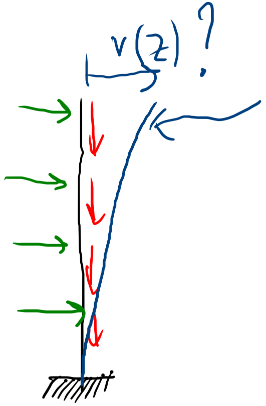
.... $\boxed{\Pi(\underline{u})}$, NELLA TEORIA LINEARE 'SOLITA' DELLA S.D.C. 7/10/22

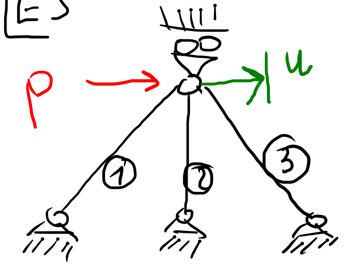
LA SOLUZ. DEL PROBLEMA ELASTICO È UNICA E COINCIDE CON UN MINIMO DELL' E. POTENZIALE TOTALE.



STUDIO EQUILIBRIO RISPETTO ALLA CONFIG. DEFORMATA

(PROBL. NON LINEARE)

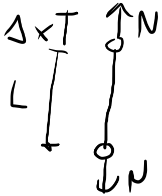
LES



EA cost

u : INCOGNITA DEL PROBLEMA

$\Pi(\underline{u})$



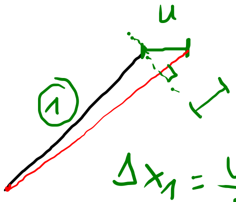
$$E_e = \frac{1}{2} K \Delta x^2 \qquad K = \frac{EA}{L}$$

$$\Delta x = \frac{N}{K}$$

$$\Pi(u) = \text{EN. ELASTICA} - Pu$$

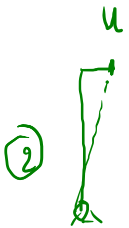
$$\Pi(u) = \frac{1}{2} \sum_{i=1}^{3} \left(\frac{EA}{L} \Delta x^2 \right)_i - Pu$$

$$E_e = \frac{1}{2} \frac{EA}{L} \Delta x^2$$

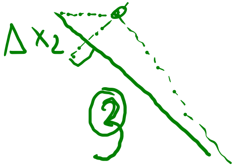


$$\Delta x_1 = f(u)$$

$$\Delta x_1 = \frac{u}{\sqrt{2}}$$



$$\Delta x_2 = 0$$



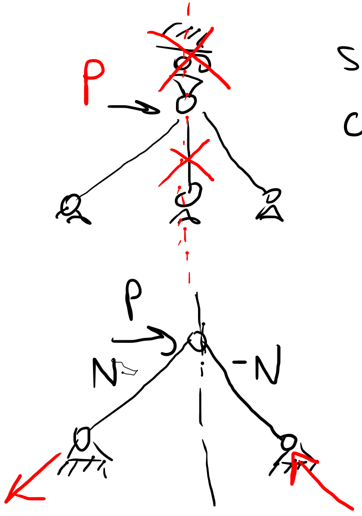
$$\Delta x_3 = -\frac{u}{\sqrt{2}}$$

$$\Pi(u) = \frac{1}{2} EA \left(\frac{u^2}{L\sqrt{2}\cdot 2} + 0 + \frac{u^2}{L\sqrt{2}\cdot 2} \right) - Pu$$

$$\Pi(u) = \frac{1}{2} EA \frac{u^2}{L\sqrt{2}} - Pu$$

$$\frac{\partial \Pi}{\partial u} = 0 \; ; \quad \frac{EAu}{L\sqrt{2}} - P = 0 \implies \boxed{u = \frac{PL\sqrt{2}}{EA}}$$

...CON LA SIMM.



STR SIMM

CARIC ANTISIMM } RISP. STR. ANTISIMM.

ISOST. DELLE STRUTTURE.

LA STAZION. DELL' E, POT. TOTALE CONDUCE ALLA FORMULAZ. DEL METODO DEGLI SPOSTAMENTI PER LA RIS.

$$\delta\Pi(\underline{u})=0$$

SPOST. INCOGNITI

RICORDIAMOCI CHE ESISTE L' EN POT. COMPLEMENTARE

$$\Pi = \frac{1}{2}\int_B \underset{\sim}{\sigma}\cdot\underset{\sim}{\varepsilon}\, dV - \int_B \underline{b}\cdot\underline{u}\, dV - \int_{\partial B}\underline{p}\cdot\underline{u}\, dS \qquad \Pi = \Pi^c \qquad \underset{\sim}{\sigma} = \mathbb{C}\,\underset{\sim}{\varepsilon}$$

$$\Pi(\underline{u}) = \frac{1}{2}\int_B \mathbb{C}\,\underset{\sim}{\varepsilon}\cdot\underset{\sim}{\varepsilon}\, dV - \qquad \Pi^c(\underset{\sim}{\sigma}) = \frac{1}{2}\int_B \underset{\sim}{\sigma}\cdot\mathbb{C}^{-1}\underset{\sim}{\sigma}\, dV - \dots$$

EN POT. TOTALE PER STR. INFLESSE (EULERO - BERNOULLI)

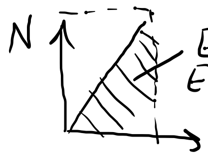
$$\Pi = \frac{1}{2}\underbrace{\int_B \tilde{\sigma} \cdot \tilde{\varepsilon} \, dV}_{} = \dots\dots$$

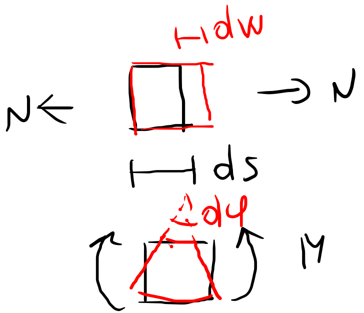
ENERGIA POTENZIALE

Φ ELASTICA DEL CORPO

$$\Phi = \frac{1}{2}\int_S N\varepsilon + M\chi \, ds$$

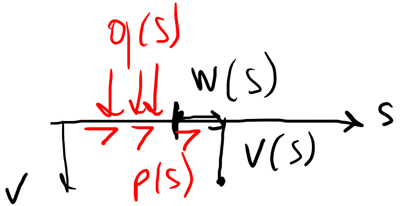
DEF. ASSIALE   CURV.





$$\frac{dw}{ds} = \varepsilon = \frac{N}{EA}$$

$$\frac{d\varphi}{ds} = \chi = \frac{M}{EI}$$



$$\Phi(\varepsilon,\chi) = \frac{1}{2}\int EA\,\varepsilon^2 + EI\,\chi^2 \, ds$$

TRAVI SNELLE

$$\varepsilon = w'(s)$$

$$\chi = -v''(s)$$

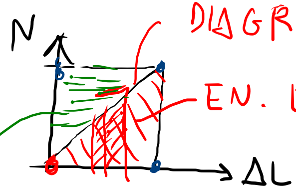
SOLO FLESSIONE

$$\Phi(w,v) = \frac{1}{2}\int_S EA(w')^2 + EI(v'')^2 \, ds$$

$$\Pi(w,v) = \frac{1}{2}\int_S EA(w')^2 + EI(v'')^2 \, ds - \int_S (qv + pw)\, ds$$

TEOREMI ENERGETICI (O DEL LAVORO DI DEFORMAZIONI)

$\Delta L = \frac{NL}{EA}$



DIAGR. CARICO

EN. ELASTICA $= \boxed{\frac{1}{2} N \Delta L}$ VALORI FINALI

$\parallel$

LAVORO DI DEFORMAZIONE

EN ELASTICA COMPLEMENTARE

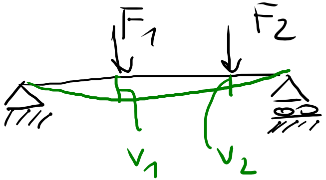
$$\left(\Phi = \frac{1}{2} \int_S \frac{N^2}{EA} + \frac{M^2}{EI} \, ds \right) = \Phi^c$$

Φ COMPLEMENTARE

TEOREMA DI CLAPEYRON

ENUNCIATO

. . . . . .



- CARICAMENTO QUASI-STATICO
- MATERIALE ELASTICO LINEARE
- VINCOLI PERFETTI

$\Big\}$ IPOTESI

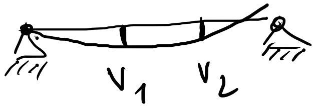
$$L_{ve} = F_1 v_1 + F_2 v_2$$

$$\boxed{\Phi = \frac{1}{2}\left(F_1 v_1 + F_2 v_2\right)} \quad \text{TESI}$$

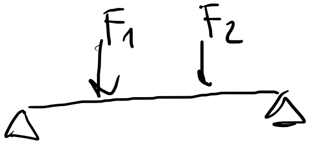
$$L_{vi} = \int_S N\varepsilon + M\chi \, ds = \int \frac{N^2}{EA} + \frac{M^2}{EI} ds$$

DIM. (CON IL PLV)

$$L_{ve} = L_{vi}$$



ε, χ : CINEM. AMMISSIBILE (CONGRUENTE)



N, M : STAT. AMMISS (EQUILIBRATO)

$$\frac{1}{2}\left(F_1 v_1 + F_2 v_2\right) = \underbrace{\frac{1}{2}\int \frac{N^2}{EA} + \frac{M^2}{EI} ds}_{\Phi}$$

$$'' \quad = \quad \Phi$$

$$L_{ve} = L_{vi}$$

OSSERVAZ.

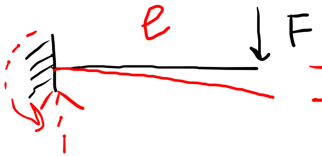
— $\Phi \to$ VIA INTERNA $= \frac{1}{2}\int \dots$

$\searrow$ VIA ESTERNA $= \frac{1}{2}(F_1 v_1 + \dots)$ (CLAPEYRON)

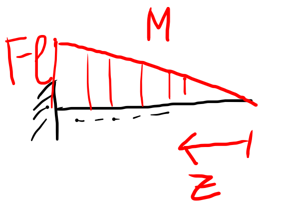
— UTILIZZO DEL TEOREMA PER CALCOLARE E

$$\Phi = \frac{1}{2}\int \frac{N^2}{EA} + \frac{M^2}{EI}\, ds \Rightarrow E$$

ES.



$I\ \delta = \frac{F\ell^3}{3EI}$



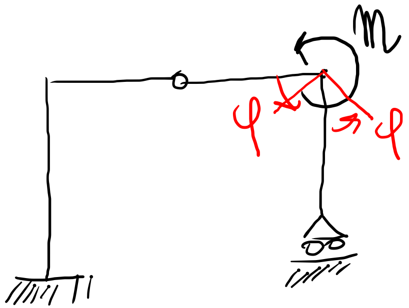
δ? CON TH. DI CLAPEYRON

$$\frac{1}{2} F\delta = \frac{1}{2}\int \frac{M^2}{EI}\, ds$$

$$M(z) = -Fz$$

$$F\delta = \frac{1}{EI}\int_0^{\ell} (Fz)^2\, dz$$

$$\cancel{F}\delta = \frac{F^{\cancel{2}}}{EI}\frac{\ell^3}{3} \quad ; \quad \delta = \frac{F\ell^3}{3EI}$$



CON TH DI CLAPEYRON

POSSO SOLO CALCOLARE $\varphi \leftrightarrow m$