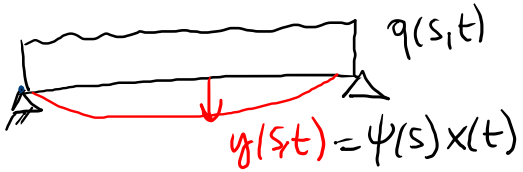
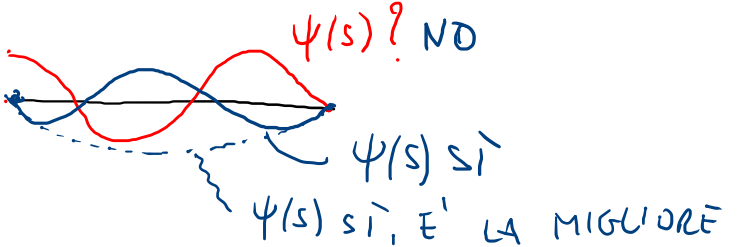
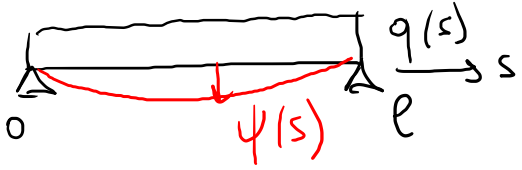




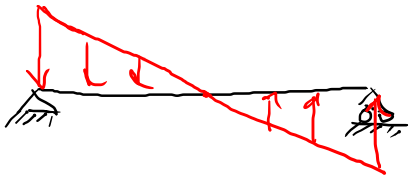
OSSERVAZ.: $\psi(s)$ può essere scelta in prima approssimazione come la deformata elastica quando il carico q agisce staticamente. UNA CONDIZ. INDISPENSABILE per la $\psi(s)$ è che soddisfi i vincoli (COMPATIBILITÀ CINEMATICA)



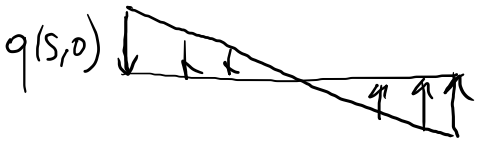
$$\begin{cases} EI\,\psi^{IV} = q(s) & \text{L. ELASTICA DEL IV ORDINE} \\ \text{c.c.}\quad \psi(0) = \psi(\ell) = 0 \;;\; \psi''(0) = \psi''(\ell) = 0 \end{cases}$$

(*)

NOTO CHE IN QUESTO PROBL LE ψ'' SODDISFANO LE CONDIZ. (*), MA PER LA SCELTA DI $\psi(s)$ QUESTE CONDIZ. NON SONO INDISPENSABILI.

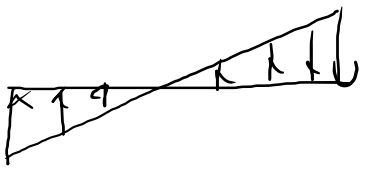


$q(s)\cos Kt = q(s,t)$

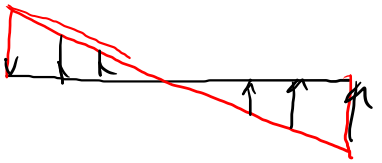


$q(s,0)$ $t=0$

$Kt=\pi/2$

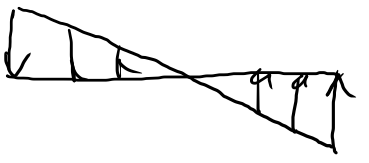


$Kt=\pi$

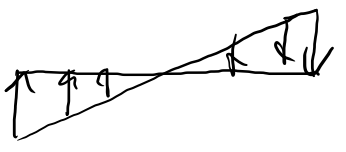


$q(s)\sin Kt$

$q(s,0)=0$ $t=0$

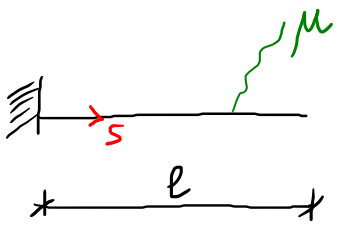


$Kt=\frac{\pi}{2}$



$Kt=\frac{3\pi}{2}$

ESEMPIO



μ : DENSITÀ DI MASSA PER UNITÀ DI LUNGH. (COSTANTE)

STUDIO LE OSCILLAZ. LIBERE

STIMO LA PULSAZ. FONDAMENTALE ω

$$\Rightarrow \omega^2 = \frac{K_{eq}}{m_{eq}}$$

SOLUZ. ESATTA È $\omega_{ES.} = \frac{3.516}{\ell^2}\sqrt{\frac{EI}{\mu}}$ NOTA DALLA TEORIA GENERALE

$$\psi_{ES} = f\left(\cos, \sin, \cosh, \sinh\left(\frac{s}{\ell}\right)\right)$$

VERIFICA DIM. ω : $[\omega] = \frac{1}{L^2}\sqrt{\frac{FL^2 L}{M}} = \frac{1}{L^2}\sqrt{\frac{ML}{T^2}\frac{L^3}{M}} = \frac{1}{T}$ OK

1) I APPROSS. $\psi_1(s) = \frac{3s^2}{2\ell^2} - \frac{s^3}{2\ell^3}$

$\psi_1(0) = 0$; $\psi_1'(s) = 3\frac{s}{\ell^2} - \frac{3s^2}{2\ell^3}$; $\psi_1'(0) = 0$

COMPATIBILITÀ ..... CINEMATICA

$$m_{eq}^{(1)} = 0.236\,\mu\ell$$
$$K_{eq}^{(1)} = 3\frac{EI}{\ell^3}$$

$$\Rightarrow \omega^{(1)} = \frac{3.568}{\ell^2}\sqrt{\frac{EI}{\mu}}$$

errore 1.5%

2) II APPROSS. $\psi_2(s) = 1 - \cos\frac{\pi s}{2\ell}$

$\psi_2(0) = \psi_2'(0) = 0$

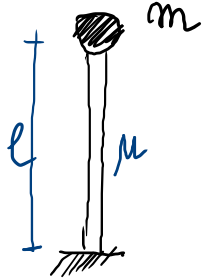
$m_{eq}^{(2)} = 0.227\,\mu\ell$; $K_{eq}^{(2)} = 3.044\frac{EI}{\ell^3}$

$$\Rightarrow \omega^{(2)} = \frac{3.664}{\ell^2}\sqrt{\frac{EI}{\mu}}$$

errore 4%

OSS. LE $\psi_K(s)$ CHE ABBIAMO SCELTO SODDISFANO LA CONDIZ. $\psi_K''(\ell)=0$ CHE HA IL SIGNIFICATO DI ANNULLARE IL MOMENTO FLETTENTE ALL'ESTREMO LIBERO. QUESTO PERÒ NON È OBBLIGATORIO PER LA FUNZIONE DI FORMA.
REQUISITO

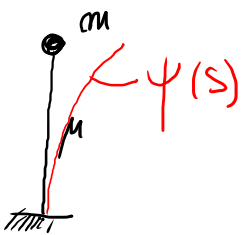
ESEMPIO DI APPLIC.



MENSOLA CHE SI RIDUCE AD UN GDL.

$K = \frac{3EI}{\ell^3}$

TRASCURO μ



MENSOLA CHE SI STUDIA COME UNA STR. AD 1 G.D.L. GENERALIZZATO

IMPORTANTE SCEGLIERE UNA $\psi(s)$ COMPATIBILE

METODO E.F.