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$$\binom{n}{k} = \boxed{\frac{n!}{k!\,(n-k)!}} = \frac{n \cdot \ldots \cdot (n-(k-1))}{k!}$$

$$\binom{n}{0} = 1$$

Teorema (Formula di Newton) $\forall a, b \in \mathbb{R}$ e $\forall n \geq 0$, si ha

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Lemma Vale l'uguaglianza

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

per $1 \leq k \leq n-1$

Dim. $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(k-1)!\,[(n-\cancel{1})-(k-\cancel{1})]!} + \frac{(n-1)!}{k!\,(n-1-k)!}$$

$$= \frac{(n-1)!}{(k-1)!\,(n-k)!} + \frac{(n-1)!}{k!\,(n-1-k)!}$$

$$= \frac{(n-1)!}{(k-1)!\,(n-k-1)!\,(n-k)} + \frac{(n-1)!}{(k-1)!\,k\,(n-1-k)!}$$

$$= \frac{(n-1)!}{(k-1)!\,(n-k-1)!}\left[\frac{1}{n-k} + \frac{1}{k}\right]$$

$$= \frac{(n-1)!}{(k-1)!\,(n-k-1)!} \quad \frac{\cancel{k} + n - \cancel{k}}{k\,(n-k)}$$

$$= \frac{n!}{k!\,(n-k)!} = \binom{n}{k}$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \qquad 1 \le k \le n-1$$

Triangolo di Tartaglia

$(a+b)^0$	1
$(a+b)^1 = a+b$	1 1
$(a+b)^2 = a^2+2ab+b^2$	1 2 1
$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$	1 3 3 1
$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$	1 4 6 4 1
$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$	1 5 10 10 5 1

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

Per induzione

$n=0$ $\quad (a+b)^0 = 1$

$$\sum_{k=0}^{0} \binom{0}{k} a^k b^{0-k} = \binom{0}{0} a^0 b^0 = 1 \cdot 1 \cdot 1 = 1$$

$n-1 \Rightarrow n$ $\quad$ Supponiamo vero $(a+b)^{n-1} = \sum_{k=0}^{n-1} \binom{n-1}{k} a^k b^{n-1-k}$

e dimostriamo che allora che è vero $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

$$(a+b)^n = (a+b)(a+b)^{n-1} = (a+b) \sum_{k=0}^{n-1} \binom{n-1}{k} a^k b^{n-1-k}$$

$$= a \sum_{k=0}^{n-1} \binom{n-1}{k} a^k b^{n-1-k} + b \sum_{k=0}^{n-1} \binom{n-1}{k} a^k b^{n-1-k}$$

$$= \sum_{k=0}^{n-1} \binom{n-1}{k} a^{k+1} b^{n-(k+1)} + \sum_{k=0}^{n-1} \binom{n-1}{k} a^k b^{n-k}$$

pongo $h = k+1$

$$= \sum_{h=1}^{n} \binom{n-1}{h-1} a^h b^{n-h} + \sum_{k=0}^{n-1} \binom{n-1}{k} a^k b^{n-k}$$

$$= \sum_{k=1}^{n} \binom{n-1}{k-1} a^k b^{n-k} + \sum_{k=0}^{n-1} \binom{n-1}{k} a^k b^{n-k}$$

$$= \binom{n-1}{n-1} a^n b^{n-n} + \sum_{k=1}^{n-1} \binom{n-1}{k-1} a^k b^{n-k} + \sum_{k=1}^{n-1} \binom{n-1}{k} a^k b^{n-k} + \binom{n-1}{0} a^0 b^{n-0}$$

$$= a^n + b^n + \sum_{k=1}^{n-1} \overbrace{\left[\binom{n-1}{k-1} + \binom{n-1}{k} \right]}^{\binom{n}{k}} a^k b^{n-k}$$

$$= a^n + b^n + \sum_{k=1}^{n-1} \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$