ASPETTI ENERGETICI 25/11/22

$$E_{mecc} = E_{el} + E_{cin}$$

→ SIST. CONSERVATIVO $\overbrace{\frac{d}{dt} E_{mecc}}^{\text{POTENZA}} = 0$

→ SIST. NON CONSERVATIVO CON FORZANTE ESTERNA $F(t)$

$$\underbrace{\frac{d}{dt} E_{mecc}}_{\text{POTENZA}} = \underbrace{F(t)\,\dot{x}(t)}_{\text{POTENZA DELLA FORZA AL TEMPO } t}$$

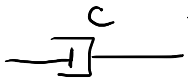
$F, \dot{x}$ CONCORDI $\frac{d}{dt} E_{mecc} > 0$

$F, \dot{x}$ DISCORDI " < 0

IN GENERALE:

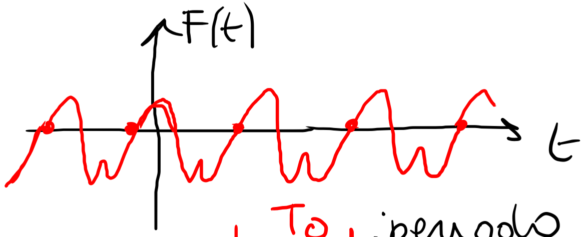
$$\frac{d}{dt} E_{mecc} = P_F - P_D$$

P_F: POTENZA DELLA FORZANTE

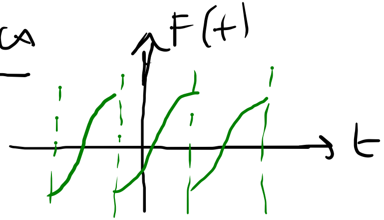
P_D: " DISSIPATA (

) ; $P_D = c\dot{x}^2$ $\left(\underbrace{c\dot{x}}_{\text{FORZA}} \cdot \underbrace{\dot{x}}_{\text{VELOCITÀ}} \right)$

FORZANTE PERIODICA GENERICA



T_0 : periodo della forzante



$$F(t+kT_0) = F(t) \quad ; \quad k \in \mathbb{Z}$$

$$T_0 \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

SERIE DI FOURIER

$$F(t) = F_0 + \sum_{n=1}^{\infty} \left[A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t) \right]$$

$$F_0 = \frac{1}{T_0} \int_{\bar{t}}^{\bar{t}+T_0} F(t)\, dt$$

$$\left\{ \begin{matrix} A_n \\ B_n \end{matrix} \right\} = \frac{2}{T_0} \int_{\bar{t}}^{\bar{t}+T_0} F(t) \begin{matrix} \cos(n\omega_0 t) \\ \sin(\quad) \end{matrix}\, dt$$

Solitamente si sceglie un n° FINITO DI ARMONICHE

$$F(t) \simeq F_0 + \sum_{n=1}^{N} \left[\quad \right]$$

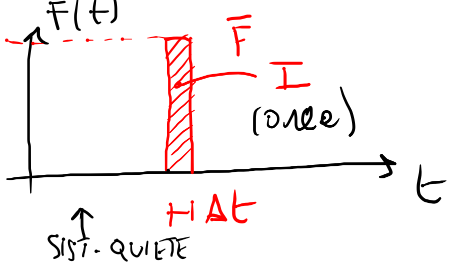
La risposta dell'oscillatore la ottengo sommando le risposte ottenute dai singoli termine

$$\boxed{x(t) = e^{-\nu\omega t}\left(c_1 \cos\omega_0 t + c_2 \sin\omega_0 t\right) + \underbrace{\frac{F_0}{K} \sum_{n=1}^{N} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]}_{x_{part}}}$$

INTEGRALE DI DUHAMEL

Fornisce la risposta dinamica di un oscillatore eccitato da una forzante generica $F(t)$

FORZA IMPULSIVA



$$I = \int F(t)\,dt$$

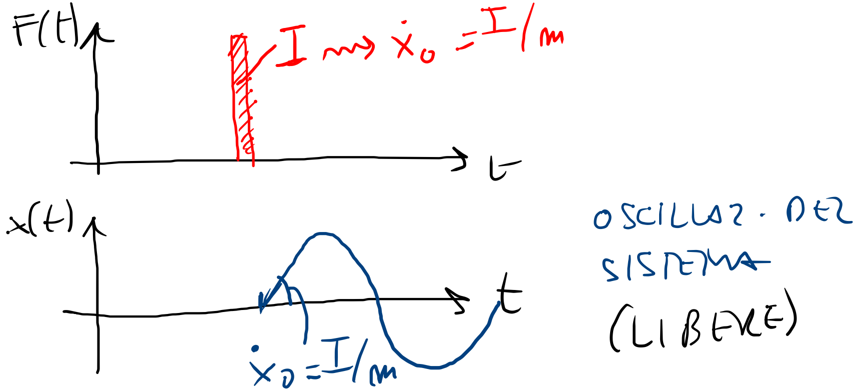
$$I = F\Delta t$$

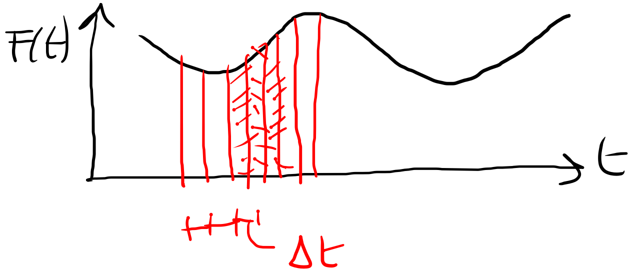
TEOREMA DELL'IMPULSO

$$\underbrace{\Delta Q}_{\text{VARIAZ. QUANTITA' DI MOTO}} = \underbrace{I}_{\text{IMPULSO}}$$

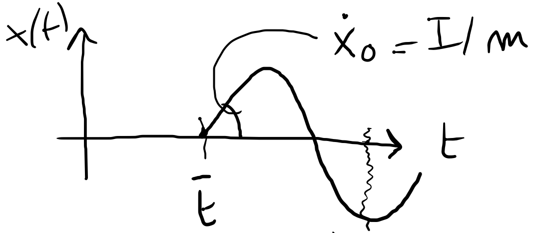
$$m\dot{x}(t+\Delta t) = I$$

$$\dot{x}(t+\Delta t) = \frac{I}{m}$$





La risposta del sistema è data dalla somma delle risposte dei singoli impulsi.



OSCILL LIBERE

$$x(t)=e^{-\nu\omega(t-\bar{t})}\left[B_1\cos\omega_D(t-\bar{t})+B_2\sin\omega_D(t-\bar{t})\right] \rightsquigarrow \dot{x}(t)=\dots\dots$$

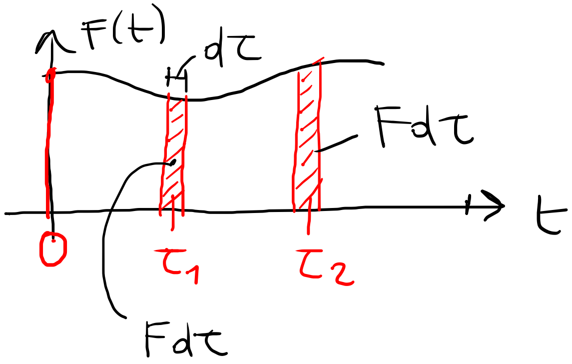
Le condiz iniziali di oscillat.

$$\begin{cases} x(\bar{t})=0 \\ \dot{x}(\bar{t})=\dfrac{I}{m} \end{cases} \Rightarrow \begin{cases} B_1=0 \\ B_2=\dfrac{I}{m\omega_D} \end{cases} \longrightarrow x(t)=e^{-\nu\omega(t-\bar{t})}\frac{I}{m\omega_D}\sin\omega_D(t-\bar{t}) \qquad (t\geq\bar{t})$$

FUNZIONE DI RISPOSTA AD UN IMPULSO UNITARIO: $h(t-\bar{t})=x(t)\big|_{I=1}$

$$h(t-\bar{t})=\frac{1}{m\omega_D}e^{-\nu\omega(t-\bar{t})}\sin\omega_D(t-\bar{t})$$

$$\Rightarrow x(t)=I\,h(t-\bar{t})$$



$$x(t) = \int_0^t \underbrace{F\,d\tau}_{I(\tau)}\, h(t-\tau) = \int_0^t F(\tau)\, \underbrace{h(t-\tau)}\, d\tau$$

$$\underbrace{x(t) = \int_0^t F(\tau)\, \frac{1}{m\omega_D}\, e^{-\nu\omega(t-\tau)} \sin \omega_D (t-\tau)\, d\tau}_{\text{INTEGRALE DI DUHAMEL}}$$

t: tempo al quale voglio studiare la risposta dell'oscillatore.

τ: variabile di integrazione