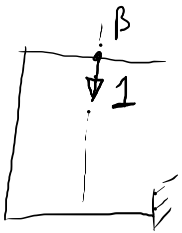
↑ v_A $\Rightarrow$ SPOST. ASSOLUTI

SCH. FITTIZIO PER v_A:



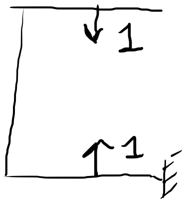
v_A

SCH. FITTIZIO PER v_B



v_B

SCH. FITTIZIO δ_{AB}

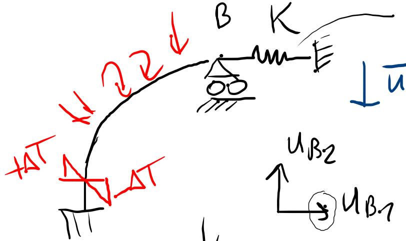


δ_{AB}

$L_{ve} = L_{vi}$ $\rightarrow$ $\delta_{AB} > 0$: I PUNTI SI AVVICINANO

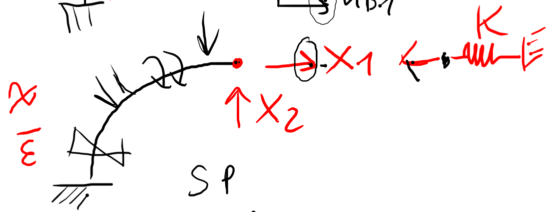
< 0: " " ALLONTANANO

ESTENSIONE AL CASO DI VINCOLI CEDEVOLI E DISTORS. TERMICHE



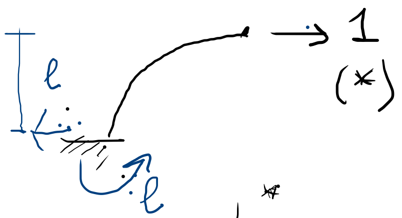
CEDIM. ELASTICO

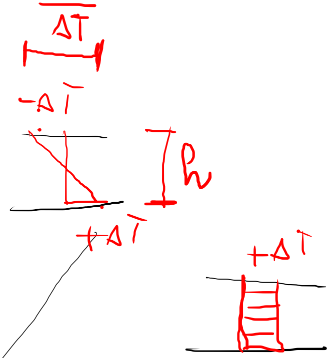
$\bar{u}$ → CEDIMENTO ANELASTICO



$$\begin{cases} u_{B_1} = -\dfrac{X_1}{K} \\ u_{B_2} = -\bar{u} \end{cases}$$

CALCOLO u_{B_1}





L_{ve}^* L_{vi}^*

$$1\cdot u_{B_1} + 1\cdot 0 + \ell\cdot 0 = \int_{STR} M^* \chi\, ds + N^* \bar\varepsilon\, ds$$

$$1\cdot u_{B_1} = \int_{STR} M^*\left(\underbrace{\frac{M}{EJ}}_{\chi^{el}} \pm \underbrace{\frac{\alpha\Delta T}{h}}_{\chi^{TERM}}\right) + N^*\left(\underbrace{\frac{N}{EA}}_{\bar\varepsilon_{el}} \pm \underbrace{\alpha\Delta T}_{\bar\varepsilon_{TERM}}\right) ds$$

CONGR.:

$$-\frac{X_1}{K} = \int M^*\left[\frac{M_0+X_1M^*+X_2M^{**}}{EJ} \pm \frac{\alpha\Delta T}{h}\right] + N^*\left[\frac{N^0+X_1N^*+X_2N^{**}}{EA} \pm \alpha\Delta T\right] ds$$

$\bar{u}_1 = u_{10} + \eta_{11} X_1 + \eta_{12} X_2$; $\bar{u}_1 \left(= -\frac{X_1}{k} \right)$: CEDIMENTO VINCOLARE IN CORRISPONDENZA DELLO SPOSTAMENTO CORRELATO A X_1

$$u_{10} = \int_{STR} M^* \left(\frac{M_0}{EJ} \pm \frac{\alpha \Delta T}{h} \right) + N^* \left(\frac{N^0}{EA} \pm \alpha \Delta T \right) ds$$

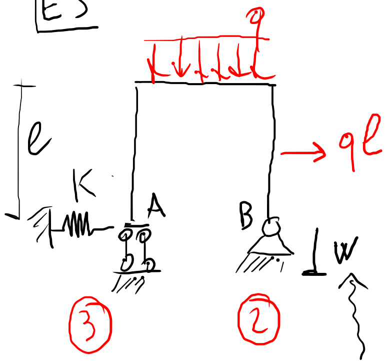
: LO SPOSTAMENTO NELLA DIREZ. CORRELATA A X_1 DOVUTO A CARICHI ESTERNI E DISTORSIONI DIFFUSE.

$$\bar{u}_i = u_{i0} + \sum_{j=1}^{n} \eta_{ij} X_j \quad (i = 1, \dots, n)$$

$$u_{i0} = \int_{STR} M^{*i} \left(\quad \square \quad \right) + N^{*i} \left(\quad \triangle \quad \right) ds$$

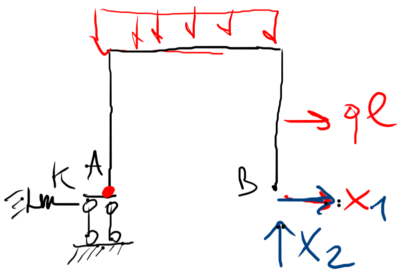
Nel nostro esercizio $(n = 2)$; $\bar{u}_2 = -\bar{u}$

ES

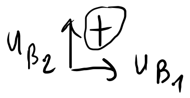


2 v. IPER

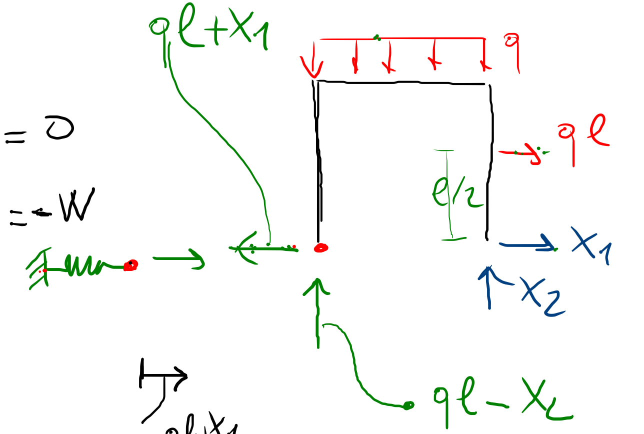
CEDIM. ANELASTICO



S.P.



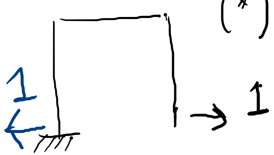
$$\begin{cases} u_{B_1} = 0 \\ u_{B_2} = -W \end{cases}$$



$$u_{A_1} = \frac{ql + X_1}{K}$$

$$q\frac{l^2}{2} + ql \cdot \frac{l}{2} - X_2 l$$

CALCOLO DI u_{B_1} NELLA S.P.

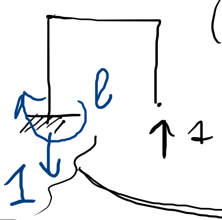


(*) M^*

$$L^*_{ve} = L^*_{vi}$$

$$1 \cdot u_{B_1} - 1 \cdot \frac{ql + X_1}{K} = \int M^* \frac{M}{EJ} ds$$

CALCOLO DI u_{B_2} NELLA SP



(**) $L^{**}_{ve} = L^{**}_{vi}$

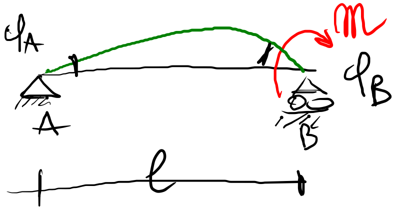
$$1 \cdot u_{B_2} + 1 \cdot 0 + l \cdot 0 = \int M^{**} \frac{M}{EJ} ds$$

CON CONGRUENZA

$$-\frac{ql + X_1}{K} = u_{10} + \eta_{11} X_1 + \eta_{12} X_2$$

$$-W = u_{20} + \eta_{21} X_1 + \eta_{22} X_2$$

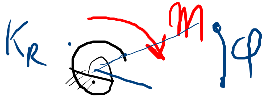
SCHEMI ELEMENTARI PER IL METODO DEGLI SPOSTAMENTI



$$\varphi_A = \frac{m\ell}{6EJ} \Rightarrow m = \frac{6EJ}{\ell}\varphi_A \sim K_R$$

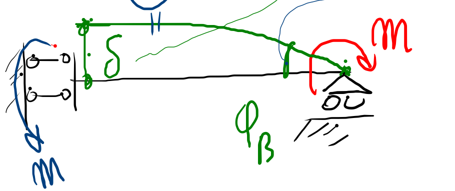
RIGIDEZZA ALLA ROTAZ.

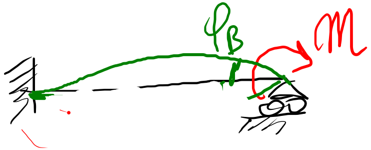
$$\varphi_B = \frac{m\ell}{3EJ} \Rightarrow m = \frac{3EJ}{\ell}\varphi_B$$



$$m = K_R\varphi$$

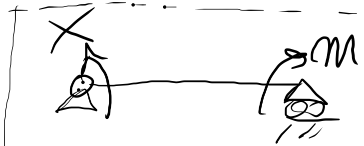
$$\varphi_B = \frac{m\ell}{1 \cdot EJ} \Rightarrow m = \frac{EJ}{\ell}\varphi_B$$





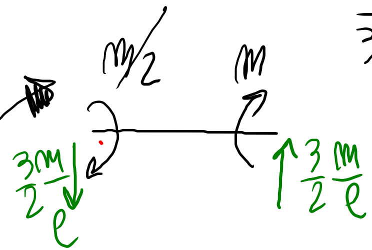
IV. IPERST.

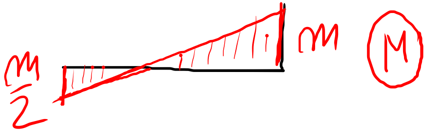
$$\varphi_B\,? = \frac{m\ell}{4EJ}$$

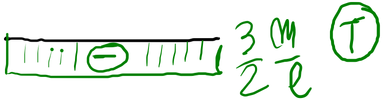


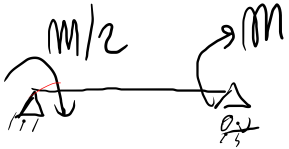
$$\varphi_A = 0 \quad \circlearrowleft +$$

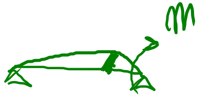
$$\frac{X\ell}{3EJ} + \frac{m\ell}{6EJ} = 0 \; ; \quad X = -\frac{m}{2}$$



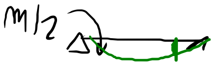








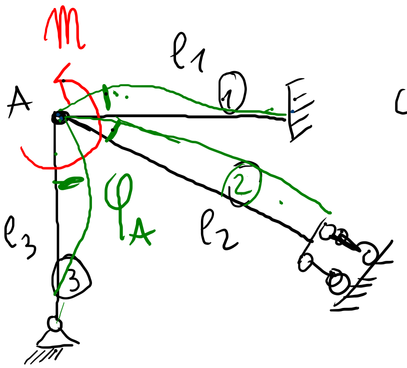
$$\varphi_B = \frac{m\ell}{3EJ} - \frac{m/2\;\ell}{6EJ} = +\frac{m\ell}{4EJ}$$



INTRODUZ. AL METODO DEGLI $\boxed{\text{SPOSTAMENTI}}$ (TELAIO CON UN NODO FISSO)

(METODO DELLE $\boxed{\text{FORZE}}$)

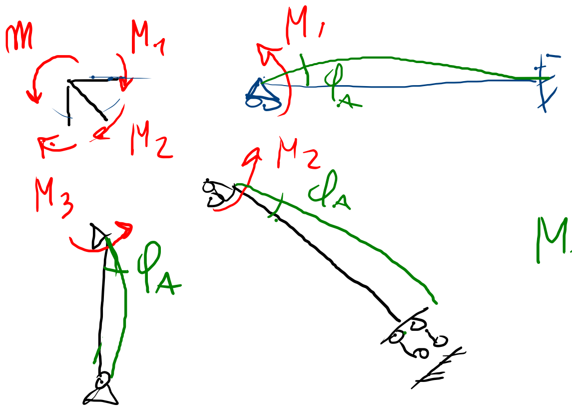
INCOGNITE



$\varphi_A(\mathfrak{m})$?

↖ INCOGNITA

$R_i = \left(\frac{EJ}{\ell}\right)_i \doteq$ RIGIDEZZA ALLA ROTAZIONE



$M_1 = 4 R_1 \varphi_A$

$M_2 = 1 \cdot R_2 \varphi_A$

$M_3 = 3 \cdot R_3 \varphi_A$

$(M_i = c_i R_i \varphi_A)$

COEFF DELLO SCHEMA CONSIDERATO)

EQUILIBRIO DEL NODO

(COME QUELLA DI CONGRUENZA NEL METODO DELLE FORZE)

$\circlearrowleft: \ \mathfrak{m} - M_1 - M_2 - M_3 = 0$

$\mathfrak{m} - 4R_1\varphi_A - R_2\varphi_A - 3R_3\varphi_A = 0$

$\mathfrak{m} = \left(\sum_{i=1}^{3} c_i R_i\right) \varphi_A$;

$$\boxed{\varphi_A = \frac{\mathfrak{m}}{\sum_{i=1}^{3} c_i R_i}}$$

se $R_i = R$ $\quad \varphi_A = \frac{\mathfrak{m}}{8R}$

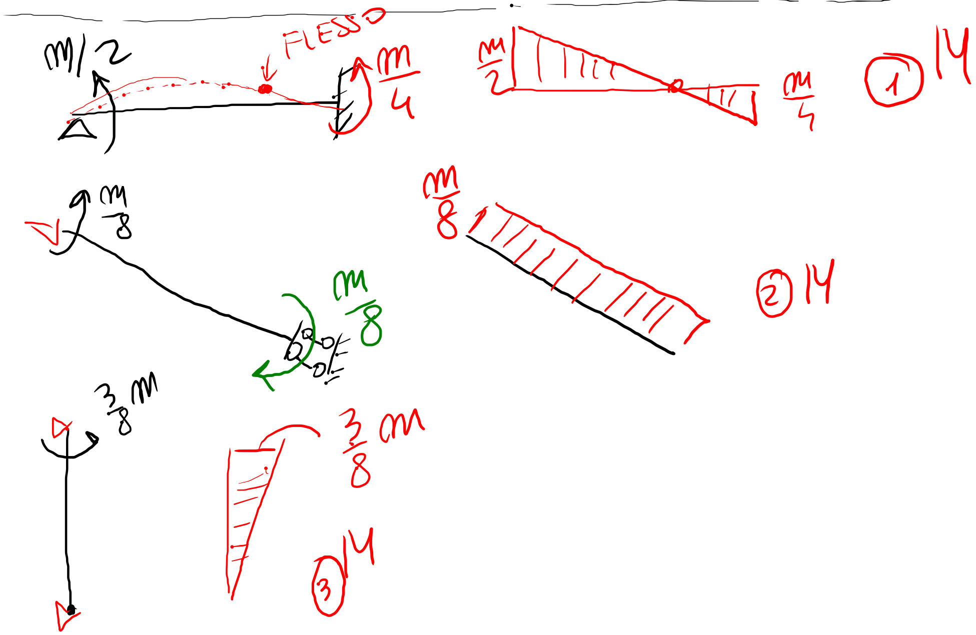
$$M_i = c_i R_i \frac{\mathfrak{m}}{\sum_{k=1}^{3} c_k R_k} = \frac{c_i R_i}{\sum_k c_k R_k} \mathfrak{m}$$

ρ_i

ρ_i : COEFF. DI RIPARTIZIONE

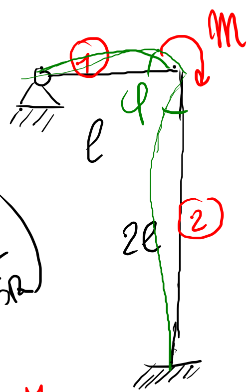
$\sum_i \rho_i = 1$

$$\ldots M_1 = \frac{4R}{8R}\, m = \frac{m}{2} \; ; \quad M_2 = \frac{1R}{8R}\, m = \frac{m}{8} \; ; \quad M_3 = \frac{3R}{8R}\, m = \frac{3}{8}\, m$$



DAL PUNTO DI VISTA OPERATIVO ABBIAMO "RIPARTITO" UN MOMENTO APPLICATO AL NODO.

ES EJ: cost



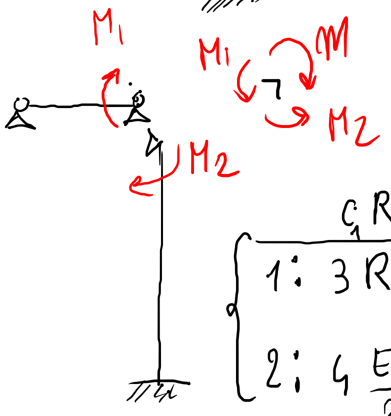
$\left(\varphi = \frac{m}{5R}\right)$

METODO DINAMICO: SFRUTTO φ, SCRIVO L'EQUILIBRIO DEL NODO, CALCOLO φ, SOSTITUISCO φ NEGLI SCHEMI STATICI, CALCOLO LE REAZ. VINCOLARI E TRACCIO I DIAGR.

METODO VELOCE: RIPARTISCO m $\propto$ AI COEFF DI RIGIDEZZA

$$M_1 = \frac{3R}{5R}\, m = \underbrace{\frac{3}{5}}_{p_1} m \quad ; \quad M_2 = \frac{2R}{5R} = \underbrace{\frac{2}{5}}_{p_2} m$$

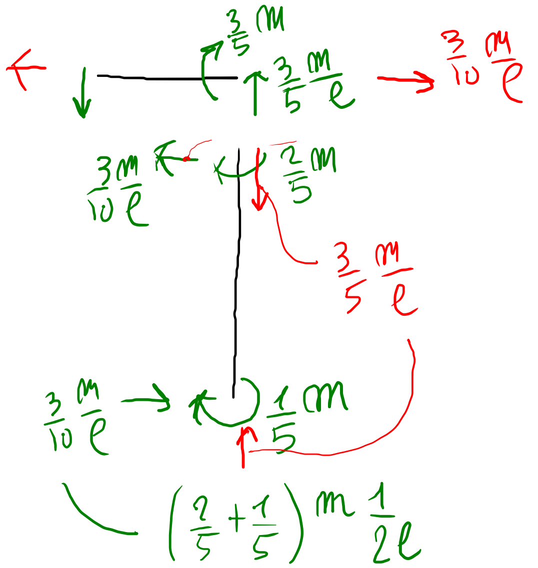
$p_1 + p_2 = 1$

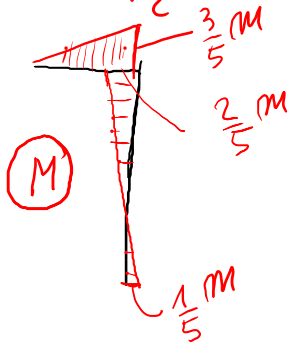


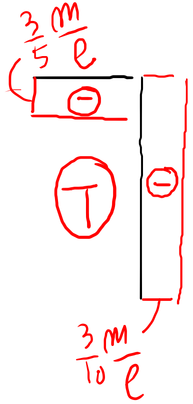
$\left(R = \frac{EJ}{l}\right)$

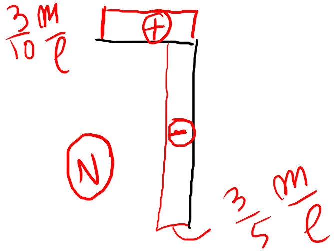
	$c_i R_i$
1:	$3R$
2:	$4\,\frac{EJ}{2l} = 2R$

$$\sum c_i \cdot R_i = 3R + 2R = 5R$$

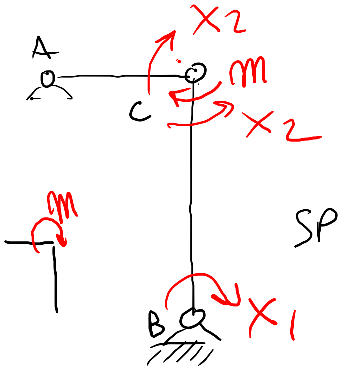


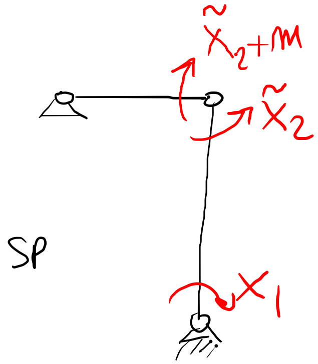






OSSERVAZ. : CON IL METODO DELLE FORZE...





σ σ

$$\begin{cases}\varphi_B = 0 \\ \varphi_{CA} = \varphi_{CB}\end{cases}$$

$$X_2 = -\frac{3}{5} m \qquad \tilde{X}_2 = -\frac{2}{5} m$$