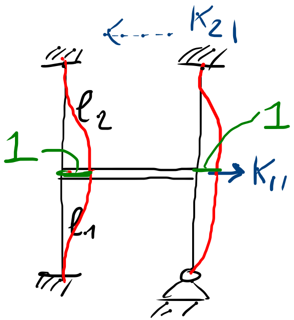
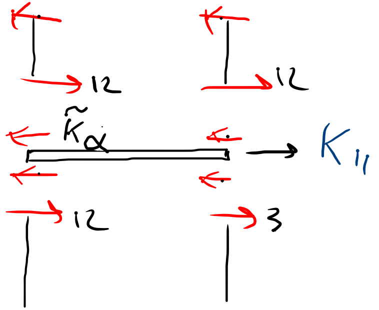
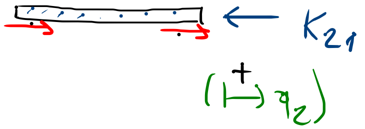
30/3/23





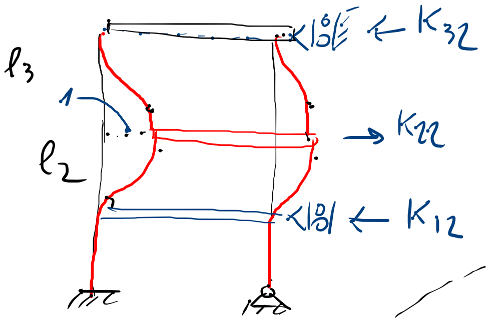


$$K_{11} = \sum_\alpha \tilde{K}_\alpha = \frac{12EI}{\ell_1^3} + \frac{3EI}{\ell_1^3} + 12\frac{EI}{\ell_2^3} + 12\frac{EI}{\ell_2^3}$$

$$K_{21} = -(12+12)\frac{EI}{\ell_2^3}$$

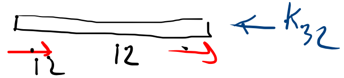
$$\boxed{q_2 = 1,\ q_1 = q_3 = 0}$$

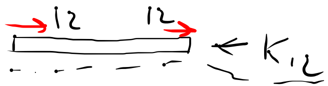
K_{22}, K_{12}, K_{32}



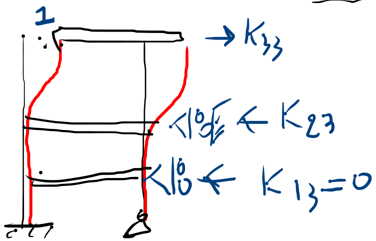
$$K_{22} = (12+12)\frac{EI}{\ell_2^3} + (12+12)\frac{EI}{\ell_3^3} = 24\,EI\left(\frac{1}{\ell_2^3} + \frac{1}{\ell_3^3}\right)$$

$$K_{32} = -(12+12)\frac{EI}{\ell_3^3} \quad ; \quad K_{12} = -(12+12)\frac{EI}{\ell_2^3}$$





$$q_3 = 1 \ ,\ q_1 = q_2 = 0$$

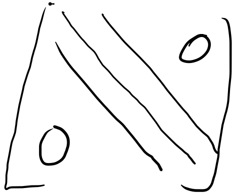


$$K_{33} = 24\frac{EI}{\ell_3^3}$$

$$K_{23} = -24\frac{EI}{\ell_3^3}$$

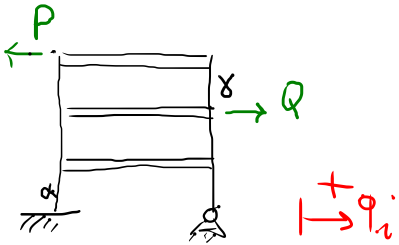
$$K_{11} = \frac{15EI}{\ell_1^3} + \frac{24EI}{\ell_2^3} \quad ; \quad K_{21} = -\frac{24EI}{\ell_2^3}$$

$$\underset{\sim}{K} = \begin{bmatrix} 15\frac{EI}{\ell_1^3} + 24\frac{EI}{\ell_2^3} & -24\frac{EI}{\ell_2^3} & 0 \\ -24\frac{EI}{\ell_2^3} & 24EI\left(\frac{1}{\ell_2^3}+\frac{1}{\ell_3^3}\right) & -24\frac{EI}{\ell_3^3} \\ 0 & -24\frac{EI}{\ell_3^3} & 24\frac{EI}{\ell_3^3} \end{bmatrix}$$



POZZATI: TEORIA E TECNICA DELLE STRUTTURE.

NOTA: COME "USO" $\underset{\sim}{K}$?

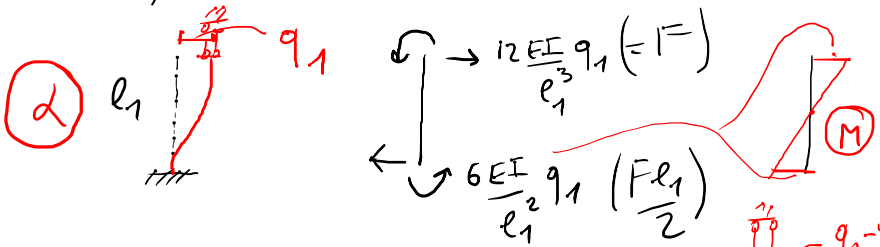


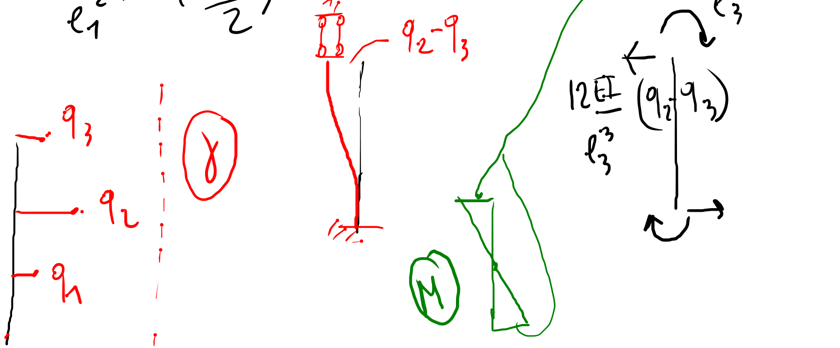
PROBLEMA: CALCOLARE LE SOLLECITAZIONI

1) CALCOLO SPOST. q_1, q_2, q_3

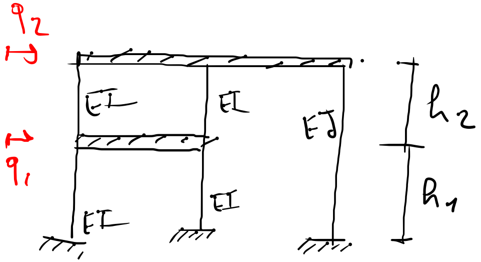
$$\begin{bmatrix} & & \\ & \underset{\sim}{K} & \\ & & \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ Q \\ -P \end{bmatrix}}_{\underline{F}} \leadsto \underline{q} = \underset{\sim}{K}^{-1}\underline{F}$$

2) CALCOLO SOLLECITAZIONI (PILASTRO α, PILASTRO γ)



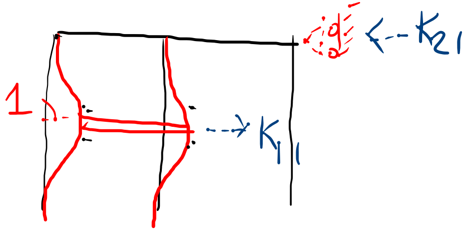


ESEMPIO



$\underset{\sim}{K}$: 2x2

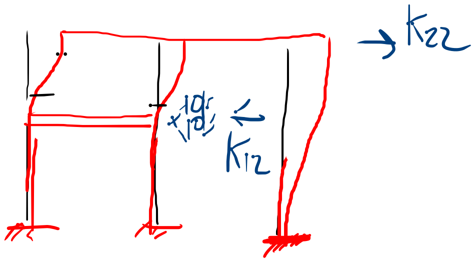
$q_1 = 1, \quad q_2 = 0$



$$K_{11} = 12\,EI \cdot \left(\frac{2}{h_1^3} + \frac{2}{h_2^3} \right)$$

$$K_{21} = -24 \frac{EI}{h_2^3}$$

$q_1 = 0, \quad q_2 = 1$



$$K_{22} = 24 \frac{EI}{h_2^3} + 12 \frac{EJ}{(h_1+h_2)^3}$$

$$K_{12} = -24 \frac{EI}{h_2^3}$$

In questi problemi di telai shear-type mi aspetto che le dimensioni dei K_{ij} siano tutte $\frac{F}{L}$.

VERIFICA:

$$\frac{EI}{\ell^3} = \frac{F}{L^2} L^4 \frac{1}{L^3} \overset{\text{ok}}{=} \frac{F}{L}$$

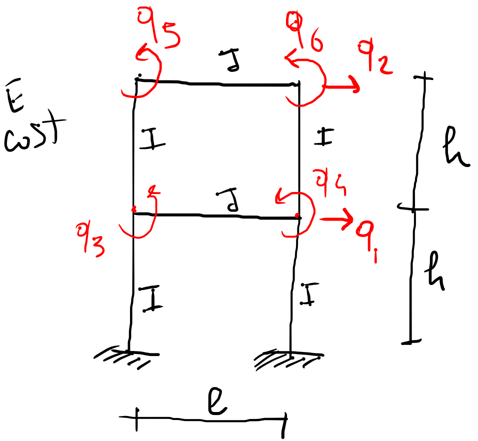
Se considero $q_i = 1$ cm

$\Rightarrow K_{ij} \Rightarrow \frac{N}{cm}$

e quando risolvo il problema $\underset{\sim}{K}\,\underline{q} = \underline{F}$ devo scegliere $\underline{q}$ e $\underline{F}$ in modo opportuno: $\underline{F} \Rightarrow N$

$\underline{q} \Rightarrow cm$

MATRICE DI RIGIDEZZA DI STRUTTURA CON GDL ROTAZIONALI E TRASLAZIONALE

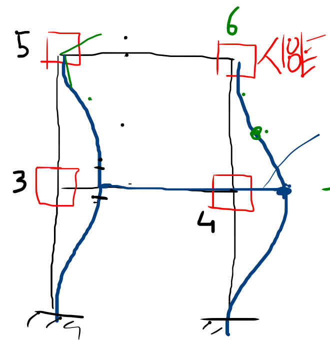


TRASC. LA DEF. ASSIALE
(PILASTRI E TRAVI INESTENSIBILI)

6 GDL $\Rightarrow$ $\underset{\sim}{K}$ è 6×6

STUDIAMO 2 "COLONNE" DELLA $\underset{\sim}{K}$

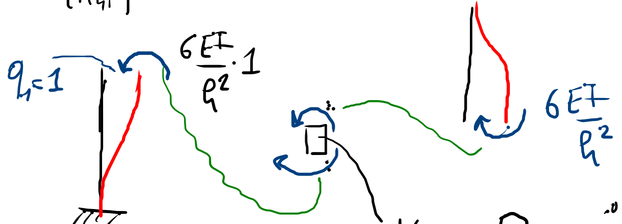
$q_1=1,\quad q_2=\dots=q_6=0$



$K_{21}=-24\frac{EI}{h^3}$

$K_{11}=48\frac{EI}{h^3}$

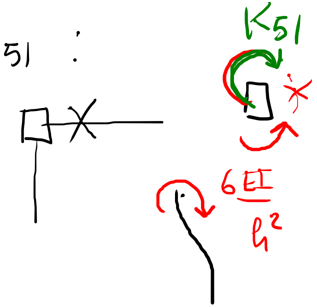
K_{31}? : REAZ. AL MORSETTO 3 (MOMENTO)
(K_{41})



$K_{31}=0$; ANCHE $K_{41}=0$

(NON È NECESS. APPLICARE NESSUN MOM. ESTERNO)

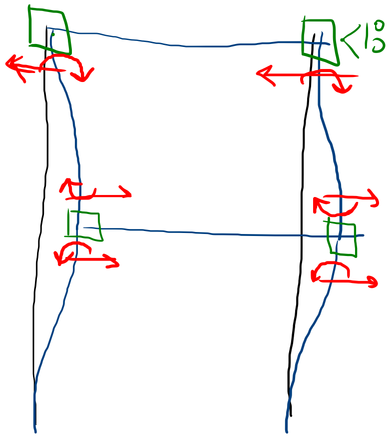
K_{51} :



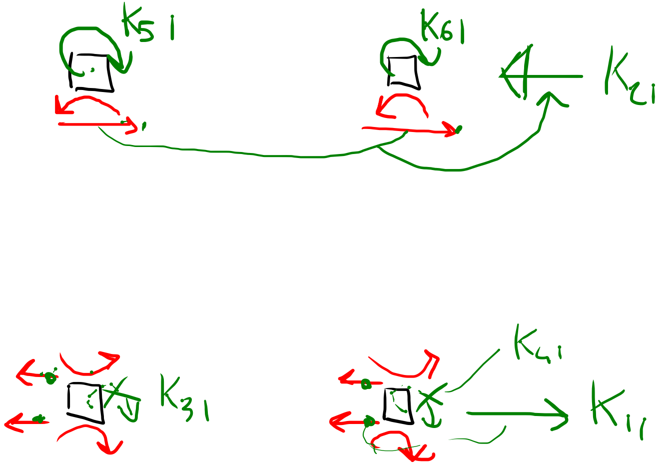
$K_{51}=-\frac{6EI}{h^2}$

ANCHE $K_{61}=-6\frac{EI}{h^2}$

1° COLONNA DI $\underset{\sim}{K}$



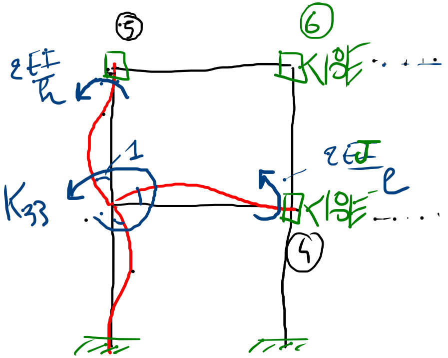
IN ROSSO LE AZIONI
SUI PILASTRI



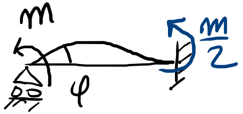
IN ROSSO LE AZIONI
SUI MORSETTI AUSILIARI

IN VERDE LE REAZIONI
VINCOLARI (K_{21}, K_{31} ...) E
LA FORZA K_{11}

$$q_3 = 1, \quad q_1 = q_2 = q_4 = q_5 = q_6 = 0$$

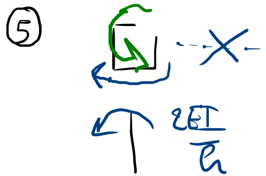


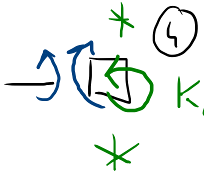
$$K_{33} = 8\frac{EI}{h} + 4\frac{EJ}{\ell}$$



$$m = \frac{4EI}{\ell}\varphi$$

$$K_{53} = +\frac{2EI}{h}$$

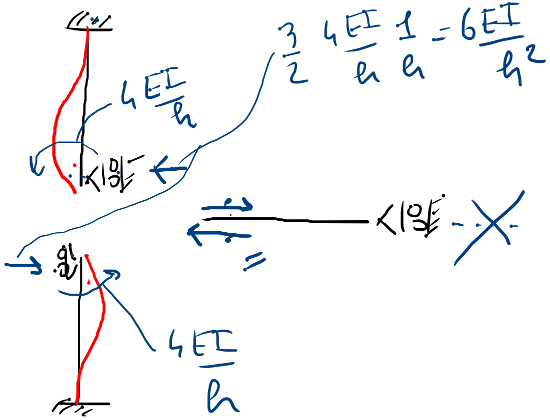




$$K_{43} = \frac{2EJ}{\ell}$$

$$K_{63} = 0$$

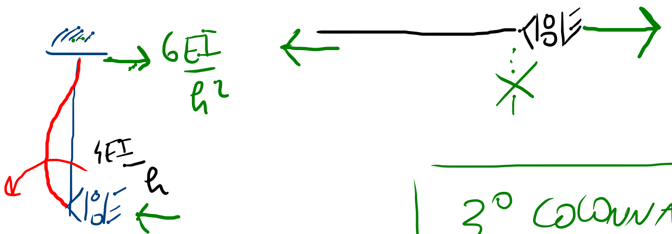
$$\frac{3}{2}\,\frac{4EI}{h}\,\frac{1}{h} = \frac{6EI}{h^2}$$



$$K_{31} = 0$$

Per K_{23}:

$$K_{23} = +\frac{6EI}{h^2}$$



$$\boxed{\text{3° COLONNA DI } \underset{\sim}{K}}$$

$$\underset{\sim}{K} = \begin{bmatrix} 48\,EI/h^3 & \boxed{0} & \cdots & \cdots & \cdots & \cdots \\ -24\,EI/h^3 & +6\,EI/h^2 & & & & \\ \boxed{0} & 8\,EI/h + 4\,EJ/l & & & & \\ 0 & 2\,EJ/l & & & & \\ -6\,EI/h^2 & 2\,EI/h & & & & \\ -6\,EI/h^2 & 0 & & & & \end{bmatrix}$$

ATTENZ.: GIUSTO CHE LE DIM. DEI TERMINI K_{ij} SIANO DIVERSE PERCHÈ QUESTO È COERENTE CON LE VARIABILI DEL SIST. LINEARE

$$\underset{\sim}{K}\,\underline{q} = \underline{F}$$

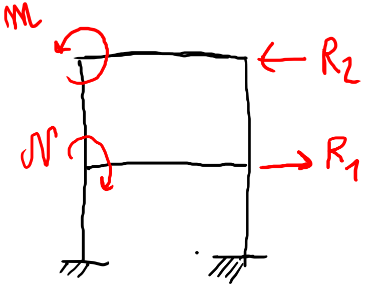
$K\,q$

$$\begin{bmatrix} & & \\ & \underset{\sim}{K} & \\ & & \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_6 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_6 \end{bmatrix}$$

$[L]$

ROTAZ. $[-]$

4/4/2023



IL SIST. LINEARE:

$$[\underset{\sim}{K}][\underline{q}] = \begin{bmatrix} +R_1 \\ -R_2 \\ -N \\ 0 \\ +M \\ 0 \end{bmatrix}$$

Osservo la II riga del sistema:

$$K_{21}q_1 + K_{22}q_2 + \qquad + K_{26}q_6 = F_2$$

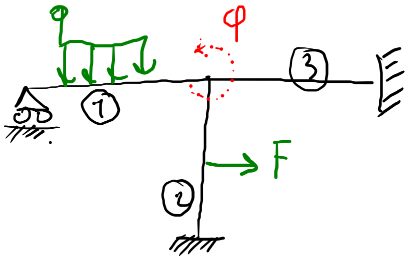
⤳ TUTTI I TERMINI HANNO LE DIM. DI UNA FORZA

SOSTITUENDO

$$\underbrace{-\underbrace{\frac{24EI}{\ell^3}}_{\left[\frac{FL^2}{L^3}\right]}\underbrace{q_1}_{[L]}}_{[F]} + \cdots + \underbrace{\underbrace{\frac{6EI}{\ell^2}}_{\left[\frac{FL^2}{L^2}\right]}\underbrace{q_3}_{[-]}}_{[F]} + \cdots\cdots = \underbrace{F_2}_{[F]}$$

$$\boxed{\text{Ok!}}$$

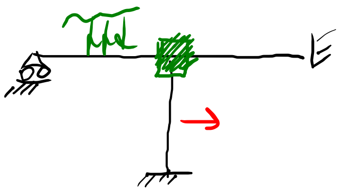
COME AFFRONTARE IN GENERALE UN PROBLEMA STRUTTURALE CON CONDIZ. DI CARICO GENERICHE?

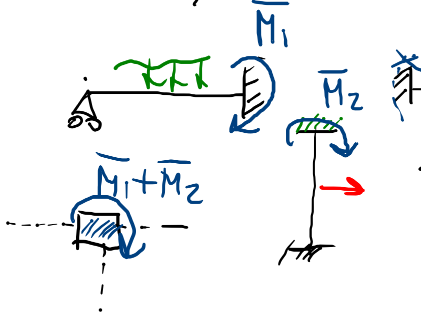


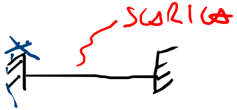
1 NODO : GDL = φ ↺+

EI cost

I) "BLOCCO" IL NODO (BLOCCO I GDL) CON UN VINCOLO AUSILIARIO

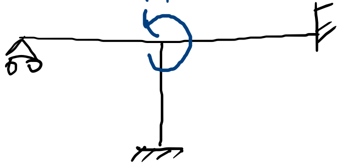






REAZIONI DI INCASTRO PERFETTO $\overline{M}_i$

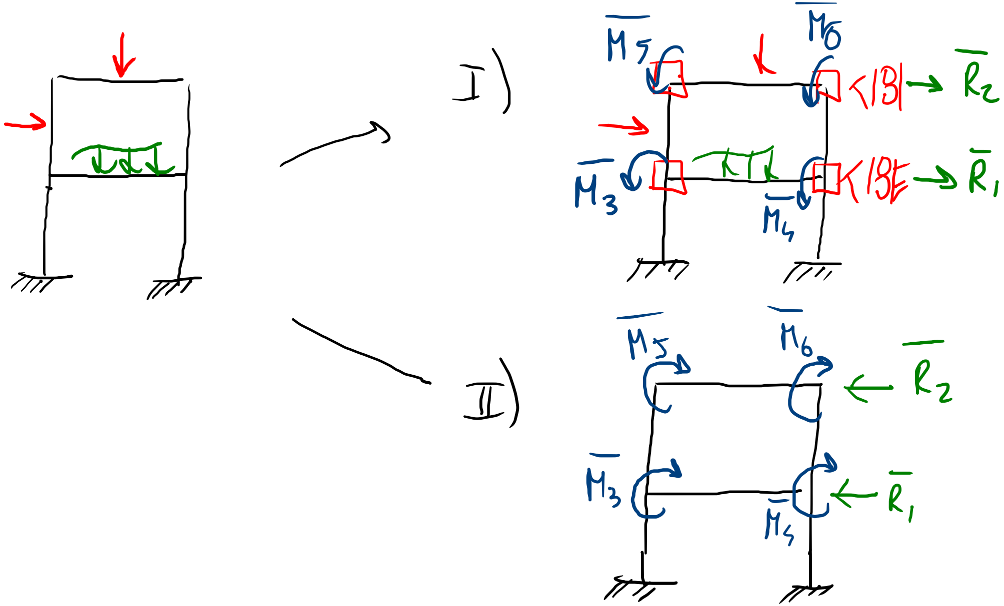
II) $\overline{M}_1+\overline{M}_2 = \overline{M}$

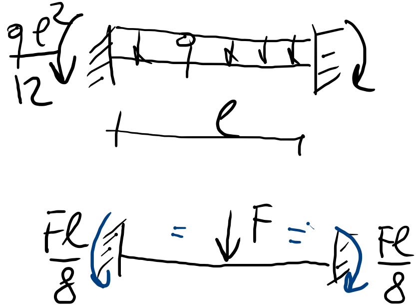


APPLICO IL MOMENTO DEL VINCOLO AUSILIARIO UGUALE ED OPPOSTA → $K\varphi = {}^{+}\overline{M}$

$$\left[K = \frac{3EI}{\ell_1} + \frac{4EI}{\ell_2} + \frac{4EI}{\ell_3}\right]$$

$$\varphi = \frac{\overline{M}}{K}$$





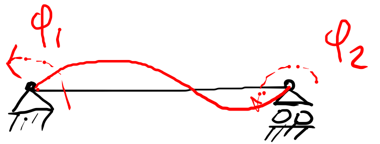
REAZ. VINCOLI

AUSILIARI APPLICATE

CON VERSO OPPOSTO

$$\underset{\sim}{K}\,\underline{q} = \underbrace{\begin{bmatrix} -\bar{R}_1 \\ -\bar{R}_2 \\ \vdots \end{bmatrix}}_{\underline{F}} \Rightarrow \underline{q} = \underset{\sim}{K}^{-1}\,\underline{F}$$

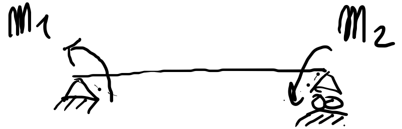
RAPPORTO TRA LA MATRICE DI FLESSIBILITA' / CEDIBILITA' E MATRICE DI RIGIDEZZA



$\varphi \circlearrowleft +$

EI cost

2 GDL



$m \circlearrowleft +$

$\varphi_i = f(m_i)$

$$\varphi_1 = \frac{m_1 l}{3EI} - \frac{m_2 l}{6EI}$$

$$\varphi_2 = -\frac{m_1 l}{6EI} + \frac{m_2 l}{3EI}$$

$$\Rightarrow \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} \frac{l}{3EI} & -\frac{l}{6EI} \\ -\frac{l}{6EI} & \frac{l}{3EI} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

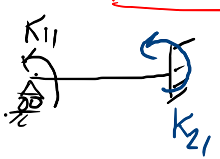
$$\boxed{\underline{\varphi} = \underset{\sim}{H}\, \underline{m}}$$

$R = \frac{EI}{l}$

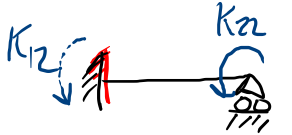
con $\underset{\sim}{K}$?

$$\boxed{\underset{\sim}{K}\, \underline{\varphi} = \underline{m}}$$

K_{11}:



$\varphi_1 = 1, \varphi_2 = 0$



$\varphi_1 = 0$

$\varphi_2 = 1$

$$\underset{\sim}{K} = \begin{bmatrix} \overset{K_{11}}{\frac{4EI}{l}} & +\frac{2EI}{l} \\ +\frac{2EI}{l} & \frac{4EI}{l} \end{bmatrix}$$

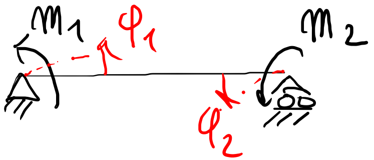
$\rightarrow$ $\underset{\sim}{K}^{-1} = \underset{\sim}{H}$? OK

$$\det \underset{\sim}{K} = \left(\frac{EI}{l}\right)^2 \cdot 12 = 12 R^2$$

$$\underset{\sim}{K}^{-1} = \frac{1}{\det \underset{\sim}{K}} \begin{bmatrix} 4R & -2R \\ -2R & 4R \end{bmatrix} = \frac{1}{12R^2} R \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{R} \begin{bmatrix} 1/3 & -1/6 \\ -1/6 & 1/3 \end{bmatrix} = \frac{l}{EI} \underbrace{\begin{bmatrix} 1/3 & -1/6 \\ -1/6 & 1/3 \end{bmatrix}}_{H}$$

VERIFICA DEF. POSITIVA DI $\underset{\sim}{K}$



ASSEGNATA $\Rightarrow$ EN. ELASTICA Φ (>0)
IMMAGAZZINATA NELLA STRUTTURA ?

$$\Phi = \frac{1}{2}\int \frac{M^2}{EI} + \frac{N^2}{EA}\, ds \quad (>0)$$

GRAZIE AL TH. DI CLAPEYRON

$$\Phi = \frac{1}{2}\left(m_1\varphi_1 + m_2\varphi_2\right) = \frac{1}{2}\, \underline{m}\cdot\underline{q} > 0 \quad (=0 \Leftrightarrow \underline{q} = \underline{0})$$

$$\left.\begin{aligned} \Phi = \frac{1}{2}\underset{\sim}{K}\,\underline{q}\cdot\underline{q} &> 0 \Leftrightarrow \underline{q} \neq \underline{0} \\ &= 0 \Leftrightarrow \underline{q} = \underline{0} \end{aligned}\right\}$$

DEFINIZ. DI DEFINITEZZA POSITIVA DELLA MATRICE $\underset{\sim}{K}$

$\Rightarrow$ $\underset{\sim}{K}$ NON È MAI SINGOLARE

LA SIMMETRIA DI $\underset{\sim}{K}$ SI DIMOSTRA ATTRAVERSO IL TH. DI BETTI.

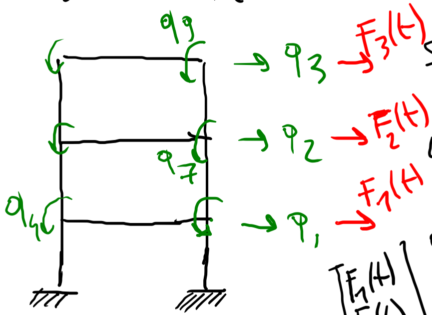
$$K_{ij} = K_{ji} \qquad i \neq j$$

$$\underset{\sim}{A}\,\underline{u}\cdot\underline{v} = \underline{u}\cdot\underset{\sim}{A}^T\underline{v}$$

$$\underset{\sim}{K}\,\underline{q}\cdot\underline{q} = \underline{q}\cdot\underset{\sim}{K}^T\underline{q}$$

$$\underset{\sim}{K}^T\underline{q}\cdot\underline{q}$$

CONDENSAZIONE DELLA MATRICE DI RIGIDEZZA



STUDIO UN PROBLEMA DI DINAMICA IN GENERALE UTILIZZANDO LA SCHEMATIZZ. A 9 GDL

$$\underline{q} = \begin{bmatrix} \underline{q}_A \\ \underline{q}_B \end{bmatrix} \qquad \underline{q}_A = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \qquad \underline{q}_B = \begin{bmatrix} q_4 \\ \vdots \\ q_9 \end{bmatrix}$$

$$\underline{F}(t) = \begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

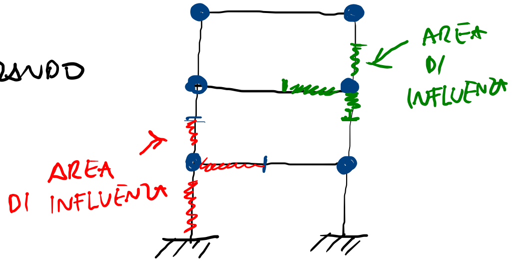
VETTORE FORZANTI

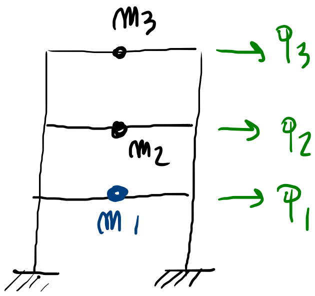
$$\underset{\sim}{M}\,\ddot{\underline{q}} + \underset{\sim}{K}\,\underline{q} = \underline{F}(t)$$

$$\begin{array}{cc} 3 & 6 \end{array}$$

$$\left[\begin{array}{c|c} \underset{\sim}{M}_A & \underline{0} \\ \hline \underline{0} & \underline{0} \end{array}\right] \left[\begin{array}{c} \ddot{\underline{q}}_A \\ \hline \ddot{\underline{q}}_B \end{array}\right] + \left[\begin{array}{c|c} \underset{\sim}{K}_{AA} & \underset{\sim}{K}_{AB} \\ \hline \underset{\sim}{K}_{BA} & \underset{\sim}{K}_{BB} \end{array}\right] \left[\begin{array}{c} \underline{q}_A \\ \hline \underline{q}_B \end{array}\right] = \left[\begin{array}{c} \underline{F}_A(t) \\ \hline \underline{0} \end{array}\right]$$

$$\underset{\sim}{M}_A = \begin{bmatrix} m_1 & & 0 \\ & m_2 & \\ 0 & & m_3 \end{bmatrix}$$





MASSE DI PIANO

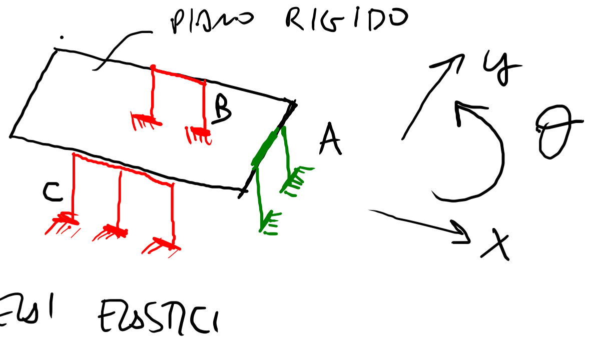
$$E_{cin} = \frac{1}{2}\left(m_1 \dot{q}_1^2 + m_2 \dot{q}_2^2 + m_3 \dot{q}_3^2\right)$$

LA "CONDENSAZIONE" PERMETTE DI ESPRIMERE IL SISTEMA LINEARE NELLE SOLE INCOGNITE $\underline{q}_A$

NEL CASO DI MOTO IMPRESSO ALLA BASE $(\ddot{y}(t))$, LE FORZANTI SI SCRIVONO

$$\underline{F}(t) = - \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \ddot{y}(t) = - \underset{\sim}{M}\, \underline{1}\, \ddot{y}(t)$$

DOVREMO ACCEDERE AL PROBLEMA DELLA MATRICE DI RIGIDEZZA PER PROBLEMI 3D:



3 gdl $\Rightarrow$ $$\underset{\sim}{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{x\theta} \\ \text{SYM} & K_{yy} & K_{y\theta} \\ & & K_{\theta\theta} \end{bmatrix}$$

IL BLOCCO IN "BASSO" FORNISCE:

$$\underset{\sim}{K}_{BA}\,\underline{q}_A + \underset{\sim}{K}_{BB}\,\underline{q}_B = \underline{0}$$

$$\boxed{\underline{q}_B = -\underset{\sim}{K}_{BB}^{-1}\,\underset{\sim}{K}_{BA}\,\underline{q}_A} \quad (*)$$

IL BLOCCO IN "ALTO" SI TRASFORMA IN:

$$\underset{\sim}{M}_A\,\ddot{\underline{q}}_A + \underset{\sim}{K}_{AA}\,\underline{q}_A + \underset{\sim}{K}_{AB}\,\underline{q}_B = \underline{F}_A(t)$$

$$\underset{\sim}{M}_A\,\ddot{\underline{q}}_A + \underbrace{\left[\underset{\sim}{K}_{AA} - \underset{\sim}{K}_{AB}\,\underset{\sim}{K}_{BB}^{-1}\,\underset{\sim}{K}_{BA}\right]}_{\underset{\sim}{K}_A}\underline{q}_A = \underline{F}_A(t)$$

$\underset{\sim}{K}_A$ MAT. RIGIDEZZA CONDENSATA

$$\underset{\sim}{M}_A\,\ddot{q}_A + \underset{\sim}{K}_A\,\underline{q}_A = \underline{F}_A(t) \Rightarrow \underline{q}_A \Rightarrow \text{OTTENGO } q_B\ (*)$$