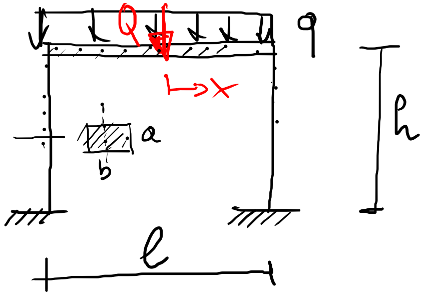
24/11/2022

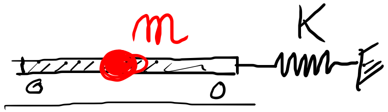


$h = 3\,m$ $\quad b = 30\,cm$ $\quad a = 30\,cm$ $\quad \ell = 5\,m$ $\quad q = 40\,kN/m$ $\quad E = 25000\,\frac{N}{mm^2}$ (cls)

$g = 9.81\,m/s^2$

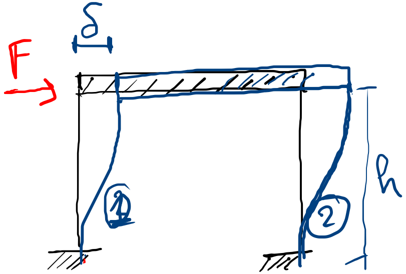
$I = \frac{ab^3}{12}$

ω, T ?



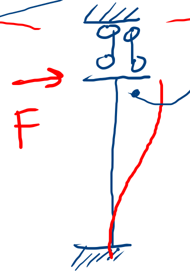
$Q = q\ell = 40 \cdot 5 = 200\,kN$

$Q = mg \Rightarrow m = \frac{Q}{g} = \frac{200000}{9.81} \simeq 20000\,kg$



$\delta = \delta(F) = \frac{F}{K}$

$\boxed{K = \frac{24EI}{h^3}}$



$\eta = \frac{Fh^3}{12EI}$

$\delta = \frac{Fh^3}{24EI}$

2 pilastri.

$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1}{m}\,24\,\frac{1}{h^3}\,E\,ab^3\,\frac{1}{12}}$

$= \hat{f}\left(\sqrt{\left(\frac{b}{h}\right)^3}\,\right)$

$\omega = 27.39\,\frac{rad}{s}$

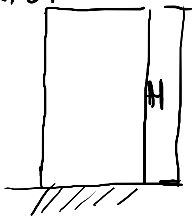
$T = \frac{2\pi}{\omega} = \frac{6.28}{27.39} \simeq 0.23\,s$

$f = \frac{1}{T} = 4.35\,Hz$

$T = \hat{g}\left(\sqrt{\frac{h^3}{b^3}}\right) \Rightarrow \bar{g}\left(h^{3/2}\right)$

"NORMATIVA" FORMULA EMPIRICA

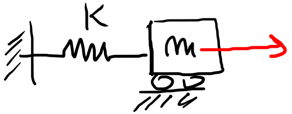
$T = \underbrace{c_1}_{0{,}05\text{ (dipende dal materiale)}} H^{3/4}$



Con $h = 4\,m$; $T = 0.35\,s$

$h = 5\,m$, $T = 0.49\,s$

OSCILLATORE SEMPLICE CON FORZANTE ARMONICA



$F(t) = F \sin \bar{\omega} t$

$\bar{\omega}$ → PULSAZIONE DELLA FORZANTE

F → AMPIEZZA

$$m\ddot{x} + Kx = F \sin \bar{\omega} t$$

$$\boxed{x(t) = x^{hom}(t) + x^{P}(t)}$$ particolare

$$x^{P}(t) = X \sin \bar{\omega} t$$

$$\ddot{x}^{P}(t) = -\bar{\omega}^2 X \sin \bar{\omega} t$$

$$m(-\bar{\omega}^2 X)\cancel{\sin \bar{\omega} t} + KX\cancel{\sin \bar{\omega} t} = F\cancel{\sin \bar{\omega} t}$$

$$-m\bar{\omega}^2 X + KX = F$$

$$X = \frac{F}{K - m\bar{\omega}^2} = \frac{F}{K}\frac{1}{1 - \frac{m}{K}\bar{\omega}^2}$$

Ricordo che $\omega^2 = \frac{K}{m} \Rightarrow X = \frac{F}{K}\frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2}$

$$x^{P}(t) = \frac{F}{K}\frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2}\sin \bar{\omega} t$$

$$x(t) = B_1 \sin \omega t + B_2 \cos \omega t + \frac{F}{K}\frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2}\sin \bar{\omega} t$$

B_1 e B_2 si determinano dalle condiz. iniziali $x(0)$, $\dot{x}(0)$

$$x(0) = x_0 \Rightarrow x(0) = \boxed{B_2 = x_0}$$

$$\dot{x}(t) = B_1 \omega \cos \omega t - B_2 \omega \sin \omega t + \left(\frac{F}{K}\frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2} \cdot \bar{\omega} \cos \bar{\omega} t \right)$$

$$\dot{x}(0) = \dot{x}_0 = B_1 \omega + \frac{F}{K}\frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2}\bar{\omega}$$

$$\boxed{B_1 = \frac{\dot{x}_0}{\omega} - \frac{F}{K}\frac{\bar{\omega}}{\omega}\frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2}}$$

Consideriamo la legge $x(t)$ con $x_0=\dot{x}_0=0$

$$x(t)=\underbrace{\frac{F}{K}}_{X_{ST}}\,\frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2}\left[\sin\bar{\omega}t-\frac{\bar{\omega}}{\omega}\sin\omega t\right]$$

$\bar{\omega}\to\omega,\ x(t)\to\infty$

1) QUESTA SOLUZ. NON VALE PER $\boxed{\frac{\bar{\omega}}{\omega}=1}$

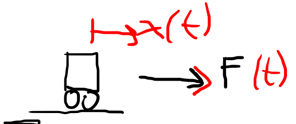
2) $\frac{F}{K}=X_{ST}$: SPOST. DELLA MASSE SOTTO L'AZIONE STATICA DELLA FORZA F

Concentriamoci su:

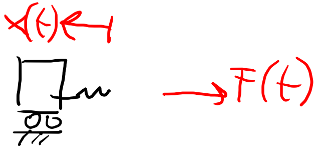
$$x(t)=X_{st}\,\frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2}\sin\bar{\omega}t$$

se $\bar{\omega}<\omega$ allora $x(t)>0$

se $\bar{\omega}>\omega$ allora $x(t)<0$



RISPOSTA IN FASE



RISPOSTA IN OPPOSIZIONE DI FASE

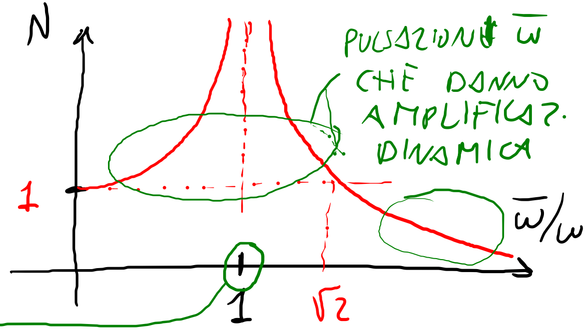
$$x(t)=N\,x_{st}\sin(\bar{\omega}t-\xi)$$

$\xi=0$ se $\bar{\omega}<\omega$

$\xi=\pi$ se $\bar{\omega}>\omega$

$$\boxed{N=\frac{1}{\left|1-\left(\frac{\bar{\omega}}{\omega}\right)^2\right|}}$$ è FATTORE DI AMPLIFICAZIONE

$$x(t)_{MAX}=N\,x_{st}\,\overbrace{\sin(\quad)}^{1}\ \rightarrow\ N=\frac{x(t)_{MAX}}{x_{st}}$$



$\bar{\omega}/\omega=1$: CONDIZIONE DI RISONANZA

QUANDO $\overline{\omega} = \omega$ È NECESSARIO DETERMINARE UNA SOLUZ. DIVERSA DELL'EQUAZ. DIFFERENZIALE.