FONDAMENTI DI DINAMICA DELLE STRUTTURE

18/11/22

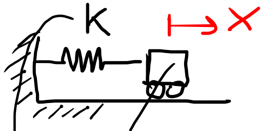
MECCANICA DELLE VIBRAZIONI

- CHOPRA "DYNAMICS OF STRUCTURES"
- CLOUGH, PENZIEN "STRUCTURAL DYNAMICS"
- VIOLA "FONDAMENTI DI DINAMICA E VIBRAZIONI"
- CASTIGLIONI, RAMASCO, GAVARINI

PROGRAMMA

- OSCILLATORE SEMPLICE
  - OSCILLAZ. LIBERE
  - SMORZAMENTO
  - FORZANTI E RISONANZA
- SPETTRO DI RISPOSTA ELASTICO

OSCILLATORE SEMPLICE

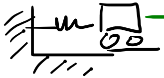


(G.D.L.)

m (MASSA)

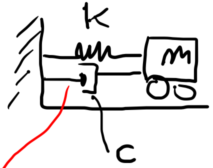
RIGIDEZZA ELASTICA LINEARE

→ OSCILLAZ. LIBERE → $F(t) = 0$



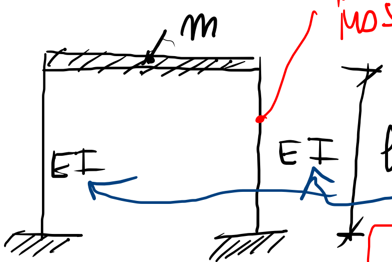
$F(t)$: OSCILLAZ. FORZATE

FORZANTE



COMPORT. DELL'OSCILLATORE CON DISSIPAZIONE DI ENERGIA

DISSIPATORE VISCOSO

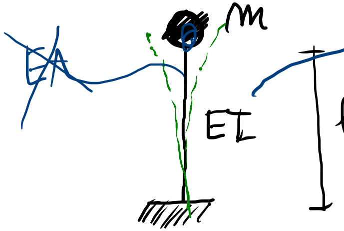


NO MASSA

m K

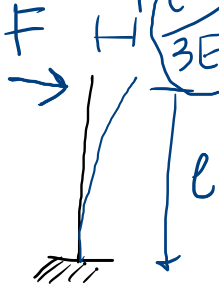
x

?

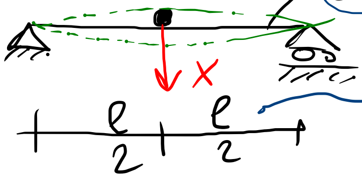


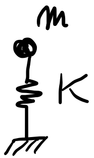
m K

$K = \dfrac{3EI}{h^3}$?



$\dfrac{Fl^3}{3EI} = \dfrac{1}{K}$

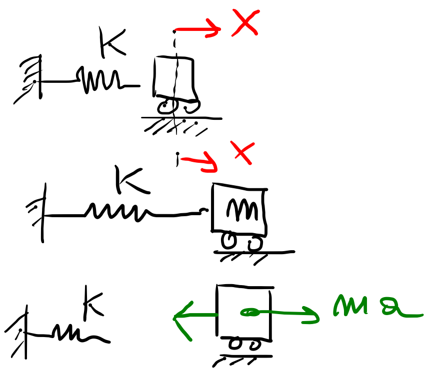




m

$K = ?\ f(EI, K_T)$

OSCILLAZ. LIBERE



$F_{el} = K x(t)$

$F = ma \;;\; a = \ddot{x}(t) = \dfrac{d^2}{dt^2} x(t)$

$$\boxed{-Kx(t) = m\ddot{x}(t)}$$

$$\boxed{m\ddot{x} + Kx = 0}$$

LEGGE DEL MOTO ARMONICO

$(*)\quad \ddot{x} + \omega^2 x = 0$

$$\boxed{\omega = \sqrt{\frac{K}{m}}}$$

PULSAZIONE NATURALE O PROPRIA

$x(t) = e^{\lambda t}$

$\lambda^2 \cancel{e^{\lambda t}} + \omega^2 \cancel{e^{\lambda t}} = 0 \;;\; \lambda^2 = -\omega^2 \;;\; \lambda = \pm i\omega$

$\rightarrow x(t) = C_1 e^{i\omega t} + C_2 e^{-i\omega t}$ I FORMA DELLA SOLUZ. DI $(*)$

$e^{i\omega t} = \cos\omega t + i \sin\omega t$

$\rightarrow x(t) = \bar{B}_1 \sin\omega t + \bar{B}_2 \cos\omega t$ II FORMA DI SOLUZ.

$\rightarrow x(t) = \underset{\text{AMPIEZZA}}{A} \sin(\omega t + \underset{\text{FASE}}{\varphi})$ III FORMA DI SOLUZ. $(*)$

COSTANTI: 2 CONDIZ. INIZIALI $(t=0)$

$x(0)$, $\dot{x}(0)$: VELOCITA'

$[\omega]$?

$$[\omega^2 x] = \left[\frac{L}{T}\,\frac{1}{T}\right] \longrightarrow [\omega^2] = \left[\frac{1}{T^2}\right]$$

ACCELERAZ.

$$[\omega] = \left[\frac{1}{T}\right]$$

$\omega = \frac{1}{s} = Hz \rightsquigarrow$ ω SI MISURA IN $\frac{RAD}{s}$

$$\boxed{f = \frac{\omega}{2\pi}} \rightarrow \boxed{f = Hz = \frac{1}{s}}$$

FREQUENZA NATURALE

$$\omega = f\,2\pi \Rightarrow Hz\,2\pi = \frac{1}{s}\,2\pi$$

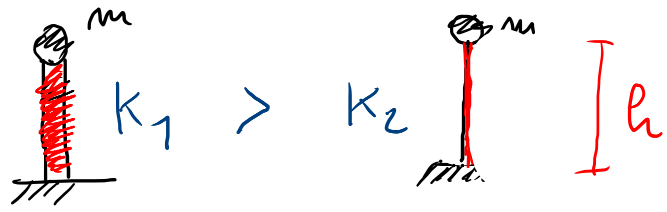
$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$ PERIODO PROPRIO O NATURALE

$[T] = [TEMPO]$ ⇒ SECONDI

$$\omega = \sqrt{\frac{K}{m}} \quad rad/s$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad Hz$$

$$T = 2\pi\sqrt{\frac{m}{K}} \quad s$$



$$\omega_1 > \omega_2$$

$$T_1 < T_2$$

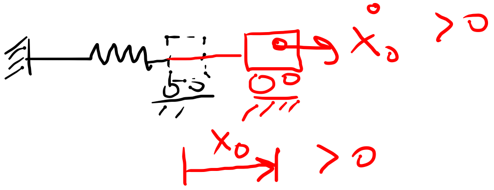
CONFRONTO $\boxed{x(t) = B_1 \sin\omega t + B_2 \cos\omega t}$ con $x(t) = A \sin(\omega t + \varphi)$

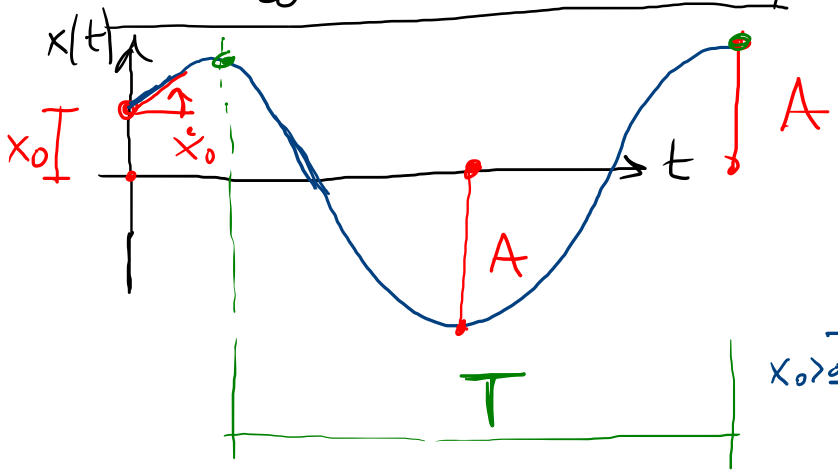
$$x(t) = \underbrace{A\cos\varphi}_{B_1} \sin\omega t + \underbrace{A\sin\varphi}_{B_2} \cos\omega t \;;\quad \frac{B_2}{B_1} = \text{tg}\,\varphi \;;\quad A = \sqrt{B_1^2 + B_2^2}$$

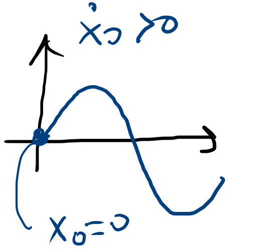
CONDIZ. INIZIALI PIÙ GENERALI

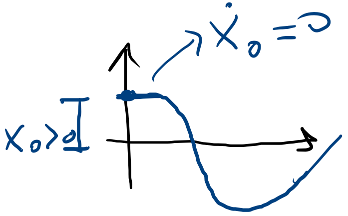
$$\begin{cases} x(0) = x_0 \;; & B_2 = x_0 \\ \dot{x}(0) = \dot{x}_0 \;; & B_1 \omega = \dot{x}_0 \end{cases} \Longrightarrow \boxed{x(t) = \frac{\dot{x}_0}{\omega} \sin\omega t + x_0 \cos\omega t}$$

$$\dot{x}(t) = B_1 \omega \cos\omega t - B_2 \omega \sin\omega t$$

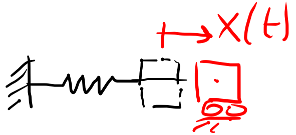








CONSIDERAZ. ENERGETICHE



ENERGIA ELASTICA $\Rightarrow$ $E_{el} = \frac{1}{2} K x^2 = \frac{1}{2} K \left[A \sin(\omega t + \varphi) \right]^2$

" CINETICA $\Rightarrow$ $E_{cin} = \frac{1}{2} m \dot{x}^2 = \frac{1}{2} m \left[A \omega \cos(\omega t + \varphi) \right]^2$

$\omega^2 = \frac{K}{m}$

$E_{mecc} = E_{el} + E_{cin} = \frac{1}{2} K A^2 \sin^2(\quad) + \frac{1}{2} m A^2 \underbrace{\omega^2}_{K/m} \cos^2(\quad)$ (*)

$= \frac{1}{2} K A^2 \left[\underbrace{\sin^2(\quad) + \cos^2(\quad)}_{1} \right] = \frac{1}{2} K A^2$

EN MECC SI CONSERVA

E_{el} MAX $(x(t))$ ma

$\max\{x(t)\} = A$ AMPIEZZA DELL'ONDA

MASSIMO SPOST.

(*) $\frac{1}{2} m \omega^2 A^2 \left[\sin^2 + \cos^2 \right] = \frac{1}{2} m \underbrace{\omega^2 A^2}_{v^2_{MAX}}$

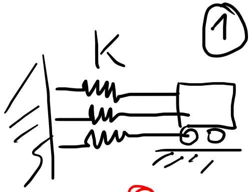
Qual è LA MAX VELOCITÀ DELLA MASSA:

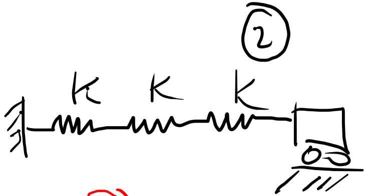
$\dot{x}(t) = A \omega \overbrace{\cos(\quad)}^{1}$

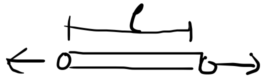
$\boxed{\max\{\dot{x}(t)\} = A \omega}$

MASSIMA VELOCITÀ

ES

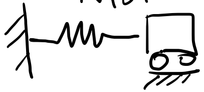






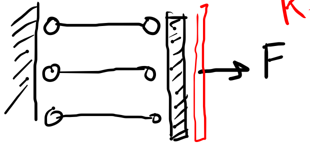
$$K = \frac{EA}{l}$$

$$K_{TOT}^{(1)} = 3K$$

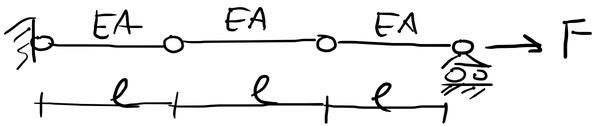


$$K_{TOT}^{(2)}$$

$$\frac{1}{K_{TOT}^{(2)}} = \frac{1}{K} + \frac{1}{K} + \frac{1}{K} = \frac{3}{K}$$



$$\delta_H = \frac{F}{K_{TOT}^{(1)}}$$



$$\delta = 3\,\frac{Fl}{EA} = F\,\frac{3l}{EA} \qquad \frac{3l}{EA} = \frac{1}{K_{TOT}^{(2)}}$$

$$K = \frac{EA}{l}$$

$$\frac{1}{K_{TOT}^{(2)}} = 3\,\frac{1}{K}$$

$$K_{TOT}$$