Testi del Syllabus

Resp. Did.	**CUCCAGNA SCIPIO**	**Matricola: 015277**
Docente	**CUCCAGNA SCIPIO, 6 CFU**	

Anno offerta:	**2022/2023**
Insegnamento:	**529SM-1 - ADVANCED ANALYSIS - mod. A**
Corso di studio:	**SM34 - MATEMATICA**
Anno regolamento:	**2022**
CFU:	**6**
Settore:	**MAT/05**
Tipo Attività:	**B - Caratterizzante**
Anno corso:	**1**
Periodo:	**Annualità Singola**
Sede:	**TRIESTE**

Testi in italiano

Lingua insegnamento	English
Contenuti (Dipl.Sup.)	a) Fundamental principles of Functional Analysis: Banach and Hilbert spaces. Continuous linear operators. b) Basic function spaces: continuous functions, Lebesgue spaces (L^p). c) Weak topologies.
Testi di riferimento	- H. Brezis, Functional Analysis, Springer - K. Yosida, Functional analysis, Springer. - M. Reed, B. Simon, Functional analysis, Academic Press. - W. Rudin, Analisi reale e complessa, Boringhieri. The instructor will post in Moodle his personal lecture notes of course.
Obiettivi formativi	By the end of the course the student will be able to manage the fundamental tools of Functional Analysis and to approach the further steps of this area of Mathematics, like the theory of Distributions and of Sobolev spaces, in order to face the first arguments in Partial Differential Equations and Calculus of Variations. The student will be able to read autonomously advanced monographs of Functional Analysis, to understand proofs and applications of theorems of any level, and to apply them to various fields of Mathematics. He/she will be able to solve problems of basic level.
Prerequisiti	Calculus I and II. Basic notions in measure theory. Basic notions in general topology.
Metodi didattici	Lectures and solutions of problems.
Modalità di verifica dell'apprendimento	The final exam is in two parts. The written part (2 hours) consists in the solution of some problems, concerning the contents of the course. The oral part is devoted to ascertain the comprehension and the managing of the topics reached by the candidate.

Programma esteso	0. Metric and normed spaces. Converging and Cauchy sequences; completeness, compactness, precompactness and relative compactness; density and separablity. Banach spaces and space of continuous linear operators on Banach spaces, dual space and extension theorem. 1. Analytic form of Hahn-Banach theorem, gauge of a convex set, first and second geometric form of Hahn-Banach theorem. Baire theorem, Banach-Steinhaus theorem, open mapping theorem, inverse mapping theorem, closed graph theorem. Diagonal procedure. Convex sets and convex functions. Examples: space of continuos functions on a real interval and L^p spaces. 2. Inner product spaces, orthogonality and orthonormality, orthonormal systems. Pitagorean theorem, Bessel inequality, Schwarz inequality, polarization identity. Hilbert spaces, orthogonal complement, projection operators, direct sum. Riesz theorem. Hilbert bases, countablity of the basis and separability, Parseval identity. Lax-Milgram theorem. Continuous linear operators on Banach and Hilbert spaces, adjoint operator, selfadjoint operators. 3. Space of continuous functions on a compact metric space. Completeness and separability. Partition of unity. Equicontinuity and Ascoli-Arzelà theorem. 4. Topology generated by a family of functions; weak topology in a Banach space, basis of a weak topology; properties of weakly converging sequences, weak and strong closure, Mazur lemma. Bidual space and reflexive spaces; strong and weak continuity of linear operators. Weak* topology, properties of weak* converging sequences, Banach-Alaoglu theorem. Helly lemma, Kakutani theorem. Properties of reflexive (and separable) Banach spaces. Sequential relative compactness theorem in a reflexive Banach space. Weiertrass theorem on the minimum of a sequential weakly lower semicontinuous functional. Uniform convexity and Millman theorem, weak-strong convergence in a uniform convex space. 5. L^p spaces. Definition, Holder inequality, Minkowsky inequality, separability, interpolation, Clarkson inequality and uniform convexity, reflexivity. Duality and Riesz theorem. Convolution, Young inequality, function with compact support, mollifiers, strong compactness criterion. Weak convergence.
Obiettivi Agenda 2030 per lo sviluppo sostenibile	

Obiettivi per lo sviluppo sostenibile

Codice	Descrizione

Testi in inglese

	English
	a) Fundamental principles of Functional Analysis: Banach and Hilbert spaces. Continuous linear operators. b) Basic function spaces: continuous functions, Lebesgue spaces (L^p). c) Weak topologies.
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By the end of the course the student will be able to manage the fundamental tools of Functional Analysis and to approach the further steps of this area of Mathematics, like the theory of Distributions and of Sobolev spaces, in order to face the first arguments in Partial Differential Equations and Calculus of Variations. The student will be able to read autonomously advanced monographs of Functional Analysis, to understand proofs and applications of theorems of any level, and to apply them to various fields of Mathematics. He/she will be able to solve problems of basic level.

Calculus I and II. Basic notions in measure theory. Basic notions in general topology.

Lectures and solutions of problems.

The final exam is in two parts. The written part (2 hours) consists in the solution of some problems concerning the contents of the course. The oral part is devoted to ascertain the comprehension and the managing of the topics reached by the candidate.

0. Metric and normed spaces. Converging and Cauchy sequences; completeness, compactness, precompactness and relative compactness; density and separablity. Banach spaces and space of continuous linear operators on Banach spaces, dual space and extension theorem.
1. Analytic form of Hahn-Banach theorem, gauge of a convex set, first and second geometric form of Hahn-Banach theorem. Baire theorem, Banach-Steinhaus theorem, open mapping theorem, inverse mapping theorem, closed graph theorem. Diagonal procedure. Convex sets and convex functions. Examples: space of continuos functions on a real interval and L^p spaces.
2. Inner product spaces, orthogonality and orthonormality, orthonormal systems. Pitagorean theorem, Bessel inequality, Schwarz inequality, polarization identity. Hilbert spaces, orthogonal complement, projection operators, direct sum. Riesz theorem. Hilbert bases, countablity of the basis and separability, Parseval identity. Lax-Milgram theorem. Continuous linear operators on Banach and Hilbert spaces, adjoint operator, selfadjoint operators.
3. Space of continuous functions on a compact metric space. Completeness and separability. Partition of unity. Equicontinuity and Ascoli-Arzelà theorem.
4. Topology generated by a family of functions; weak topology in a Banach space, basis of a weak topology; properties of weakly converging sequences, weak and strong closure, Mazur lemma. Bidual space and reflexive spaces; strong and weak continuity of linear operators. Weak* topology, properties of weak* converging sequences, Banach-Alaoglu theorem. Helly lemma, Kakutani theorem. Properties of reflexive (and separable) Banach spaces. Sequential relative compactness theorem in a reflexive Banach space. Weiertrass theorem on the minimum of a sequential weakly lower semicontinuous functional. Uniform convexity and Millman theorem, weak-strong convergence in a uniform convex space.
5. L^p spaces. Definition, Holder inequality, Minkowsky inequality, separability, interpolation, Clarkson inequality and uniform convexity, reflexivity. Duality and Riesz theorem. Convolution, Young inequality, function with compact support, mollifiers, strong compactness criterion. Weak convergence.

Obiettivi per lo sviluppo sostenibile

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