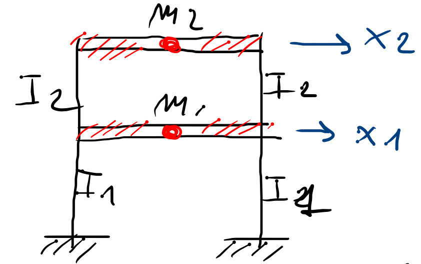
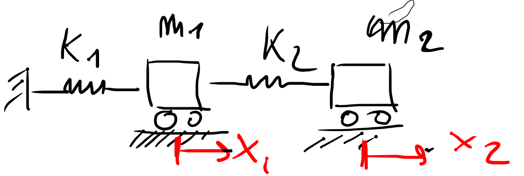
OSCILLAZ. LIBERE DI UN SIST. A 2 GDL

2/12/22

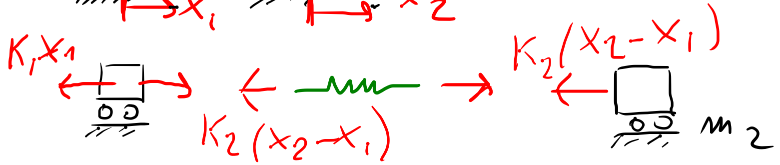


MASSE CONCENTR. AL PIANO

SCHEMA CHE VOGLIAMO MODELLARE CON 2 GDL

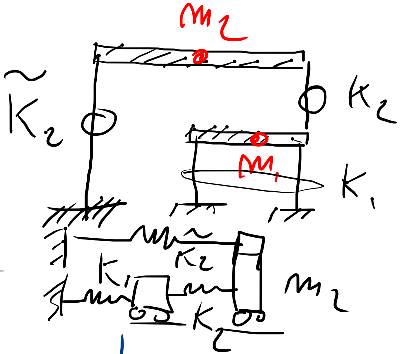


$x_2 > x_1$



$$\begin{cases} m_1 \ddot{x}_1 = +K_2(x_2 - x_1) - K_1 x_1 \\ m_2 \ddot{x}_2 = -K_2(x_2 - x_1) \end{cases}$$

$$\begin{cases} m_1 \ddot{x}_1 + K_1 x_1 - K_2(x_2 - x_1) = 0 \\ m_2 \ddot{x}_2 + K_2(x_2 - x_1) = 0 \end{cases}$$



$\underset{\sim}{M}$: MATRICE DELLE MASSE

$\underset{\sim}{K}$: MATRICE DI RIGIDEZZA

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{M} \, \underline{\ddot{x}} + \underset{\sim}{K} \, \underline{x} = \underline{0}$$

NOTAZIONE MATRICIALE CHE GENERALIZZA L'EQUAZ.

$$m \ddot{x} + K x = 0$$

(OSCILLAZ. LIBERE)

CERCO SOLUZIONI DEL TIPO: $\begin{cases} x_1 = U_1 \sin \omega t \\ x_2 = U_2 \sin \omega t \end{cases}$ ADIM. $\Longleftrightarrow \begin{cases} x_1 = D_1 e^{\lambda t} \\ x_2 = D_2 e^{\lambda t} \end{cases}$

ω?? $\left[\omega = \sqrt{\frac{K}{m}} \right]$ OSC SEMPL.

$\left[\frac{1}{s}\right]$

$$\underbrace{\left[m_1 U_1 (-\omega^2) + K_1 U_1 + K_2 (U_1 - U_2) \right]}_{0\ (\forall t)} \underbrace{\sin \omega t}_{\neq 0} = 0$$

$$\underbrace{\left[m_2 U_2 (-\omega^2) + K_2 (U_2 - U_1) \right]}_{0\ (\forall t)} \underbrace{\sin \omega t}_{\neq 0} = 0$$

$$\begin{cases} -m_1 U_1 \cdot \omega^2 + K_1 U_1 + K_2 (U_1 - U_2) = 0 \\ -m_2 U_2 \cdot \omega^2 + K_2 (U_2 - U_1) = 0 \end{cases}$$

SIST. OMOGENEO CHE AMMETTE SOLUZ. SE IL DET DELLA MATRICE ASSOCIATA È NULLO.

$$\begin{vmatrix} K_1 + K_2 - m_1 \omega^2 & -K_2 \\ -K_2 & K_2 - m_2 \omega^2 \end{vmatrix} = 0 \quad (*)$$

$\Rightarrow$ PROBL. AGLI AUTOVALORI GENERALIZZ. $(\omega^2) \to \omega_1^2, \omega_2^2$ (DUE SOLUZ.)

IN TERMINI MATRICIALI:

$$\left| \underset{\sim}{K} - \omega^2 \underset{\sim}{M} \right| = 0$$

~~PROBLEMA~~ AUTOVALORI STANDARD

$$\left| \underset{\sim}{A} - \lambda \underset{\sim}{I} \right|$$

$$\det\left| \quad \right| = 0 \Rightarrow \omega^4 - \left(\frac{k_1+k_2}{m_1} + \frac{k_2}{m_2}\right)\omega^2 + \frac{k_1 k_2}{m_1 m_2} = 0$$ BIQUADRATICA

$$\left.\begin{array}{l} 0<\omega_1^2 \\ 0<\omega_2^2 \end{array}\right\} = -b \pm \sqrt{b^2-4ac} \Rightarrow$$ NORMALMENTE SI ORDINANO IN MODO CHE $\boxed{\omega_1^2 < \omega_2^2} \Rightarrow \omega_1, \omega_2 \in \mathbb{R}$

ω_1: PULSAZIONE FONDAMENTALE DEL SISTEMA (NATURALE)

$T_1 = \dfrac{2\pi}{\omega_1}$: PERIODO " " " $(T_1 > T_2)$

$\left(T_2 = \dfrac{2\pi}{\omega_2}\right)$

SIGNIFICATO DEGLI AUTOVETTORI

- per ogni autovalore $(\omega_i^2) \Rightarrow$ UN AUTOVETTORE $U^{(i)}$

$$\left.\begin{array}{l} \omega_1^2 \Rightarrow U^{(1)} \\ \omega_2^2 \Rightarrow U^{(2)} \end{array}\right\}$$ MODI DI VIBRARE DEL SISTEMA.

CALCOLIAMO $U^{(1)}$

$$\begin{cases} [(K_1+K_2)-\omega_1^2 m_1]\,U_1^{(1)} - K_2\, U_2^{(1)} = 0 \\ -K_2\, U_1^{(1)} + [K_2 - \omega_1^2 m_2]\,U_2^{(1)} = 0 \end{cases} \Rightarrow U_1^{(1)},\ U_2^{(1)}$$

SIST. LINEARM. DIPENDENTE

$$\Rightarrow U_2^{(1)} = f(U_1^{(1)})$$

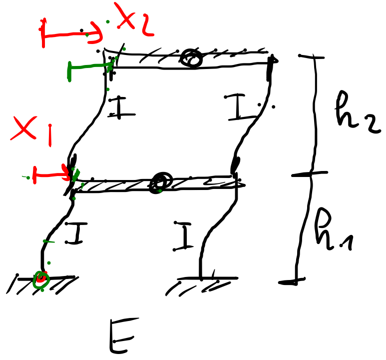
GLI AUTOVETTORI SONO A MENO DI UNA COSTANTE

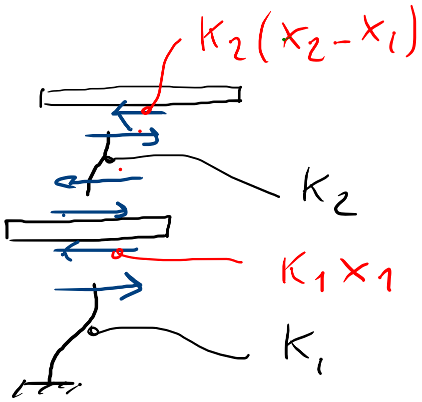
SOLO UNA EQUAZ. È SIGNIFICATIVA

SE VOGLIO CALCOLARE $U^{(2)}$

$$\begin{cases} \text{----}\ \omega_2^2\ \text{--}\ U_1^{(2)}\ \text{--}\ U_2^{(2)} = 0 \\ \text{--}\ U_1^{(2)}\ \text{---}\ \omega_2^2\ \text{--}\ U_2^{(2)} = 0 \end{cases} \Rightarrow U_2^{(2)} = g(U_2^{(1)})$$

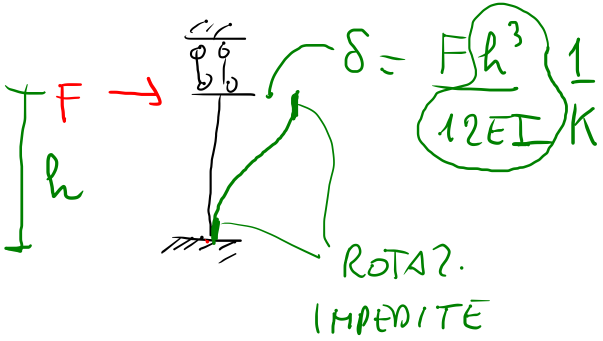
ESEMPIO





$$K_2 = 2\cdot\frac{12EI}{h_2^3}$$

$$K_1 = 2\cdot\frac{12EI}{h_1^3}$$



$h_1 = h_2 = h$; $K_1 = K_2 = K = 24\,\frac{EI}{h^3}$; $m_1 = m_2 = m$

$$\omega_1 = 0.618\sqrt{\frac{K}{m}} \qquad T_1 = 10.166\sqrt{\frac{m}{K}}$$

$$\omega_2 = 1.618\sqrt{\frac{K}{m}} \qquad T_2 = 3.88\sqrt{\frac{m}{K}}$$

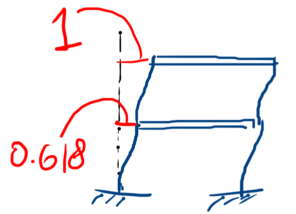
$$U^{(1)} = \begin{bmatrix} 0.618 \\ 1 \end{bmatrix} \; ; \; U^{(2)} = \begin{bmatrix} 1 \\ -0.618 \end{bmatrix}$$

I MODO DI VIBRARE

II MODO DI VIBRARE

0 NODI

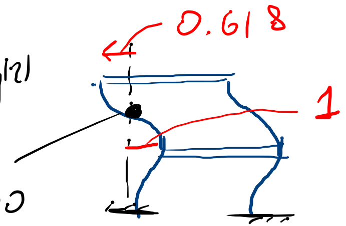
$U^{(1)}$



NORMALIZZATO

$$\max\left|U_j^{(i)}\right| = 1$$

$U^{(2)}$

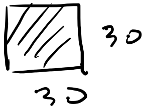


1 NODO

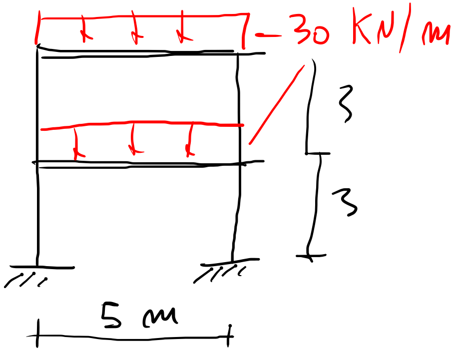
ES NUMERICO

$E = 20\ GPa$ (CLS)

$I = \frac{30^4}{12} = 67500\ cm^4$



$h = 3\ m$



$m = \frac{30000 \cdot 5}{g} \simeq 15000\ Kg$

$T_1 = 0.36\ s$ PERIODO FONDAMENTALE

$T_2 = 0.14\ s$