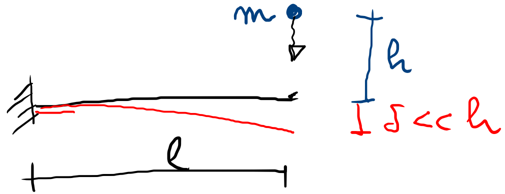
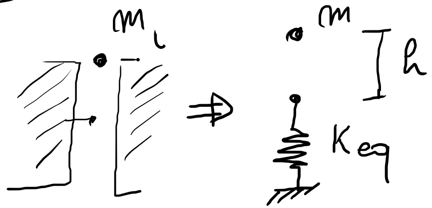
...TEOREMI ENERGETICI 17/11/2022



LASCIO LA MASSA E VOGLIO CALCOLARE δ (MASSIMO SPOST. TRAVE)

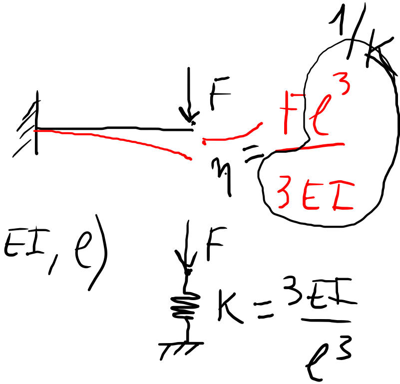
$(\delta \ll h)$ $E_{pot} = mgh$

I METODO



$E_{pot} = E_{elast}$

$mgh = \frac{1}{2} K_{eq} \delta^2 \Rightarrow \delta = f(K_{eq}) = f(EI, l)$



$\boxed{K_{eq}\,? = \frac{3EI}{l^3}}$

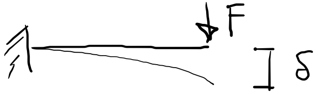
II METODO

IN UNA STR. INFLESSA

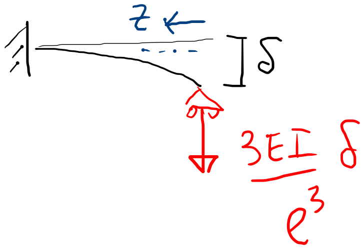
$\Phi = \frac{1}{2} \int_{str} \frac{M^2}{EI} ds$: EN ELASTICA IMMAGAZZINATA NELLA STRUTTURA

$E_{pot} = \Phi$

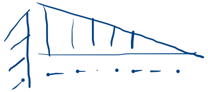
$mgh = \frac{1}{2} \int_{str} \frac{M^2}{EI} ds$ $\quad M(s)\,? \Rightarrow M(\delta)$



$$\delta = \frac{F\ell^3}{3EI}$$



$$M(z) = -\frac{3EI}{\ell^3}\delta\, z$$



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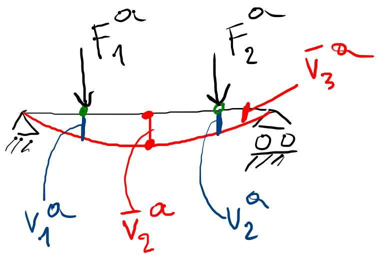
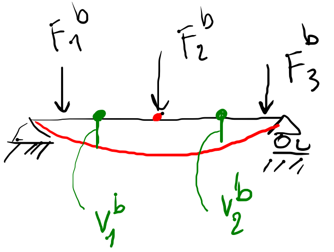
$$\frac{1}{2}\int_0^{\ell} \frac{M^2}{EI}\,dz = \frac{1}{2}\int_0^{\ell} \frac{1}{EI}\,\frac{9(EI)^2}{\ell^6}\,\delta^2 z^2\,dz = \frac{9}{2}\,\frac{EI}{\ell^6}\,\delta^2\,\frac{\ell^3}{3} = \frac{9}{6}\,\frac{EI}{\ell^3}\,\delta^2$$

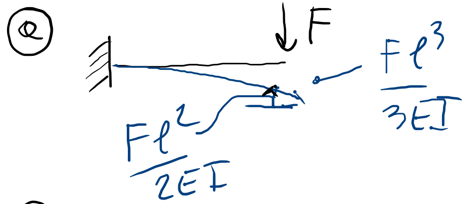
$$mgh = \frac{3}{2}\,\frac{EI}{\ell^3}\,\delta^2 \Longrightarrow \delta = \sqrt{}$$

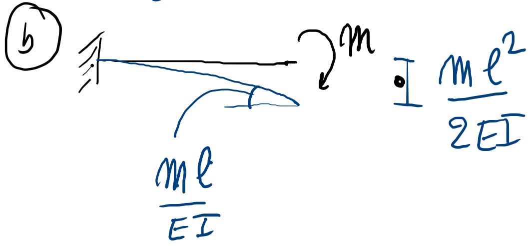
TEOREMA DI BETTI (O DI RECIPROCITA') 1872





IPOTESI
TH BETTI = TH DI CLAPEYRON





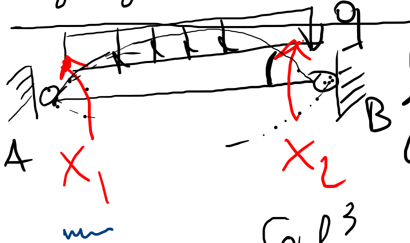
$$F_1^a v_1^b + F_2^a v_2^b = F_1^b \bar{v}_1^a + F_2^b \bar{v}_2^a + F_3^b \bar{v}_3^a$$

$$L_{ab} = L_{ba}$$

uguaglianza dei lavori mutui.

$$L_{ab} = F \frac{ml^2}{2EI}$$

$$L_{ba} = m \frac{Fl^2}{2EI}$$



$$\begin{cases} \varphi_A = 0 \\ \varphi_B = 0 \end{cases}$$

$$\begin{cases} \frac{ql^3}{24EI} - \frac{X_1 l}{3EI} - \frac{X_2 l}{6EI} = 0 \\ -\frac{ql^3}{24EI} + \frac{X_2 l}{3EI} + \frac{X_1 l}{6EI} = 0 \end{cases}$$

$$\begin{cases} -\frac{ql^3}{24EI} + \frac{X_1 l}{3EI} + \frac{X_2 l}{6EI} = 0 \\ -\frac{ql^3}{24EI} + \frac{X_2 l}{3EI} + \frac{X_1 l}{6EI} = 0 \end{cases}$$

= PER TH. DI BETTI.

$$\begin{array}{cc} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{array}$$

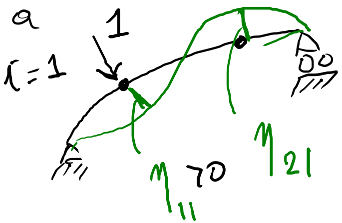
$$\begin{bmatrix} \frac{\ell}{3EI} & \frac{\ell}{6EI} \\ \frac{\ell}{6EI} & \frac{\ell}{3EI} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \frac{q\ell^3}{24EI} \\ \frac{q\ell^3}{24EI} \end{bmatrix}$$

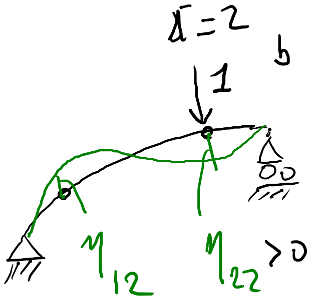
BETTI $\quad \eta_{12} = \eta_{21}$; $\eta_{11} > 0$; $\eta_{22} > 0$

COEFF DI INFLUENZA

$$\eta_{ii} > 0$$

COEFF DI INFLUENZA





$$1^a \cdot \eta_{12} = 1^b \cdot \eta_{21} \Longrightarrow \eta_{12} = \eta_{21}$$

TH. DI MAXWELL

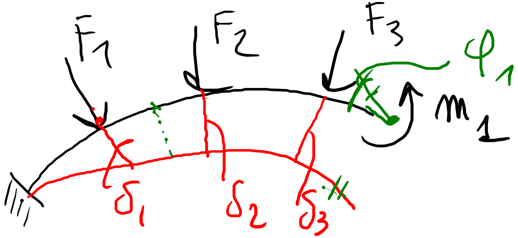
η_{ij} = SPOST. NELLA DIREZ i DOVUTO AD UNA FORZA UNITARIA NELLA DIREZ. j.

η_{12} = DIR 1 FORZA LUNGO 2

η_{21} = DIR 2 FORZA LUNGO 1

η_{11} = DIR 1 FORZA LUNGO DIR 1

TH. DI CASTIGLIANO → ATTE



$$\Phi = \frac{1}{2}\int \frac{M^2}{EI} + \frac{N^2}{EA} = \Phi(F_1, F_2, \dots, m_1)$$

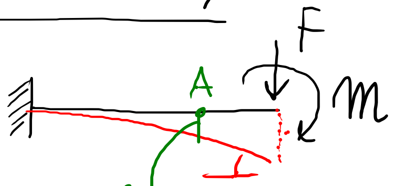
$$M, N = f(F_1, F_2, \dots m_1)$$

STESSE IPOTESI TH. CLAPEYRON $\left\{ \begin{array}{l} \text{NO ATERMICHE} \\ \text{ATT. VINCOLI ELASTICI} \end{array} \right.$

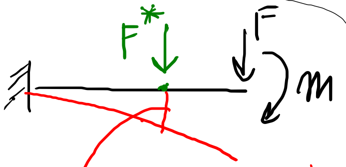
CARICHI CONCENTRATI

$$\boxed{\frac{\partial \Phi(F_1, F_2, \dots)}{\partial F_i}} = \delta_i$$: SPOSTAMENTO CORRELATO ALLA FORZA F_i

$$\boxed{\frac{\partial \Phi}{\partial m_i} = \varphi_i}$$ ROTAZ. CORRELATA AL MOMENTO M_i



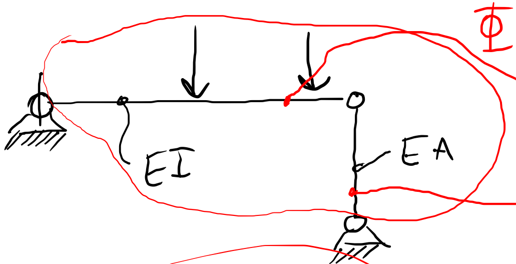
$$\alpha ? = \alpha(F, m)$$



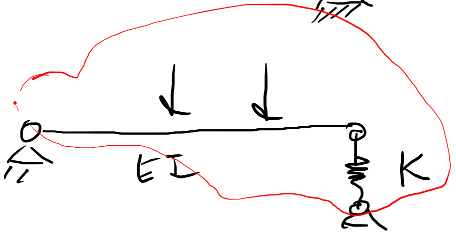
$$\alpha^* = \alpha^*(F, m, F^*)$$

$$\alpha^* = \frac{\partial \Phi}{\partial F^*}$$

$$\alpha = \lim_{F^* \to 0} \alpha^* = \lim_{F^* \to 0} \frac{\partial \Phi}{\partial F^*}$$

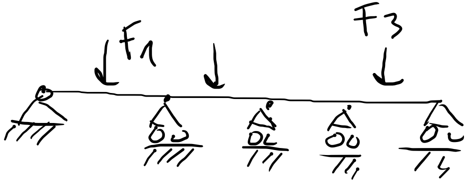


$$\Phi = \Phi(EI) + \Phi(EA)$$

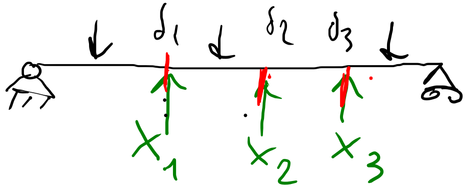


$$\Phi = \Phi(EI) + \Phi(k)$$

TUTTA L'EN. ELASTICA DEL SIST. VA CONSIDERATA ~~CON~~ NEL TH. DI CASTIGLIANO



VINCOLI FISSI
STR 3 VOLTE IPERST.



S. PRINCIPALE

$$\Phi = \Phi(F_1, \dots, X_1, X_2, X_3)$$

$$\begin{cases} \delta_1 = 0 \\ \delta_2 = 0 \\ \delta_3 = 0 \end{cases}$$

EQ DI CONGR. (X_1, X_2, X_3)

VINCOLI FISSI

$$\begin{cases} \dfrac{\partial \Phi}{\partial X_1} = 0 \\ \dfrac{\partial \Phi}{\partial X_2} = 0 \\ \dfrac{\partial \Phi}{\partial X_3} = 0 \end{cases}$$

$\Phi(\dots, X_1, X_2, X_3)$

FRA TUTTI I POSSIBILI VALORI ATTRIBUIBILI ALLE INCOGNITE X_i, L'INSIEME CHE SODDISFA LA CONGRUENZA È QUELLO CHE RENDE MINIMA L'ENERGIA ELASTICA.

TH. DI MENABREA