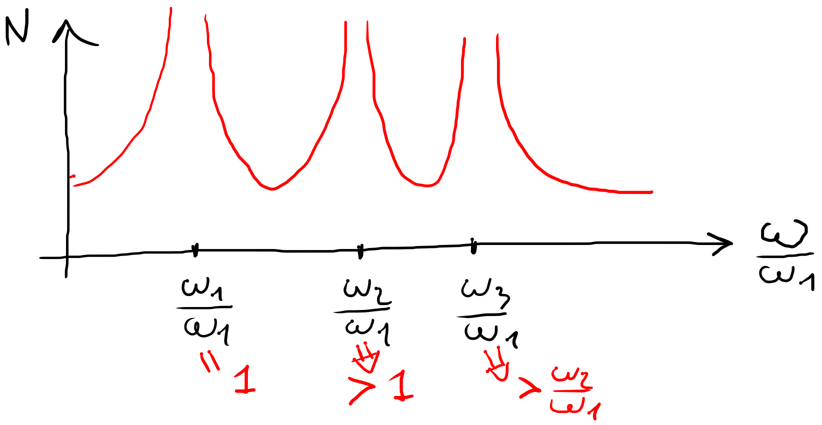
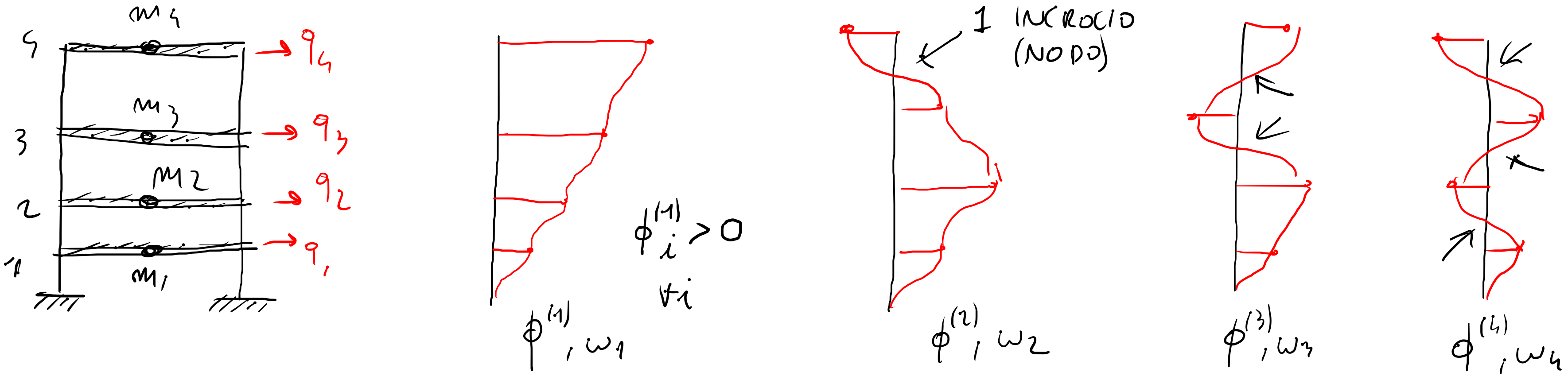
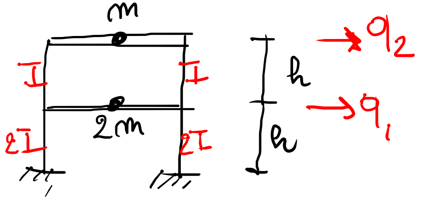
OSSERVAZIONI SUL DIAGR. DEL COEFF DI AMPLIFICAZ. 16/03/23



OSSERVAZ. SUI MODI DI VIBRARE DI TELAI SHEAR-TYPE



ES: ANALISI MODALE 2 GDL



$$\underset{\sim}{M} = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}$$

$$\underset{\sim}{K} = \frac{24EI}{h^3} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

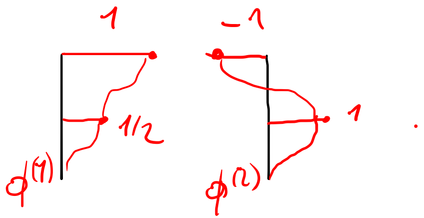
k

ANALISI AUTOVALORI / AUTOVETTORI

$$\left(\underset{\sim}{K} - \omega^2 \underset{\sim}{M}\right) \underline{\phi} = \underline{0}$$

$$\begin{cases} \omega_1^2 = \frac{1}{2} \frac{k}{m} \\ \omega_2^2 = 2 \frac{k}{m} \end{cases}$$

$$\underline{\phi}^{(1)} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} ; \quad \underline{\phi}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



$$\underset{\sim}{\Phi} = \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} \quad \left(= \underset{\sim}{\Phi}^T \text{ CASO PARTICOLARE}\right)$$

PER INTRODURRE LE COORD. PRINCIPALI: $\underline{q} = \underset{\sim}{\Phi}\, \underline{z}$

$$\hat{\underset{\sim}{M}} = \underset{\sim}{\Phi}^T \underset{\sim}{M} \underset{\sim}{\Phi} = \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} = \underline{\begin{bmatrix} 3/2\, m & 0 \\ 0 & 3m \end{bmatrix}}$$

$$\hat{\underset{\sim}{K}} = \underset{\sim}{\Phi}^T \underset{\sim}{K} \underset{\sim}{\Phi} = \quad '' \quad \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix} \quad '' \quad =$$

$$\begin{bmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{3}{2}-1 & 3+1 \\ -\frac{1}{2}+1 & -2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1/2 & 4 \\ 1/2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4}+\frac{1}{2} & 2-2 \\ \frac{1}{2}-\frac{1}{2} & 4+2 \end{bmatrix} = \underline{\begin{bmatrix} 3/4 & 0 \\ 0 & 6 \end{bmatrix} k}$$

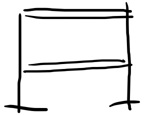
SIST. COORD. PRINCIPALI

$$\begin{cases} \overbrace{\frac{3}{2} m}^{\hat{m}_1} \ddot{z}_1 + \overbrace{\frac{3}{4} K}^{\hat{k}_1} z_1 = 0 & \Longrightarrow \omega_1^2 \\ 3 m \ddot{z}_2 + 6 K z_2 = 0 & \Longrightarrow \omega_2^2 \end{cases}$$

$$\ddot{z}_1 + \Omega_1^2 z_1 = 0 \qquad \Omega_1^2 = \frac{3}{4} K \frac{2}{3m} = \frac{1}{2} \frac{K}{m} \overset{OK}{=} \omega_1^2 \; !!$$

$$\ddot{z}_2 + \Omega_2^2 z_2 = 0 \qquad \Omega_2^2 = \frac{6K}{3m} = 2 \frac{K}{m} \overset{OK}{=} \omega_2^2 \; !!$$

APPLICHIAMO ORA UN MOTO IMPRESSO ALLA BASE



$\rightarrow \ddot{y}(t)$ (ASSEGNATO)

$$\underline{F}(t) = -\begin{bmatrix} 2 \\ 1 \end{bmatrix} m \ddot{y}(t) \quad ; \quad \hat{F}(t) = \underset{\sim}{\Phi}^T \underset{\sim}{F}(t) = -\begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} m \ddot{y}(t)$$

$$= -\begin{bmatrix} 2 \\ 1 \end{bmatrix} m \ddot{y}(t)$$

SIST. COORD. PRINCIPALI

$$\begin{cases} \frac{3}{2} m \ddot{z}_1 + \frac{3}{4} K z_1 = -2 m \ddot{y}(t) \\ 3 m \ddot{z}_2 + 6 K z_2 = - m \ddot{y}(t) \end{cases}$$

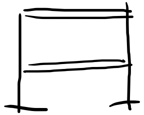
SIST. COORD. PRINCIPALI

$$\begin{cases} \overset{\hat m_1}{\frac{3}{2}m}\,\ddot z_1 + \overset{\hat k_1}{\frac{3}{4}K}\,z_1 = 0 \quad \Longrightarrow \omega_1^2 \\ 3m\,\ddot z_2 + 6K\,z_2 = 0 \quad \Longrightarrow \omega_2^2 \end{cases}$$

$$\ddot z_1 + \Omega_1^2 z_1 = 0 \qquad \Omega_1^2 = \frac{3}{4}K\,\frac{2}{3m} = \frac{1}{2}\frac{K}{m} \overset{OK}{=} \omega_1^2 \;!!$$

$$\ddot z_2 + \Omega_2^2 z_2 = 0 \qquad \Omega_2^2 = \frac{6K}{3m} = 2\frac{K}{m} \overset{OK}{=} \omega_2^2 \;!!$$

APPLICHIAMO ORA UN MOTO IMPRESSO ALLA BASE



$\rightarrow \ddot y(t)$ (ASSEGNATO)

$$\underset{\sim}{F}(t) = -\begin{bmatrix}2\\1\end{bmatrix} m\,\ddot y(t) \quad ; \quad \hat F(t) = \underset{\sim}{\Phi}^T \underset{\sim}{F}(t) = -\begin{bmatrix}1/2 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} m\,\ddot y(t)$$

$$= -\begin{bmatrix}2\\1\end{bmatrix} m\,\ddot y(t)$$

SIST. COORD. PRINCIPALI

$$\begin{cases} \frac{3}{2}m\,\ddot z_1 + \frac{3}{4}K z_1 = -2m\,\ddot y(t) \\ 3m\,\ddot z_2 + 6K z_2 = -m\,\ddot y(t) \end{cases} \qquad \begin{cases} \ddot z_1 + \omega_1^2 z_1 = -\overset{g_1}{\frac{4}{3}}\,\ddot y(t) \\ \ddot z_2 + \omega_2^2 z_2 = -\underset{g_2}{\frac{1}{3}}\,\ddot y(t) \end{cases}$$

ORTOGONALITA' DEI MODI DI VIBRARE

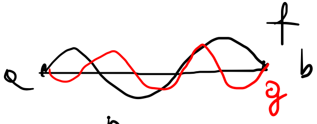
$\boxed{i \neq j}$ $\quad \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(j)} = \underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(j)} = 0$

$\underline{u} \perp \underline{v}$ se $\underline{u} \cdot \underline{v} = 0$

$\underline{u} \cdot \underset{\sim}{I}\, \underline{v} = 0$

ORTOGONALITA' GENERALIZZ. $\quad \underline{u} \overset{A}{\perp} \underline{v}$ se $\underline{u} \cdot \underset{\sim}{A}\, \underline{v} = 0$

ORTOGONALITA' DI FUNZ.



$$\int_a^b f\, g\, dx = 0$$

DIM.

$$(\underset{\sim}{K} - \omega_i^2 \underset{\sim}{M})\, \underline{\phi}^{(i)} = \underline{0} \Rightarrow \underset{\sim}{K}\, \underline{\phi}^{(i)} = \omega_i^2 \underset{\sim}{M}\, \underline{\phi}^{(i)}$$

$$\underbrace{\underline{\phi}^{(j)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)}} = \omega_i^2\, \underline{\phi}^{(j)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)} \qquad \text{I}$$

$$(\underset{\sim}{K} - \omega_j^2 \underset{\sim}{M})\, \underline{\phi}^{(j)} = \underline{0} \Rightarrow \underbrace{\underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(j)}} = \omega_j^2\, \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(j)} \qquad \text{II}$$

VISTO CHE $\underset{\sim}{K}$ ed $\underset{\sim}{M}$ SONO MATRICI SIMMETRICHE FACCIO LA DIFFERENZA TRA I e II

$$0 = (\omega_i^2 - \omega_j^2)\, \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(j)} \Rightarrow \text{se } \omega_i \neq \omega_j \Rightarrow \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(j)} = 0$$

Considero ora l'eq I: il SECONDO MEMBRO È NULLO $\Rightarrow \underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \phi^{(j)} = 0$

$(i \neq j)$ $\blacksquare$

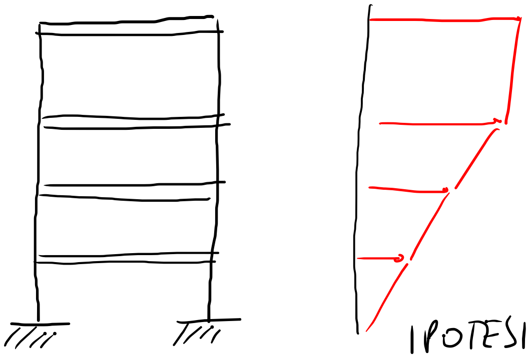
CONSIDERO L'EQ (I) MA PER $i = j$

$$\underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)} = \omega_i^2\, \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)} \Longrightarrow \omega_i^2 = \frac{\underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)}}{\underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)}} \quad (*)$$

RAPPORTO DI RAYLEIGH

(*) PUÒ ESSERE USATA PER STIMARE (SPERIMENTALMENTE) ω_i DI UNA STRUTTURA

ES.



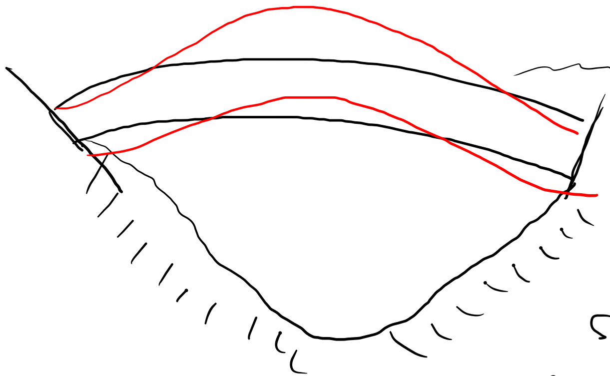
STIMA ω_1

$$\omega_{1\,APPR.}^2 = \frac{\underline{\phi}_{APPR}^{(1)} \cdot \underset{\sim}{K}\, \underline{\phi}_{APPR}^{(1)}}{\underline{\phi}_{APPR}^{(1)} \cdot \underset{\sim}{M}\, \underline{\phi}_{APPR}^{(1)}}$$

IPOTESI SUL I MODO DI VIBRARE

- ESPERIENZA DELL'INGEGNERE
- INDAGINI SPERIMENTALI

$\Longrightarrow \underline{\phi}_{APPR}^{(1)}$



DIGA DALL'ALTO

FORMA VEROSIMILE DEL I MODO DI VIBRARE

$$T = \frac{2\pi}{\omega}$$

$$\omega = 2\pi f$$

PULSAZ. (rad/sec)

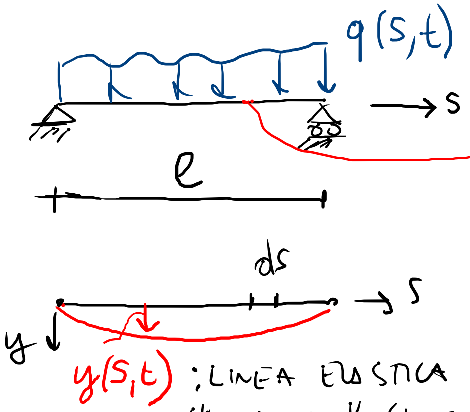
FREQUENZA (Hz)

SPERIMENTALM. SI PUÒ STIMARE ATTRAVERSO INDAGINI CON STRUMENTI VIBRANTI (VIBRODINA) O SENSORI CHE MISURANO LE "VIBRAZ. AMBIENTALE".

CHOPRA "DYNAMICS OF STRUCTURES"

⇝ INDUSTR. / MECC. / ⟹ MECCANICA DELLE VIBRAZIONI

DINAMICA DEI SISTEMI CONTINUI AD 1 G.D.L. GENERALIZZATO



t: tempo ; $q(s,t)$: FORZANTE ESTERNA

RIGIDEZZA ELASTICA "EI"

MASSA PER UNITA' DI LUNGHEZZA μ

$y(s,t)$; LINEA ELASTICA "DINAMICA" (INCOGNITA)

$$y(s,t) = \underbrace{\psi(s)}_{\text{FUNZ. DI FORMA}} x(t)$$

$$\dot{y}(s,t) = \psi(s)\,\dot{x}(t) \quad ; \quad \chi(s) = -y''(s,t) = -\psi''(s)\,x(t)$$

χ ← CURVATURA DELLA TRAVE

$$M(s,t) = -EI\,y'' = -\psi''\,x\,EI$$

MASSA EQUIV.

$$E_{cin} = \frac{1}{2}\int_0^\ell \mu\,ds\,\dot{y}^2 = \frac{1}{2}\overbrace{\int_0^\ell \mu\,\psi^2\,ds}^{m_{eq}}\;\dot{x}^2 = \frac{1}{2}\,m_{eq}\,\dot{x}^2$$

$$E_{el} = \frac{1}{2}\int_0^\ell M\chi\,ds = \frac{1}{2}\int_0^\ell (-\psi''xEI)(\underbrace{-y''}_{-\psi''x})\,ds$$

$$= \frac{1}{2}\underbrace{\int_0^\ell EI\,\psi''^2\,ds}_{K_{eq}}\;x^2$$

K_{eq} RIGIDEZZA EQUIVALENTE

$$\frac{d}{dt}\left[E_{cin} + E_{el}\right] = \text{Potenza del carico } q \quad (*)$$

BILANCIO DELLE POTENZE

POTENZA = FV

$$\int_0^\ell q\,\dot{y}\,ds = \underbrace{\int_0^\ell q\,\psi\,ds}_{F_{eq}}\;\dot{x}$$

$$\frac{d}{dt}\left[\frac{1}{2} m_{eq} \dot{x}^2 + \frac{1}{2} K_{eq} x^2\right] = F_{eq}\,\dot{x} \Rightarrow m_{eq} \cancel{\dot{x}}\, \ddot{x} + K_{eq}\, x \cancel{\dot{x}} = F_{eq} \cancel{\dot{x}}$$

$\forall t$

$$\boxed{m_{eq}\, \ddot{x}(t) + K_{eq}\, x(t) = F_{eq}(t)}$$

EQ. DELLA DINAMICA 1 G.D.L. GENERALIZZATO

CON $\psi(s)$ APPROSSIMATA POSSO OTTENERE UNA STIMA DELLA PULSAZIONE FONDAMENTALE DEL SISTEMA

$$\omega = \sqrt{\frac{K_{eq}}{m_{eq}}} \Rightarrow \omega^2 = \frac{K_{eq}}{m_{eq}} = \frac{\int_0^\ell EI\, \psi''^2\, ds}{\int_0^\ell \mu\, \psi^2\, ds}$$

RAPPORTO DI RAYLEIGH

$\psi(s)$ HA LO STESSO RUOLO DEL MODO DI VIBRARE $\underline{\phi}^{(1)}$