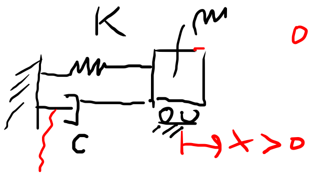
## OSCILLATORE CON SMORZAMENTO (o CON DISSIPAZIONE)

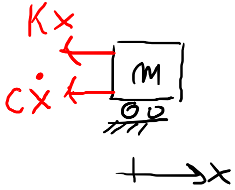
23/11/22




SMORZATORE (DASH POT)

OSCILLAZIONI LIBERE

$$\boxed{m a = F}$$




c: coeff di smorzamento viscoso

$[c\dot{x}] = [\text{FORZA}]$ $\quad [c] = \left[\frac{FT}{L}\right] \sim \frac{Ns}{m}$

$\omega = \sqrt{\frac{K}{m}}$

EQ. DELLA DINAMICA $\quad m\ddot{x} = -Kx - c\dot{x} \;;\; \boxed{m\ddot{x} + c\dot{x} + Kx = 0}$

$$\ddot{x} + 2\frac{c}{2m\omega}\omega\dot{x} + \omega^2 x = 0 \rightarrow \boxed{\ddot{x} + 2\nu\omega\dot{x} + \omega^2 x = 0} \;;\; \nu = \frac{c}{2m\omega} = \frac{c}{2\sqrt{Km}} = \frac{c}{c_{cr}}$$

$\nu$: RAPPORTO DI SMORZAMENTO ; $[\nu] = [-]$

$c_{cr} = 2\sqrt{Km}$

↳ SMORZAMENTO CRITICO

$\nu \sim 0{,}03 \div 0{,}07$ nelle strutture civili.

$\ddot{x} + 2\nu\omega\dot{x} + \omega^2 x = 0$ ; $x(t) = e^{\lambda t}$ ; $\dot{x} = \lambda e^{\lambda t}$ ; $\ddot{x} = \lambda^2 e^{\lambda t}$

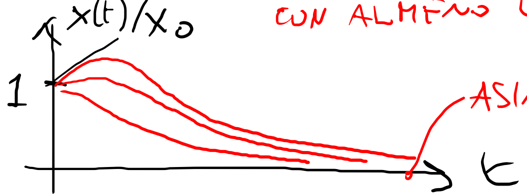
$$\big(\underbrace{\lambda^2 + 2\nu\omega\lambda + \omega^2}_{=0 \;\to\; \lambda_1,\,\lambda_2}\big) e^{\lambda t} = 0$$

$$\lambda_{1,2} = -\nu\omega \pm \sqrt{\nu^2\omega^2 - \omega^2} = \omega\left(-\nu \pm \sqrt{\nu^2 - 1}\right)$$

- $\nu > 1$ SMORZ. SUPERCRITICO
- $\nu = 1$ " CRITICO
- $\nu < 1$ " SOTTOCRITICO

$\underline{\nu > 1}$ : $\lambda_1, \lambda_2$ : REALI ; $x(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ (*)

CON ALMENO UN $\lambda$ NEGATIVO




$\underline{\nu = 1}$ : $\lambda_1 = \lambda_2 = -\nu\omega = \lambda$ ; LA SOLUZ. (*) NON VA PIÙ BENE : $x(t) = A_1 e^{\lambda t} + A_2 \lambda e^{\lambda t}$

$\boxed{\nu < 1}$: $x(t) = A_1\, e^{\underbrace{-\nu\omega t}\, + i \underbrace{\omega\sqrt{1-\nu^2}}_{\omega_D} t} + A_2\, e^{\underbrace{-\nu\omega t}\, - i \underbrace{\omega\sqrt{\phantom{1-\nu^2}}}_{\omega_D} t}$

$\omega\left(-\nu \pm \sqrt{\nu^2 - 1}\right)$

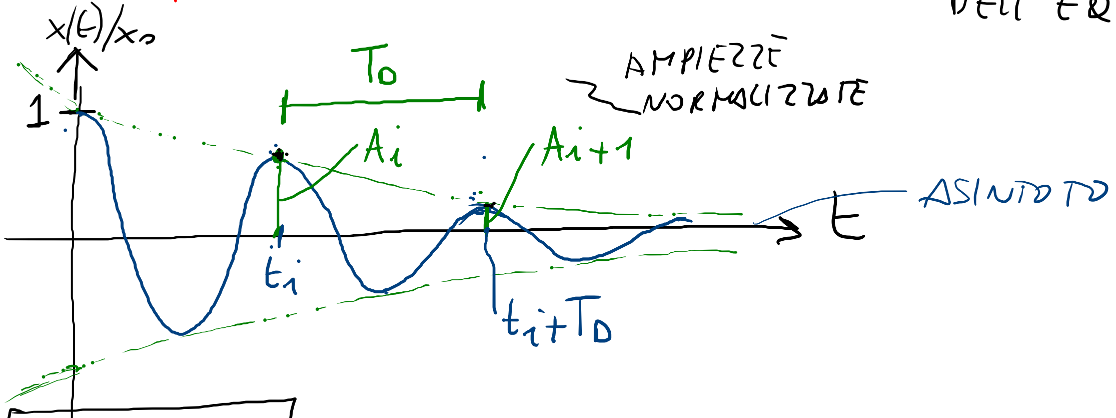
$\omega\left(-\nu \pm i\sqrt{1-\nu^2}\right)$, $\quad 1-\nu^2 > 0$

$$x(t) = \underbrace{e^{-\nu\omega t}}_{\text{MODULANTE}} \left(\underbrace{B_1 \sin\omega_D t + B_2 \cos\omega_D t}_{\text{OSCILLANTE}}\right)$$

MODO ALTERNATIVO DI SCRIVERE L'INT. GENERALE DELL'EQ.

$\omega_D = \omega\sqrt{1-\nu^2}$ PULSAZIONE RIDOTTA

$T_D = \dfrac{2\pi}{\omega_D}$ PERIODO RIDOTTO




$e^{i\omega_D t} = \cos\omega_D t + i\sin\omega_D t$

$\omega_D < \omega$ ; $T_D > T$

$\nu \to 0 \quad \omega_D \to \omega$ , $T_D \to T$

$\nu \to 1$ ; $\omega_D \to 0$ ; $T_D \to \infty$

$$\boxed{E_{mecc} = \frac{1}{2} K A^2}$$

$$\Delta E_{mecc} = \frac{1}{2} K \left(A_{i+1}^2 - A_i^2\right) < 0$$

DECREMENTO DI EN. MECC. DEL SISTEMA.

$A_i = e^{-\nu\omega t_i}$

$A_{i+1} = e^{-\nu\omega(t_i+T_D)} = e^{-\nu\omega t_i} \cdot e^{-\nu\omega T_D} = A_i \, e^{-\nu\omega T_D}$

$$\boxed{\frac{A_i}{A_{i+1}} = e^{\nu\omega T_D}} > 1$$

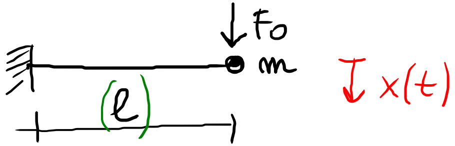
$\frac{A_i}{A_{i+1}} = e^{\delta}$ ; $\delta = \nu\omega T_D$ : DECREMENTO LOGARITMICO ; $\ln \frac{A_i}{A_{i+1}} = \delta$

STUDIO $\nu\omega T_D = \nu \cancel{\omega} \frac{2\pi}{\cancel{\omega}\sqrt{1-\nu^2}} = \frac{2\pi\nu}{\sqrt{1-\nu^2}} \simeq$ ; SE $\nu \ll 1$ $\simeq 2\pi\nu$

ALLORA $\frac{A_i}{A_{i+1}} \simeq e^{2\pi\nu}$ ; ma $e^{2\pi\nu} \simeq 1 + 2\pi\nu$ $(\nu \ll 1)$ $\Rightarrow$ $\boxed{\frac{A_i}{A_{i+1}} \simeq 1 + 2\pi\nu}$ ESPRESS. APPROSS.

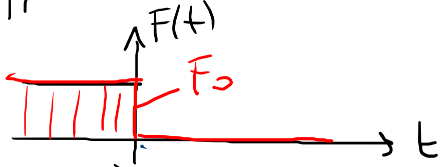
ESEMPIO DI [IDENTIFICAZIONE STRUTTURALE] $\Leftarrow$ POSSO CALCOLARE $\nu$ ATTRAVERSO IL RAPPORTO TRA DUE AMPIEZZE CONSECUTIVE DELLA FUNZIONE OSCILLANTE

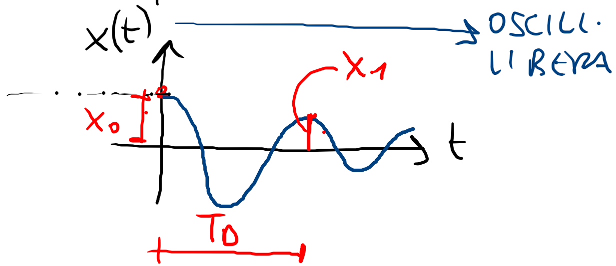
ES.




| INCOGNITE | |
|---|---|
| m ? | ok |
| c ? | |
| K ? | ok |

DATI



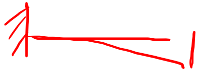




$x_0 = 5\ mm$

$x_1 = 4\ mm$

$T_D = 1.40\ s$

$F_0 = 90\ KN$



$k = \frac{3EJ}{\ell^3}$

PROPOSTA DI SOLUZ.

$\boxed{\nu = \frac{c}{2\sqrt{Km}}}$

NOTO

$\frac{x_0}{x_1} = 1 + 2\pi\nu \rightarrow \nu \simeq 0.035$

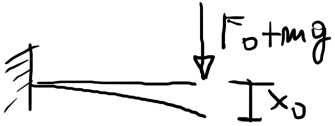
$\omega = \frac{2\pi}{T_D}\frac{1}{\sqrt{1-\nu^2}} \rightarrow \omega \simeq 4.49\ \frac{rad}{s}$

NOTI

$\omega^2 = \frac{K}{m}$

LA SOLUZ. STATICA $(t \leq 0)$ È:

$K x_0 = F_0 + mg$




$\boxed{K x_0 = F_0 + \frac{K}{\omega^2} g} \Rightarrow K$

$m = \frac{K}{\omega^2} \Rightarrow m$

NOTO

$c = \nu\, 2\sqrt{Km} \Rightarrow c$