Chapter 34

Geometric Optics

PowerPoint® Lectures for
University Physics, Thirteenth Edition
– Hugh D. Young and Roger A. Freedman

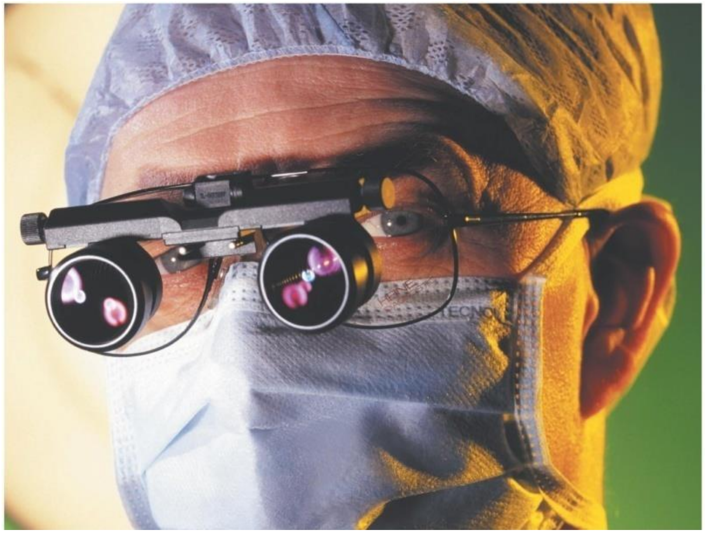
Lectures by Wayne Anderson

Goals for Chapter 34

- To see how plane and curved mirrors form images
- To learn how lenses form images
- To understand how a camera works
- To analyze defects in vision and how to correct them
- To discover how a simple magnifier works
- To understand the design of microscopes and telescopes

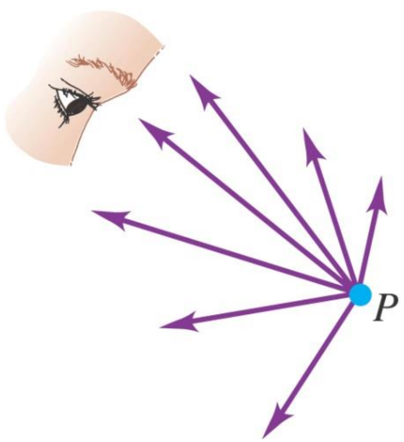
Introduction

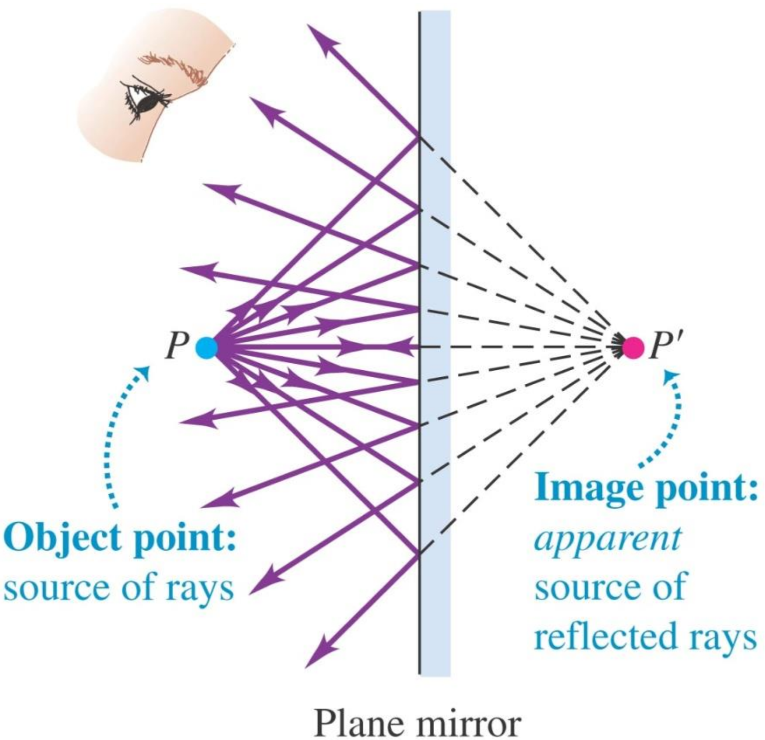
- How do magnifying lenses work?
- How do lenses and mirrors form images?
- We shall use light rays to understand the principles behind optical devices such as camera lenses, the eye, microscopes, and telescopes.



Reflection at a plane surface

- Light rays from a point radiate in all directions (see Figure 34.1 at left).
- Light rays from an object point reflect from a plane mirror as though they came from the image point (see Figure 34.2 at right).





Refraction at a plane surface

- Light rays from an object at P refract as though they came from the image point P' (see Figure 34.3 at right).

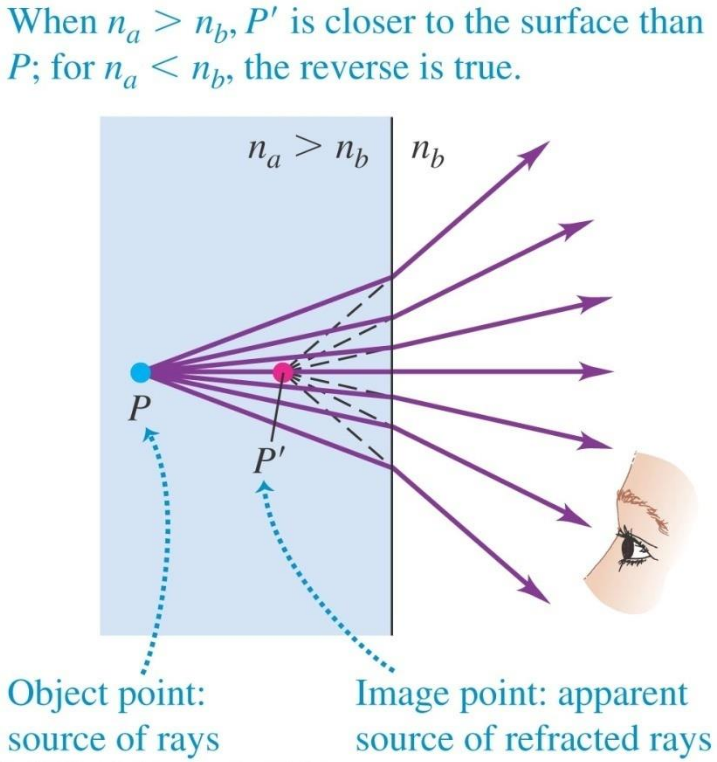
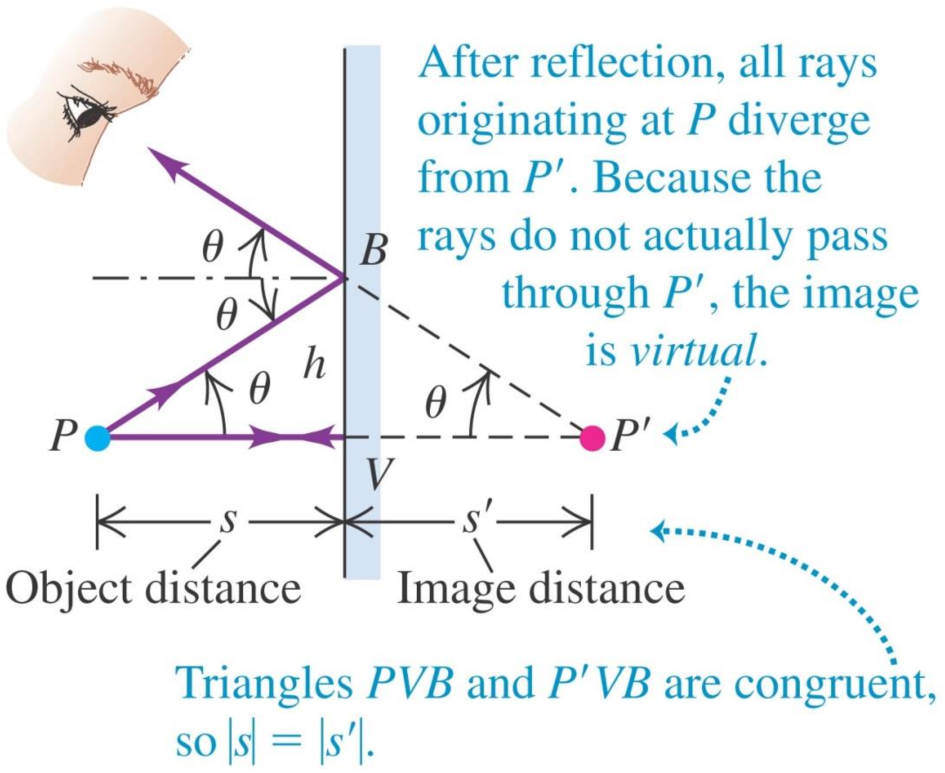
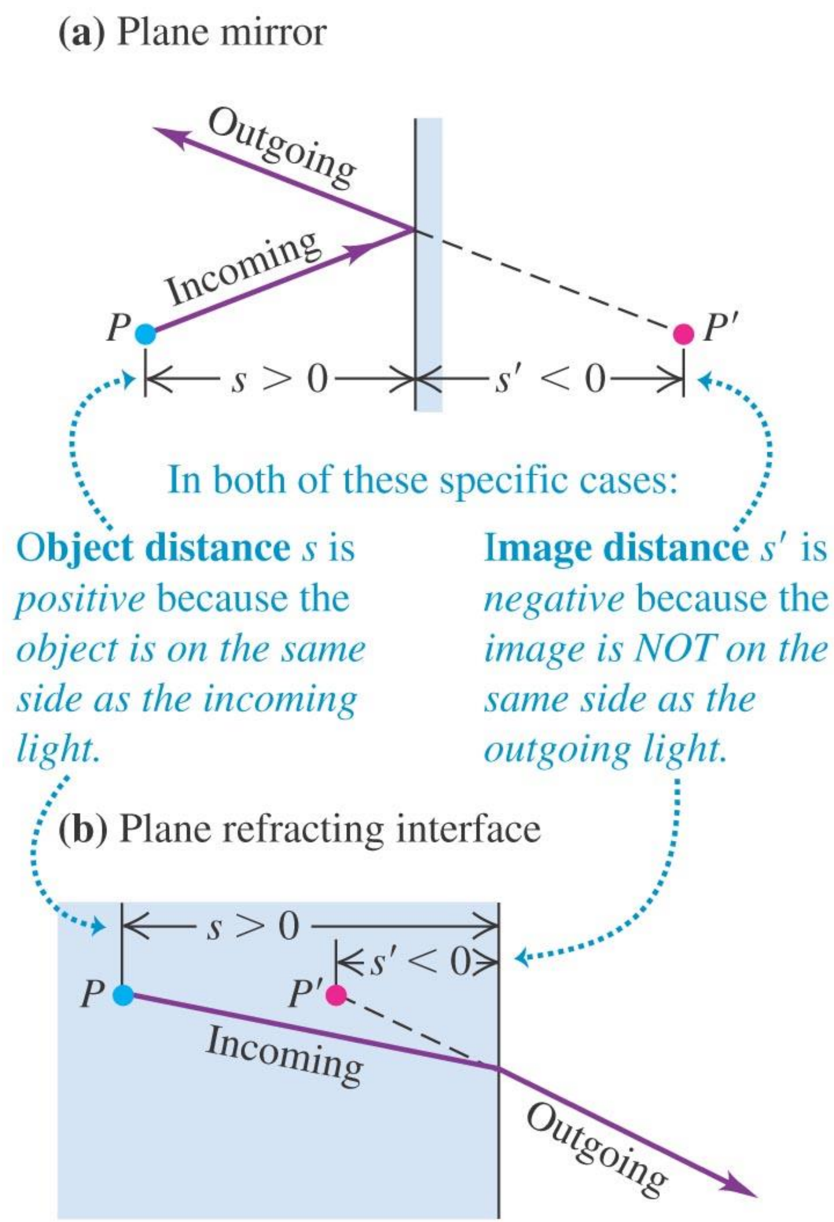


Image formation by a plane mirror

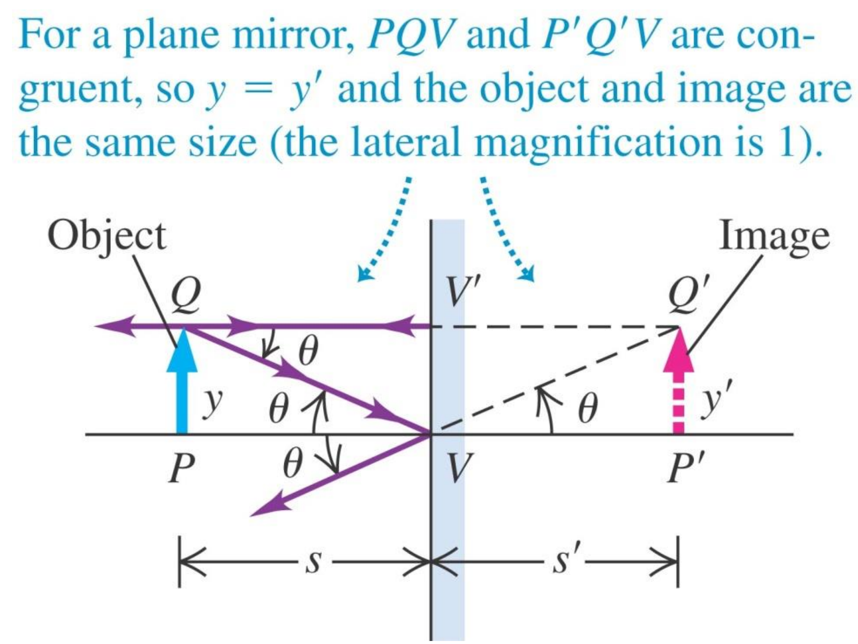
- Follow the text discussion of image formation by a plane mirror using Figures 34.4 (below) and 34.5 (right).





Characteristics of the image from a plane mirror

- The image is just as far behind the mirror as the object is in front of the mirror.
- The *lateral magnification* is $m = y'/y$.
- The image is virtual, erect, reversed, and the same size as the object (see Figure 34.6 at the right and the next slide).



The image is reversed

- The image formed by a plane mirror is reversed back to front. See Figures 34.7 (left) and 34.8 (right).

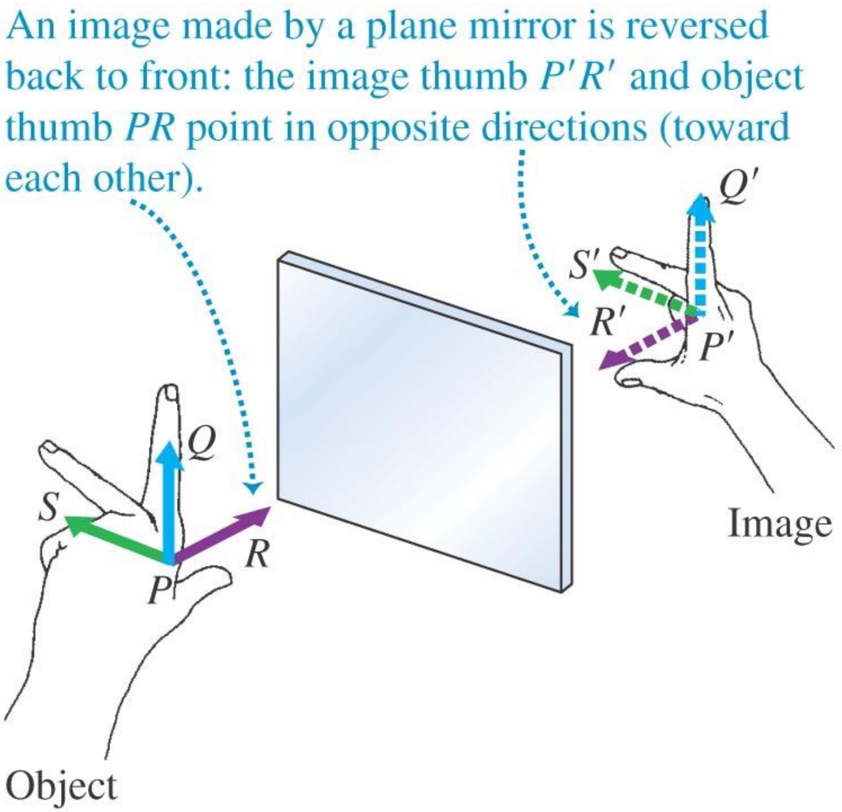
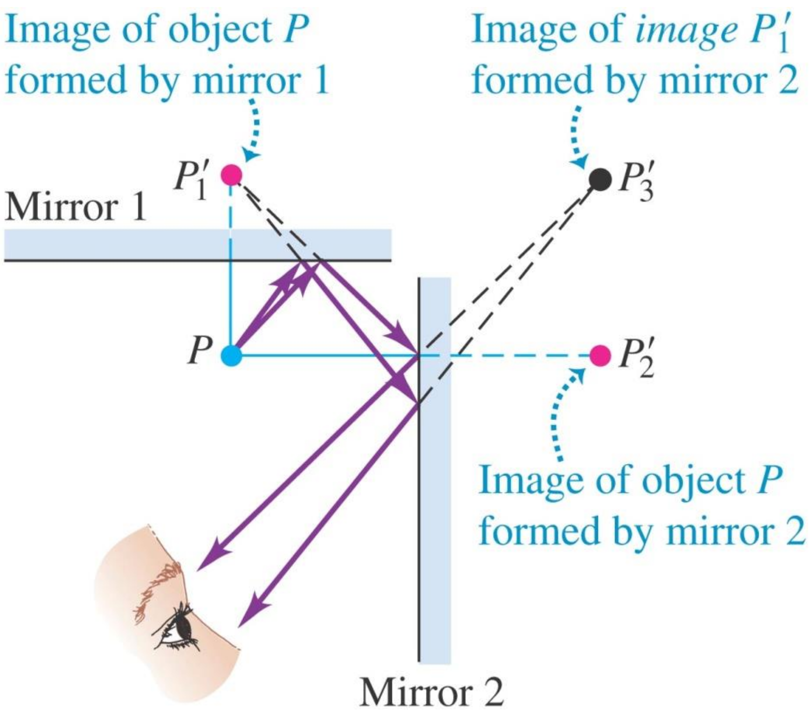


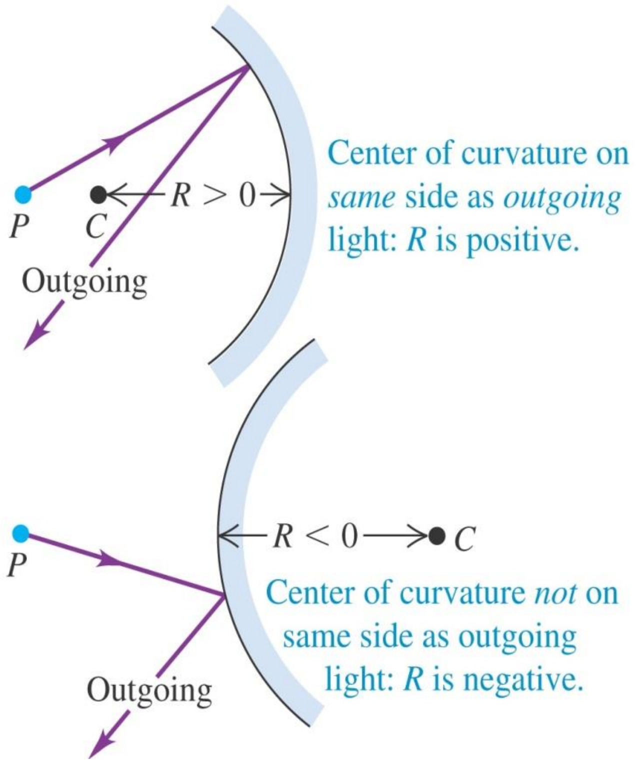
Image formed by two mirrors

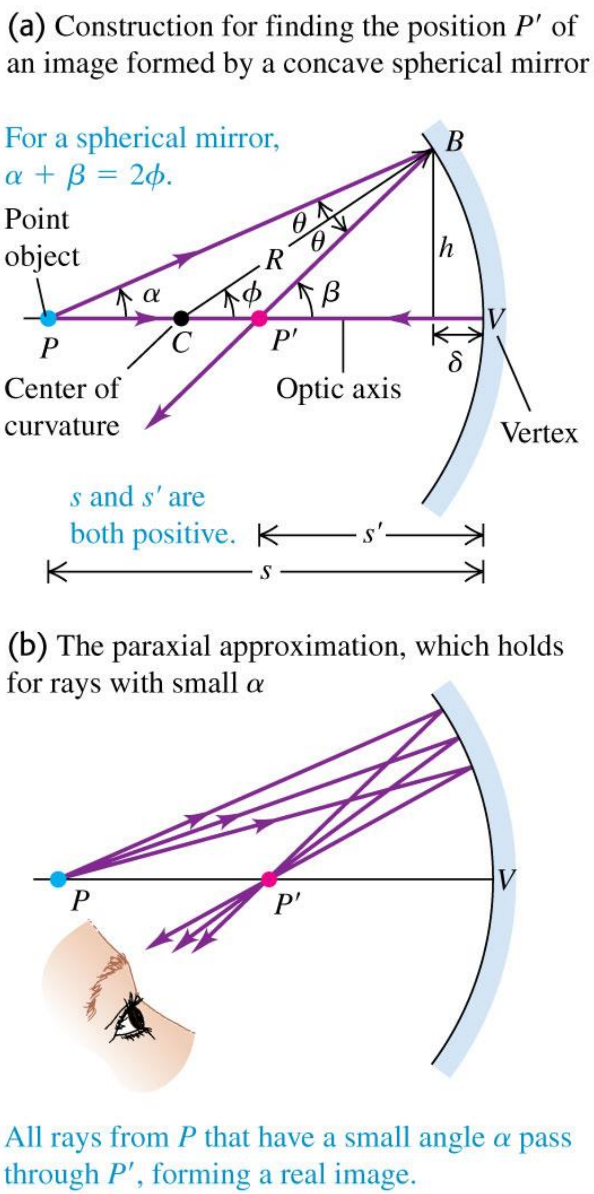
- The image formed by one surface can be the object for another surface.
- Figure 34.9 (right) shows how this property can lead to multiple images.



Spherical mirror with a point object

- Figure 34.10 (right) shows how a concave mirror forms an image of a point object.
- Figure 34.11 (below) shows the sign rule for the radius.





Paraxial Approximation

- Look at the geometry of the figure. Angles $\alpha, \beta, \phi, \theta$ have the following relationships:

$$\phi = \alpha + \theta \qquad \beta = \phi + \theta$$

$$\alpha + \beta = 2\phi$$

- The three triangles with height h have these relationships:

$$\tan\alpha = \frac{h}{s-\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$

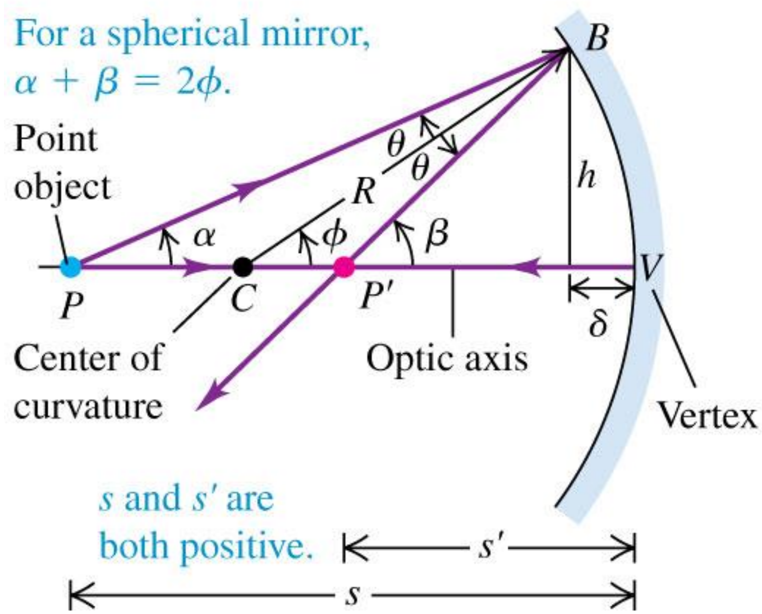
- We make the "paraxial approximation for small $\alpha, \beta,$ and ϕ

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

- Finally, we have

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$ object-image relationship for spherical mirror

(a) Construction for finding the position P' of an image formed by a concave spherical mirror



Focal point and focal length

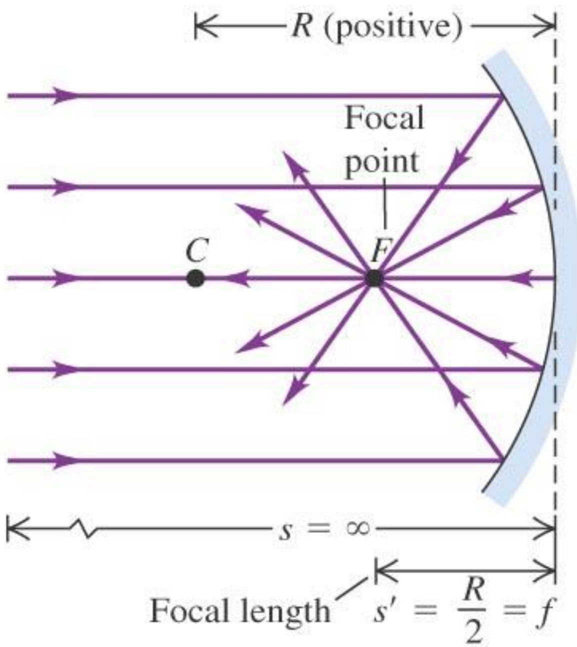
- Follow the text discussion of focal point and focal length using Figure 34.13 below.

- The focal length is half of the mirror's radius of curvature: $f = R/2$.

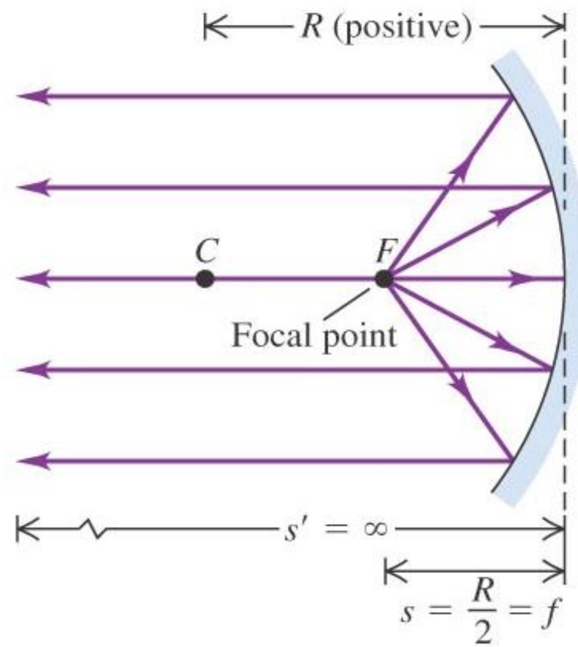
$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \quad \Rightarrow \quad s' = \frac{R}{2}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \quad \Rightarrow \quad s' = \infty$$

(a) All parallel rays incident on a spherical mirror reflect through the focal point.



(b) Rays diverging from the focal point reflect to form parallel outgoing rays.



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Image of an extended object

- Figure 34.14 below shows how to determine the position, orientation and height of the image.

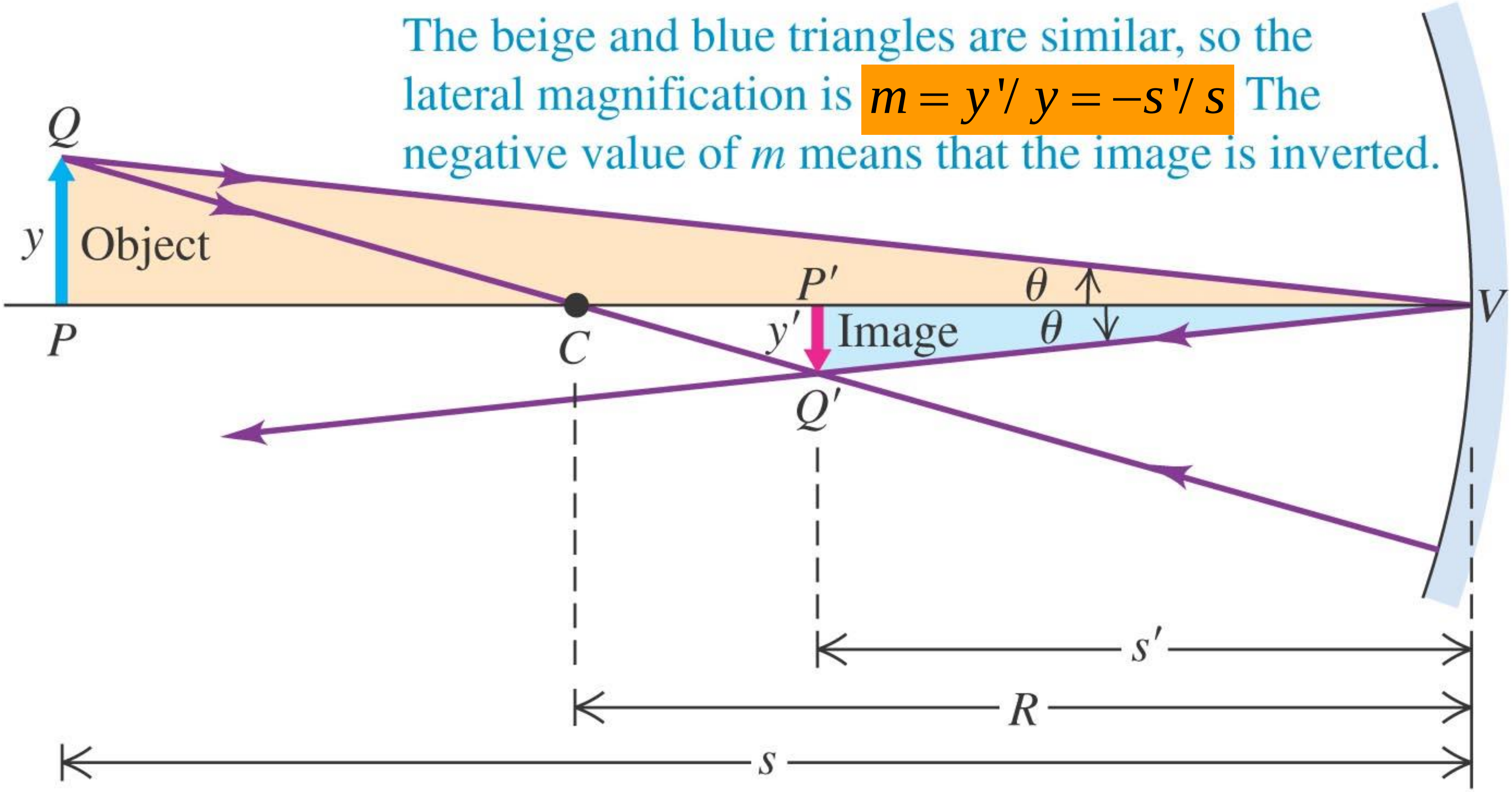


Image formed by a concave mirror

- Follow Example 34.1 using Figure 34.15 below.
- Follow Conceptual Example 34.2 for the same mirror.

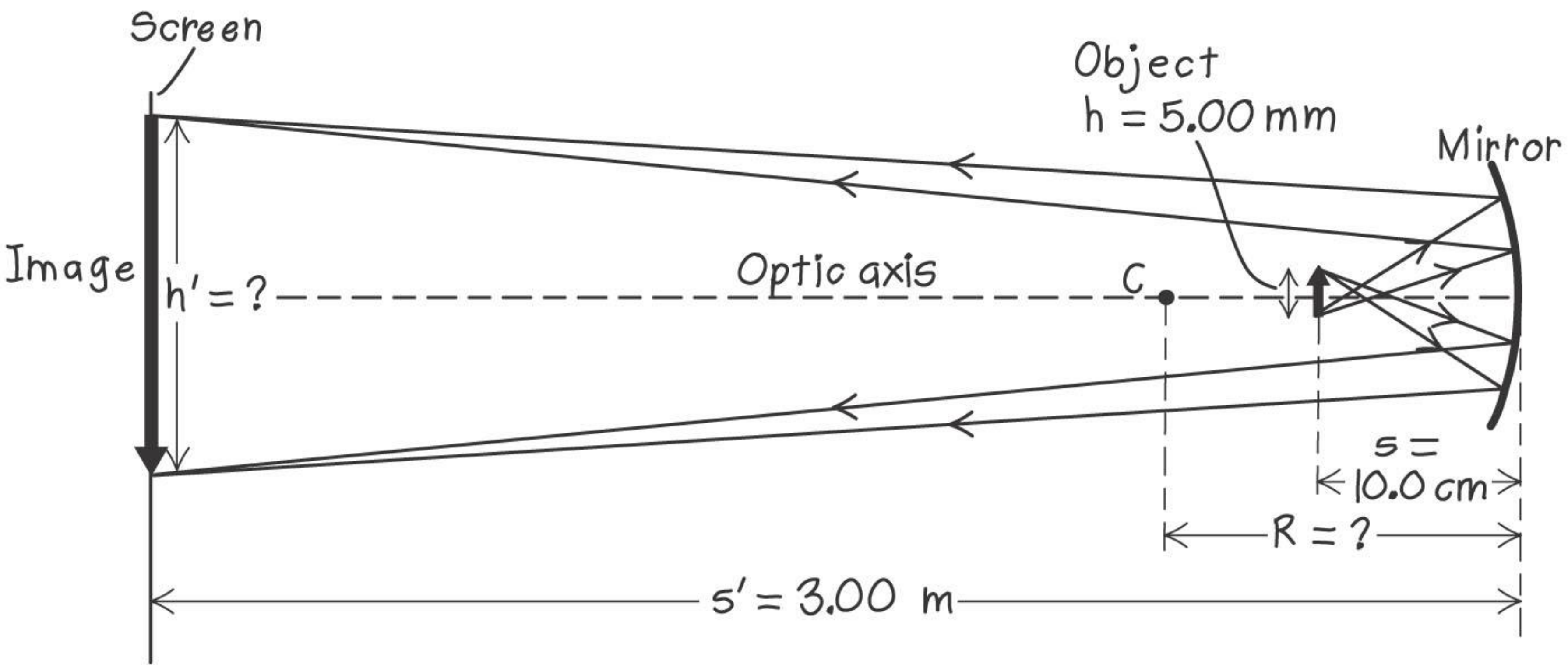
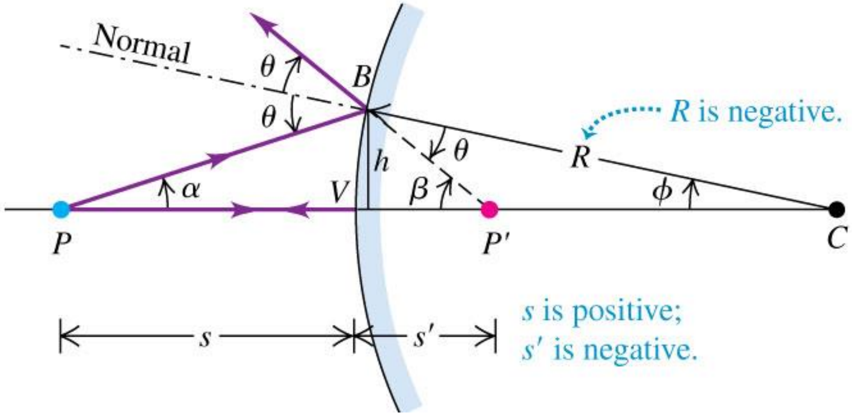


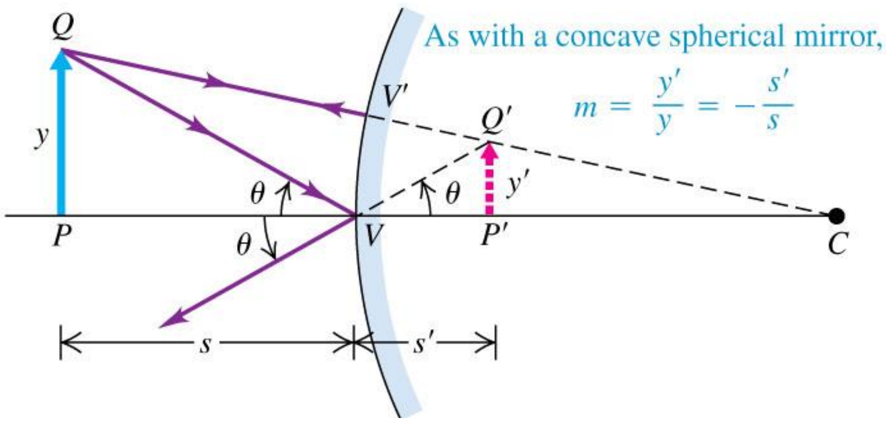
Image formation by a convex mirror

- Figure 34.16 (right) shows how to trace rays to locate the image formed by a convex mirror.

(a) Construction for finding the position of an image formed by a convex mirror

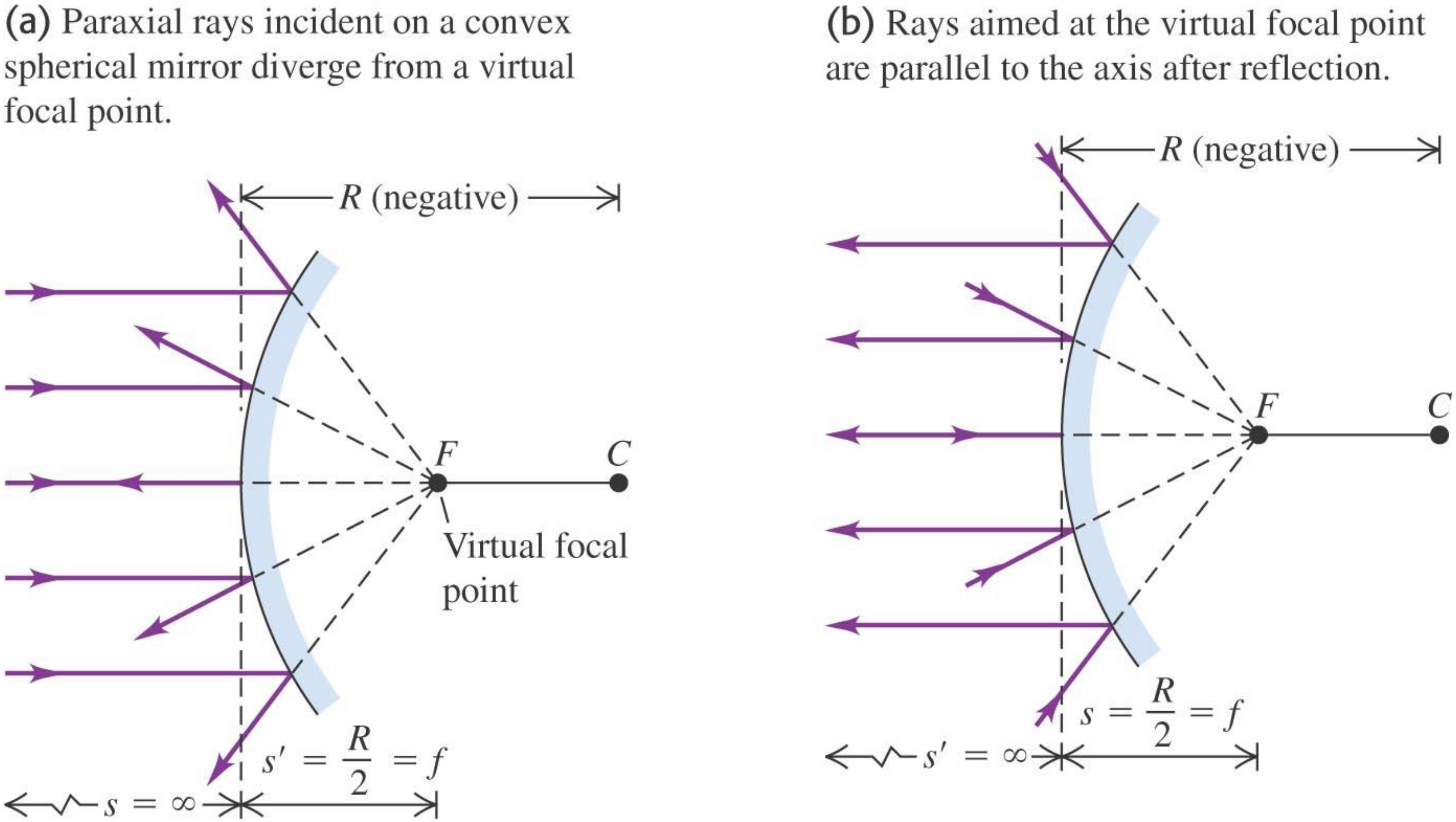


(b) Construction for finding the magnification of an image formed by a convex mirror



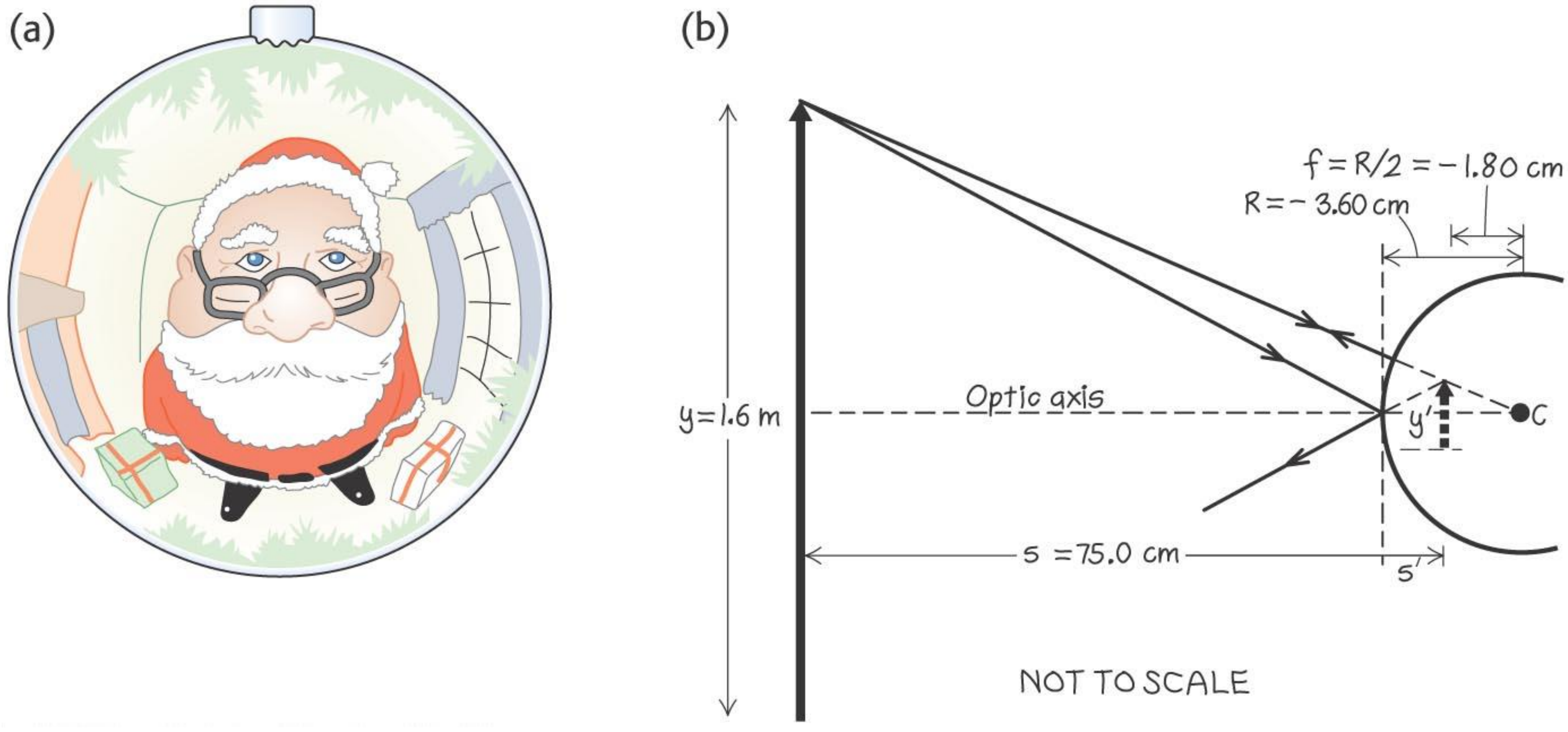
Focal point and focal length of a convex mirror

- Figure 34.17 below shows the focal point and focal length of a convex mirror.



Santa's image problem

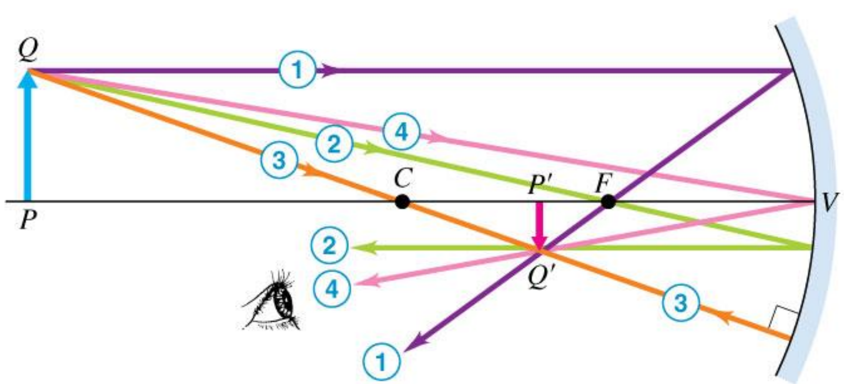
- Consider Santa's problem in Example 34.3 using Figure 34.18 below.



Graphical methods for mirrors

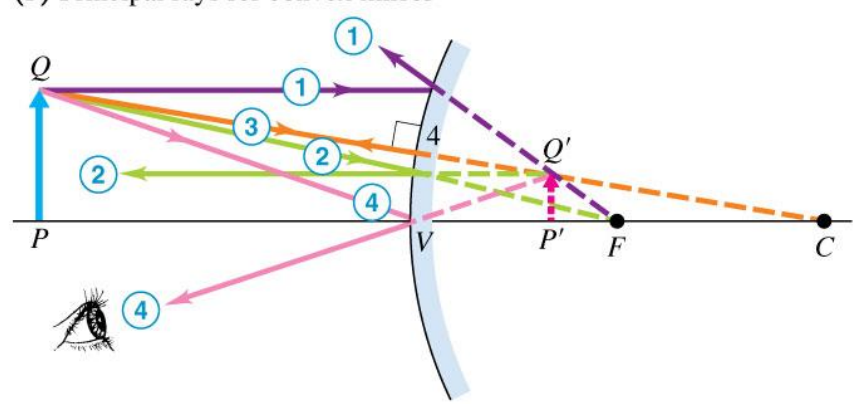
- Principle Rays
  - A ray *parallel to the axis*, after reflection, passes through the focal point *F* of a concave mirror, or appears to come from the (virtual) focal point of a convex mirror.
  - A ray *through (or proceeding toward) the focal point F* is reflected parallel to the axis.
  - A ray *along the radius through or away from the center of curvature C* intersects the surface normally and is reflected back along its original path.
  - A ray *to the vertex V* is reflected forming equal angles with the optical axis.

(a) Principal rays for concave mirror



1. Ray parallel to axis reflects through focal point.
2. Ray through focal point reflects parallel to axis.
3. Ray through center of curvature intersects the surface normally and reflects along its original path.
4. Ray to vertex reflects symmetrically around optic axis.

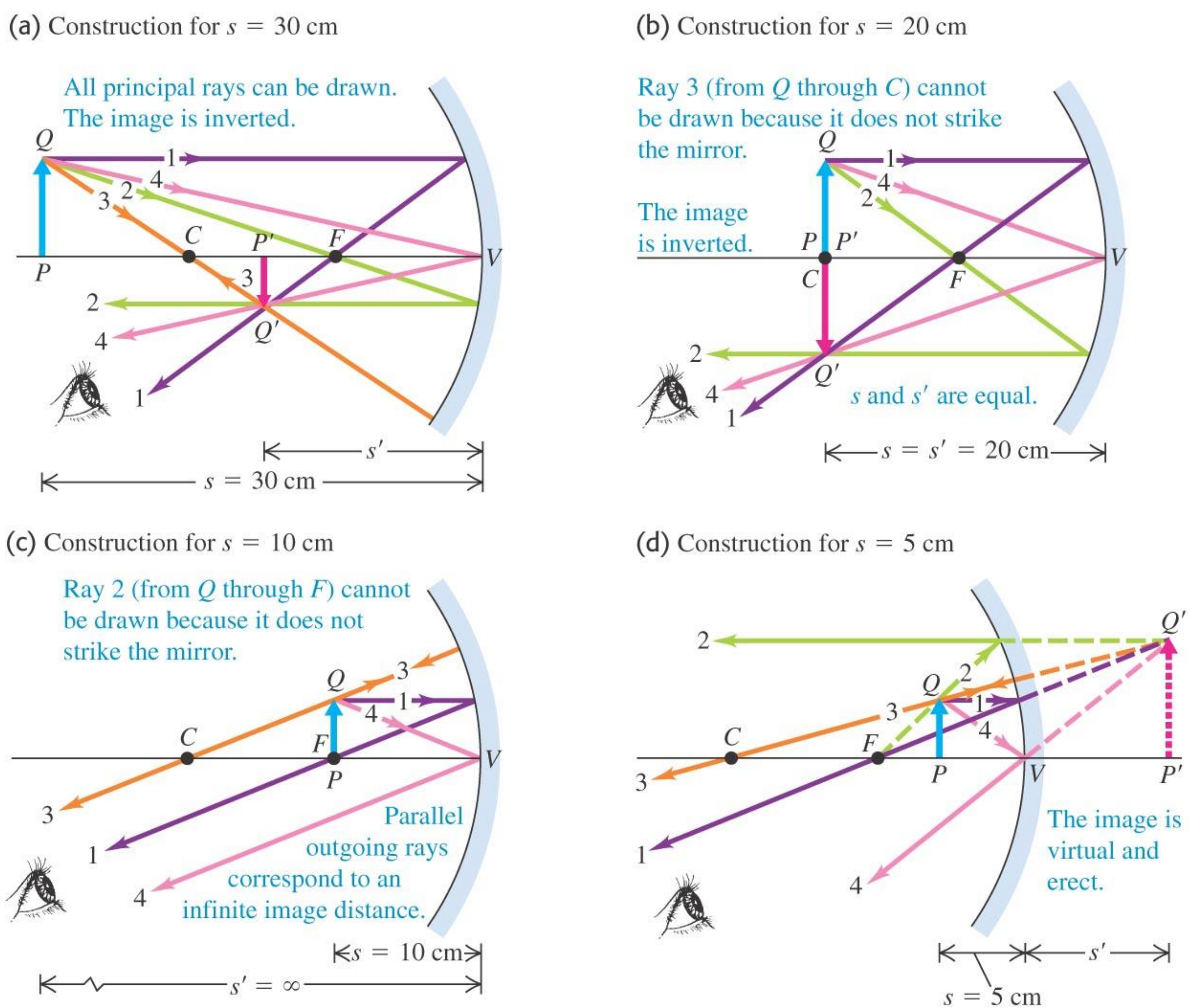
(b) Principal rays for convex mirror



1. Reflected parallel ray appears to come from focal point.
2. Ray toward focal point reflects parallel to axis.
3. As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
4. As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

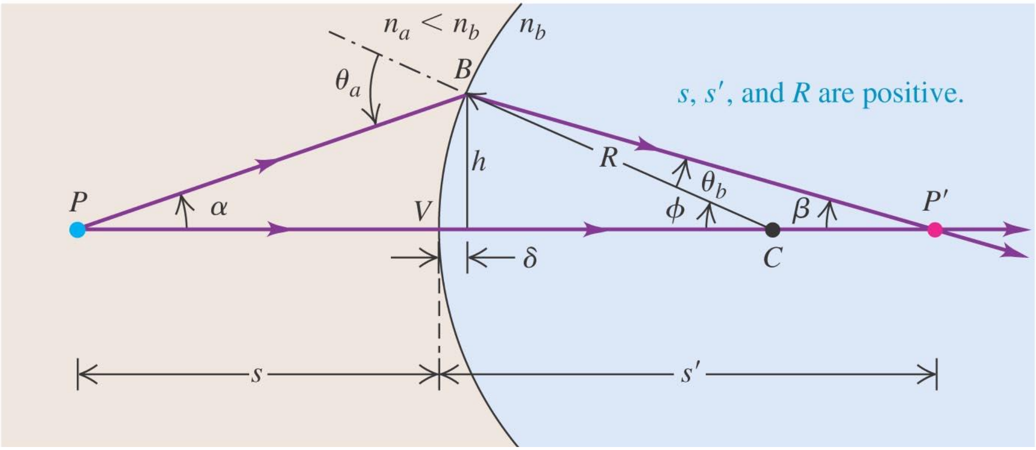
Concave mirror with various object distances

- Some example situations.



Refraction at a spherical surface

- Look at the geometry of the figure. Angles α, β, ϕ, θ_a, θ_b have the following relationships:



$$\theta_a = \alpha + \phi \qquad \phi = \beta + \theta_b$$

$$n_a \sin\theta_a = n_b \sin\theta_b$$

- The three triangles with height h have these relationships:

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$

- We make the "paraxial approximation for small θ_a, θ_b

$$\theta_a = \alpha + \phi \quad n_a\theta_a = n_b\theta_b \quad \Rightarrow \quad \theta_b = \frac{n_a}{n_b}(\alpha+\phi) \text{ and } n_a\alpha + n_b\beta = (n_b - n_a)\phi$$

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

- Finally, we have

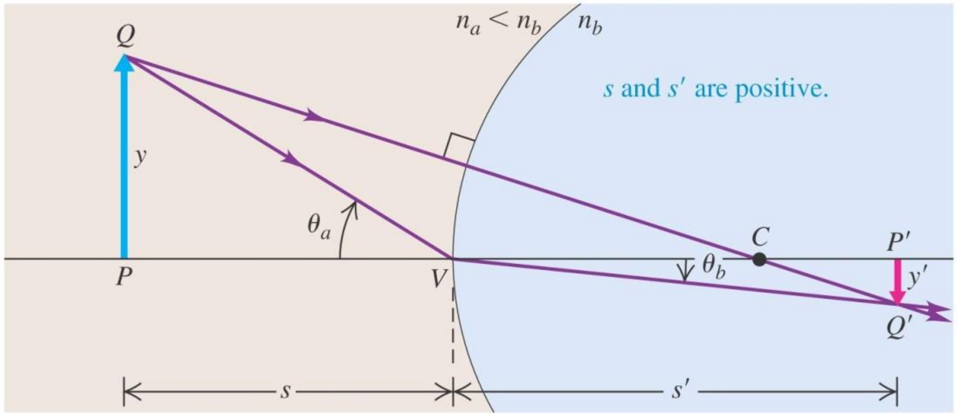
$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

object-image relationship for spherical refracting surface

Height of the image formed by a spherical surface

- Look at the geometry of the figure. Triangles PQV and P'Q'V give:

$$\tan\theta_a = \frac{y}{s} \qquad \tan\theta_b = \frac{-y'}{s}$$



- From Snell's Law of refraction:

$$n_a \sin\theta_a = n_b \sin\theta_b$$

- We make the "paraxial approximation for small θ_a, θ_b

$$\tan\theta_a = \sin\theta_a \qquad \tan\theta_b = \sin\theta_b$$

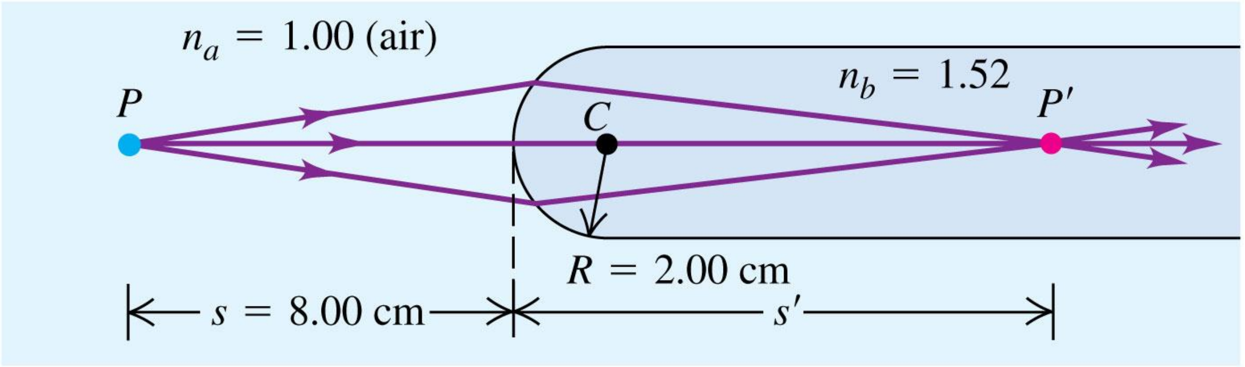
$$\frac{n_a y}{s} = -\frac{n_b y'}{s'}$$

- Finally, we have

$$m = \frac{y'}{y} = \frac{-n_a s'}{n_b s}$$ magnification for spherical refracting surface

Image formed by a glass rod in air

- Example 34.5 for a glass rod in air. Find image distance s' and lateral magnification. Use Figure 34.24 below.



Start with $$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

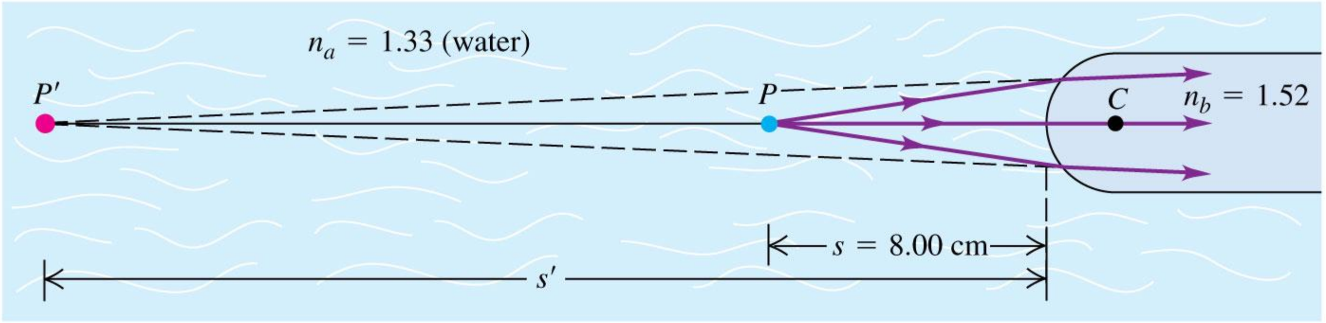
$$\frac{n_b}{s'} = \frac{n_b - n_a}{R} - \frac{n_a}{s} \quad \Rightarrow \quad \frac{n_b}{s'} = \frac{s(n_b - n_a) - Rn_a}{Rs}$$

$$s' = n_b \frac{Rs}{s(n_b - n_a) - Rn_a} = \frac{1.52(2\text{ cm})(8\text{ cm})}{(8\text{ cm})(0.52) - (2\text{ cm})} = 11.3\text{ cm}$$

Start with $$m = \frac{y'}{y} = \frac{-n_a s'}{n_b s} = \frac{-11.3\text{ cm}}{(1.52)(8\text{ cm})} = -0.926$$

Image formed by a glass rod in water

- Follow Example 34.6 for a glass rod in water. Use Figure 34.25 below.



$$s' = n_b \frac{Rs}{s(n_b - n_a) - Rn_a} = \frac{1.52(2\text{ cm})(8\text{ cm})}{(8\text{ cm})(0.19) - (2\text{ cm})(1.33)} = -21.3\text{ cm}$$

This is a virtual image!

$$m = \frac{y'}{y} = \frac{-n_a s'}{n_b s} = \frac{(1.33)(21.3\text{ cm})}{(1.52)(8\text{ cm})} = 2.33$$

So simply moving the experiment from air to water has a huge effect on the outcome.

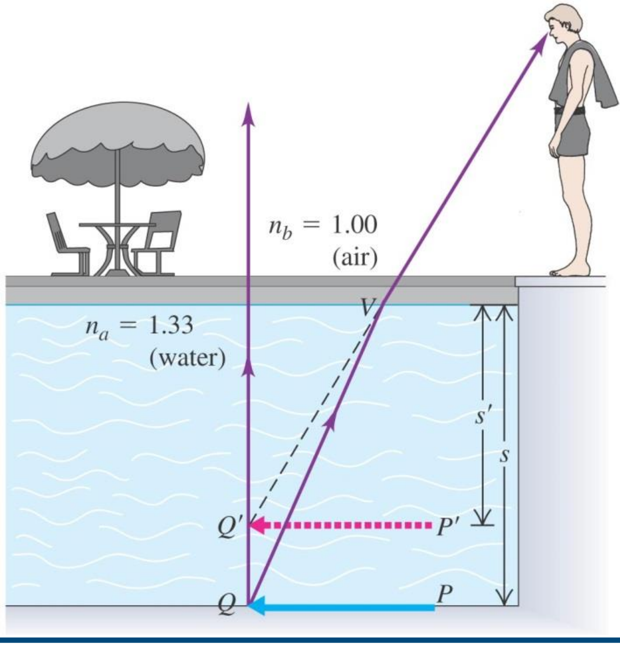
Apparent depth of a swimming pool

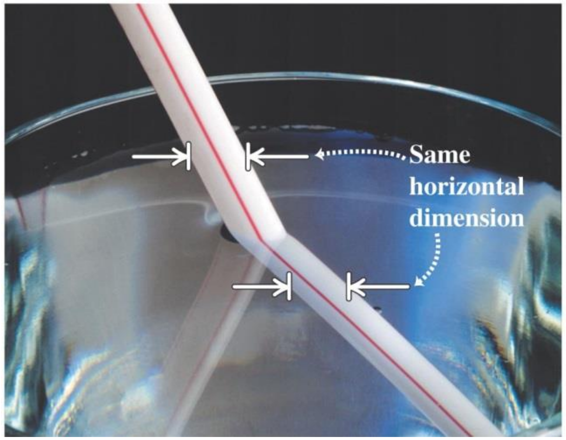
- Example 34.7 using Figure 34.26 at the left—how deep does the pool appear?

Start with $$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$ What is the radius in this case?

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \quad \Rightarrow \quad s' = -\frac{n_b s}{n_a} = -\frac{2\text{ m}}{1.33} = -1.50\text{ m}$$

- This is a virtual image—the pool appears shallower
- Figure 34.27 (right) shows that the submerged portion of the straw appears to be at a shallower depth than it actually is.





Thin converging lens

- Thin lens => images are formed in the same medium as the object. The same rules apply, but $n_a = n_b$, and we use "focal length" to provide information about the lens curvature, index of refraction, etc.

- Figure 34.28 below shows the focal points F and focal length f of a *thin converging lens*. Note, F_1 and F_2 are equidistant from lens.

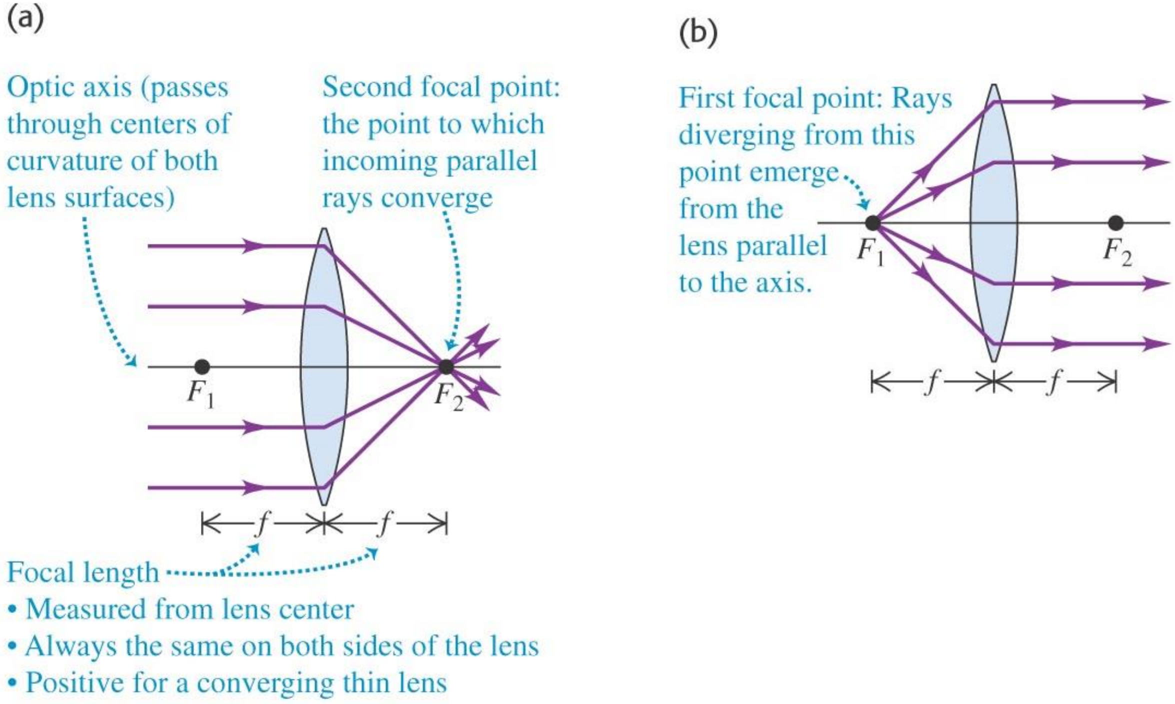


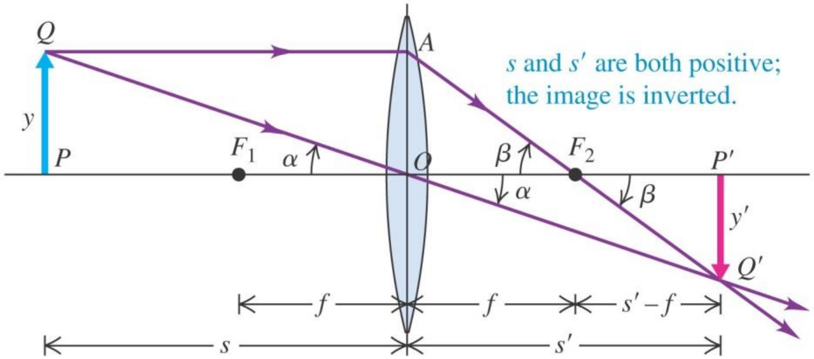
Image formed by a thin converging lens

- Look at the geometry of the figure. Triangles OPQ and OP'Q' are similar triangles, so:

$$\frac{y}{s} = -\frac{y'}{s'} \quad \Rightarrow \quad \frac{y'}{y} = -\frac{s'}{s}$$

- Also, AOF2 and Q'P'F2 are similar, so:

$$\frac{y}{f} = -\frac{y'}{s'-f} \quad \Rightarrow \quad \frac{y'}{y} = -\frac{s'-f}{f}$$



- Equating and rearranging, we have the same result as earlier for mirrors:

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

object-image relationship for thin lens

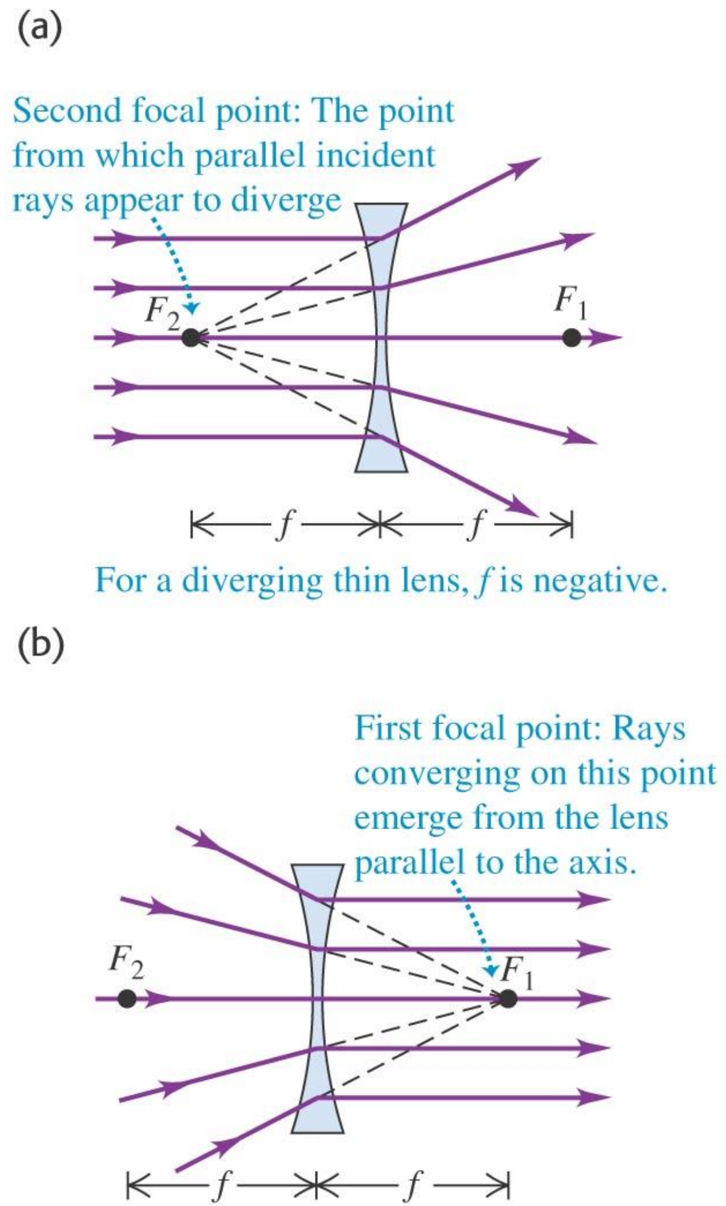
and

$$\boxed{m = \frac{y'}{y} = -\frac{s'}{s}}$$

magnification for thin lens

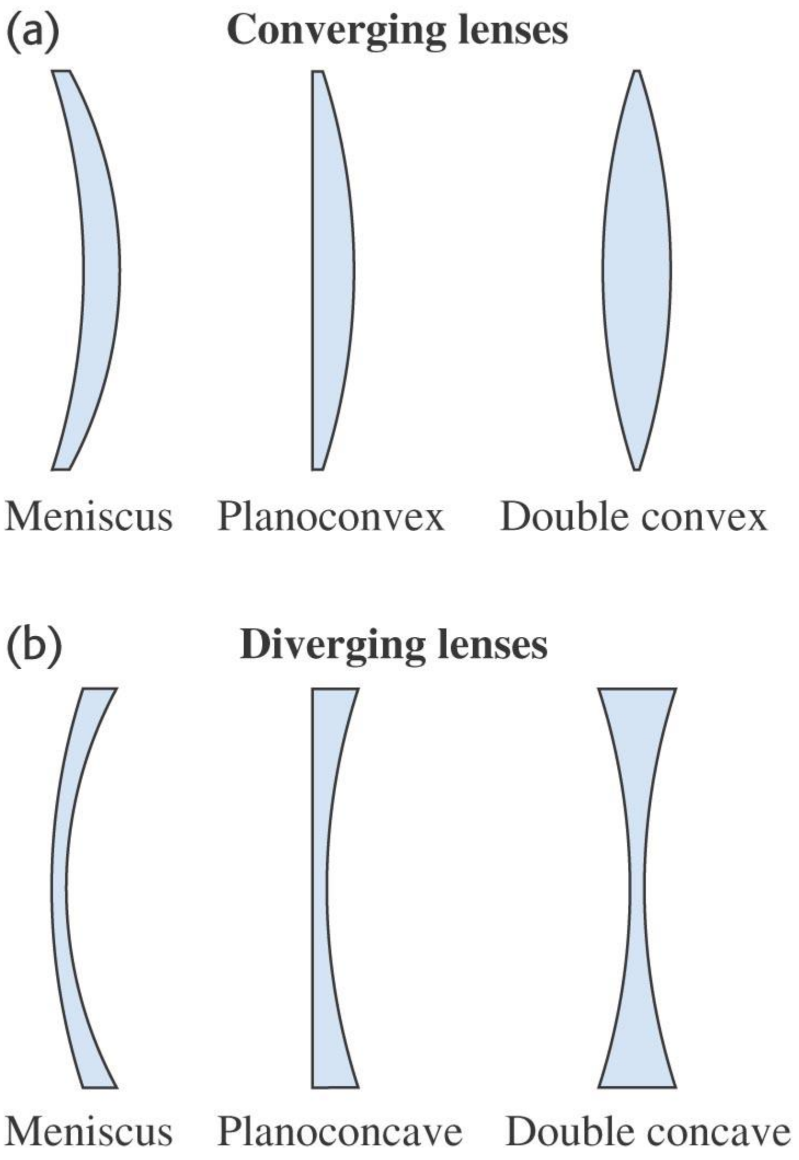
Thin diverging lens

- Figure 34.31 at the right shows the focal points and focal length for a thin diverging lens.
- The results for a converging lens also apply to a diverging lens.

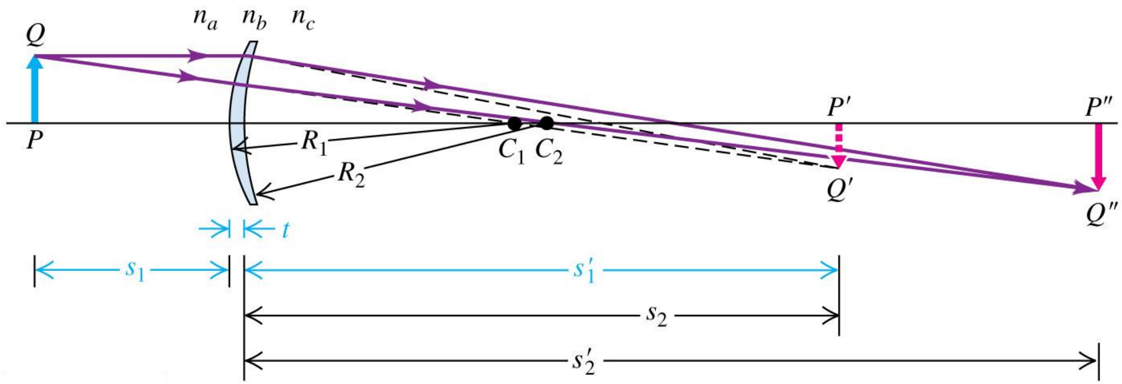


Types of lenses

- Figure 34.32 at the right illustrates various types of converging and diverging lenses.
- Any lens that is thicker in the middle than at the edges is a converging lens.
- Any lens that is thinner in the middle than at the edges is a diverging lens.
- Each of these still has equadistance foci, despite the dissimilar curvatures.



Lensmaker's equation



- We now will derive a very important general equation, called the lensmaker's equation. We will apply our earlier formula twice:

$$\frac{n_a}{s_1}+\frac{n_b}{s_1'}=\frac{n_b-n_a}{R_1}$$

$$\frac{n_b}{s_2}+\frac{n_c}{s_2'}=\frac{n_c-n_b}{R_2}$$

- Since the first and third materials are air, $n_a = n_c = 1$, so we do not need the subscript b on the remaining n. Also, $s_2 = s_1'$. The equations are now:

$$\frac{1}{s_1}+\frac{n}{s_1'}=\frac{n-1}{R_1}$$

$$\frac{n}{s_1'}+\frac{1}{s_2'}=\frac{1-n}{R_2}$$

$$\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

lensmaker's equation

$$\boxed{\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

Used to determine focal length from radii

Determining the focal length of a lens

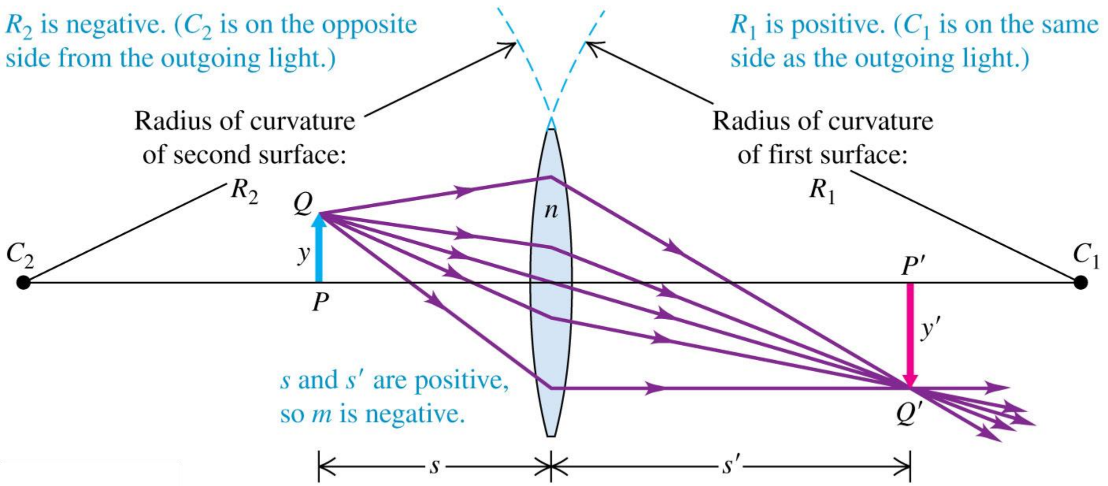
- Example 34.8: (a) Determine focal length of converging lens below if both radii are 10 cm and index of refraction is 1.52. (b) Determine focal length of a diverging lens of the same curvatures. (a)

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{1}{f}=(1.52-1)\left(\frac{1}{10\text{ cm}}-\frac{1}{-10\text{ cm}}\right)$$

$$f=9.6\text{ cm}$$

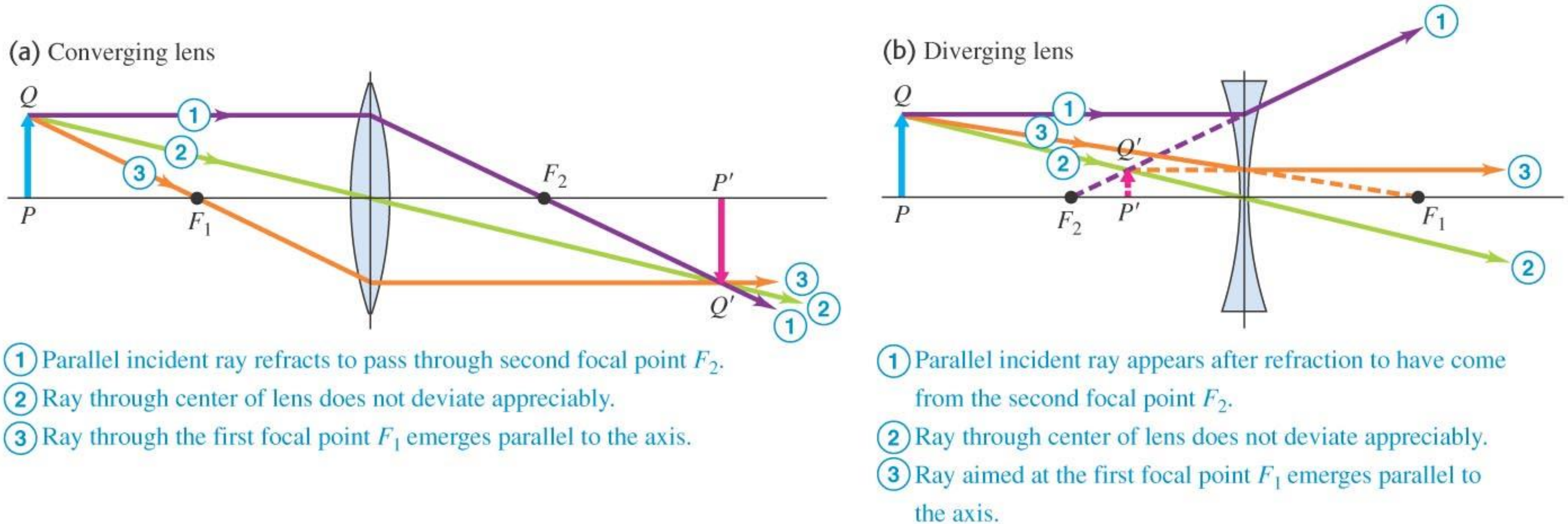
(b) $$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{1}{f}=(1.52-1)\left(\frac{1}{-10\text{ cm}}-\frac{1}{+10\text{ cm}}\right)$$

$$f=-9.6\text{ cm}$$



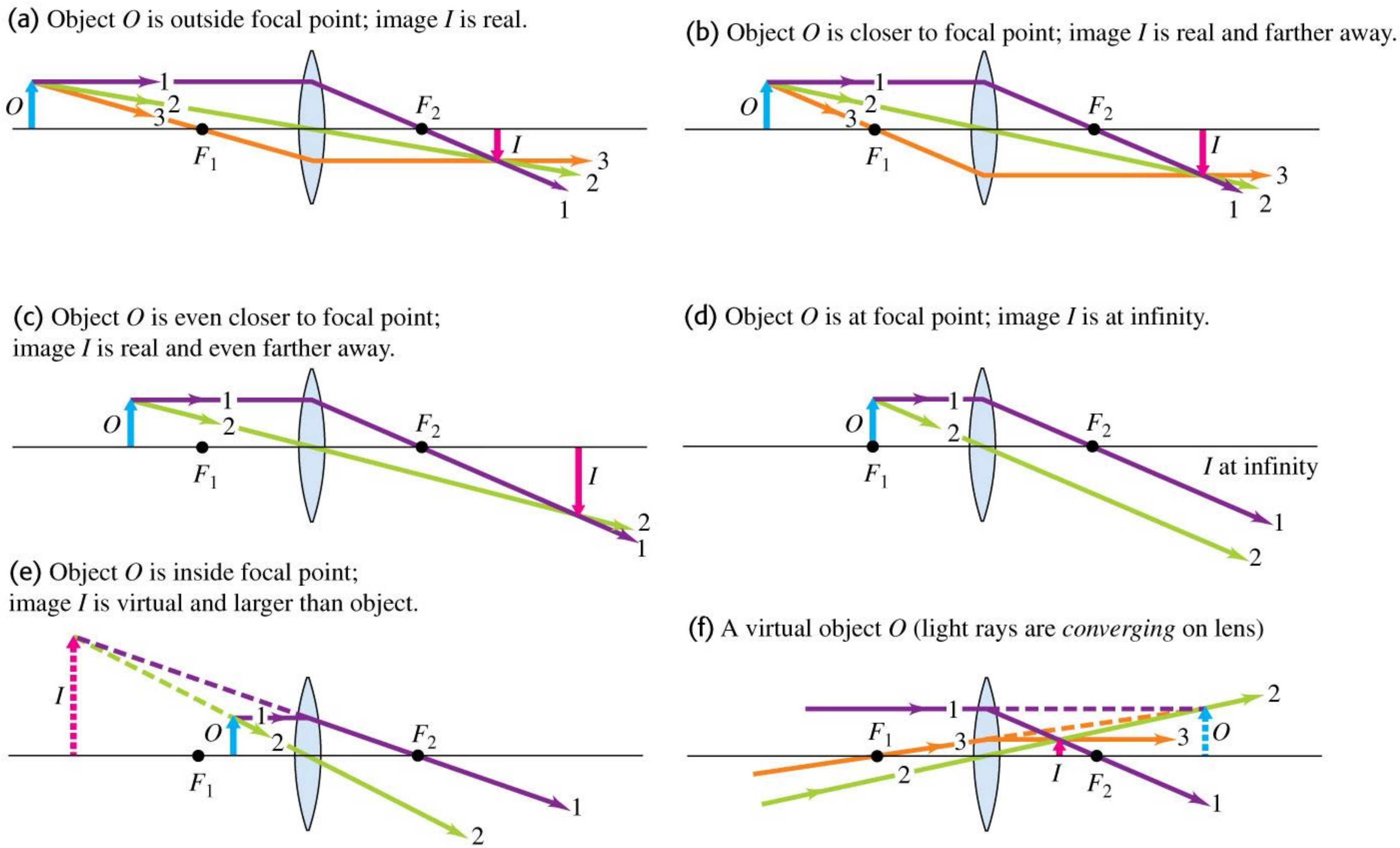
Graphical methods for lenses

- Follow the text summary of the three principal rays.
- Figure 34.36 below illustrates the principal rays for converging and diverging lenses.



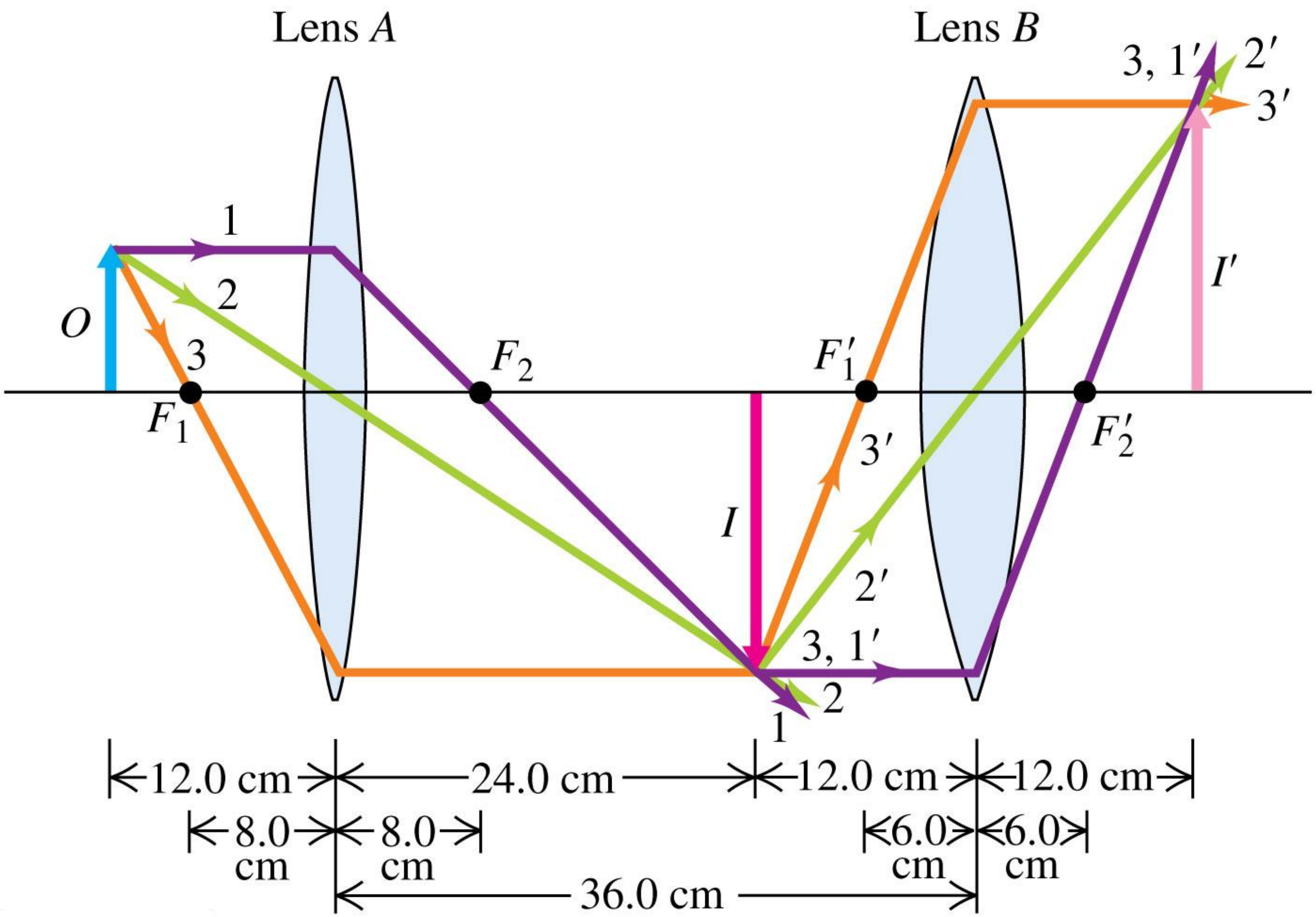
The effect of object distance

- The object distance can have a large effect on the image (Figure 34.37 below).



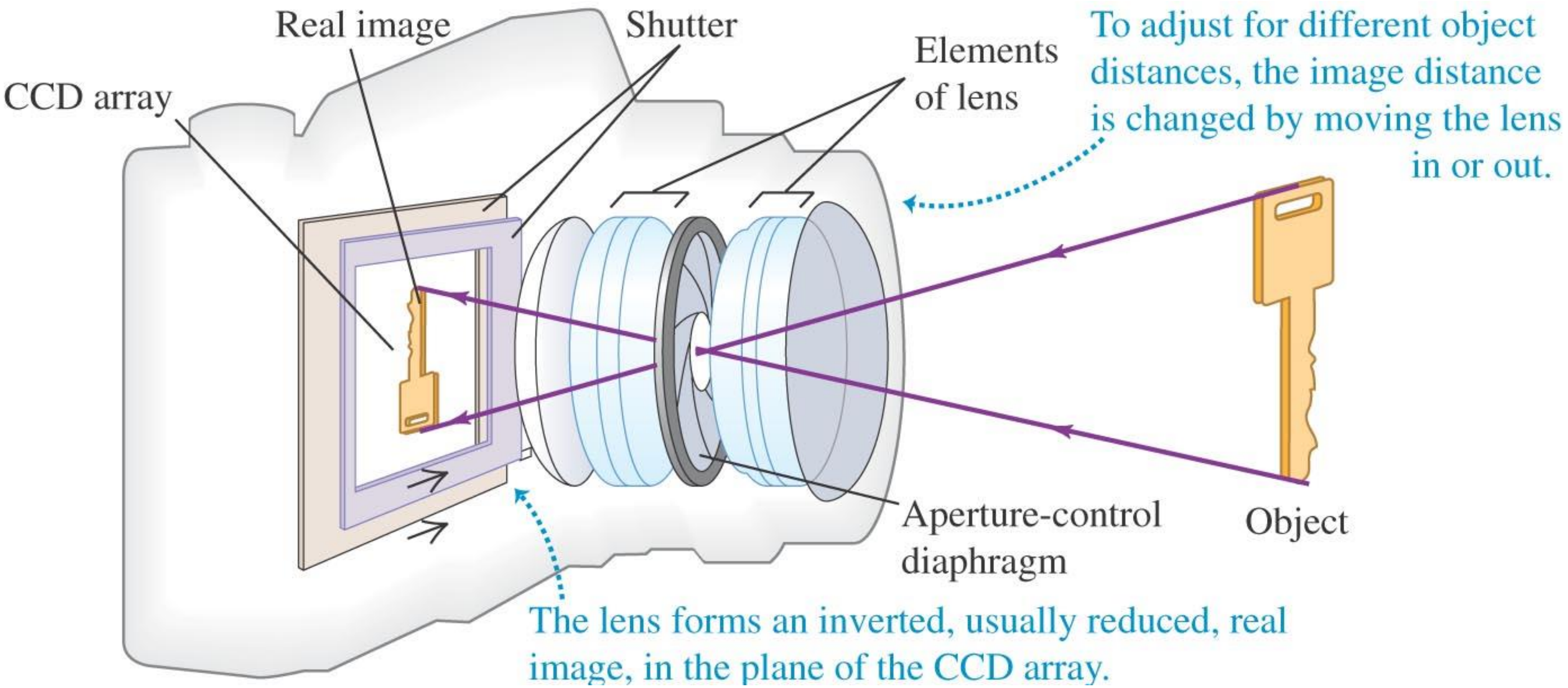
An image of an image

- Follow Example 34.11 using Figure 34.39 below.



Cameras

- Figure 34.40 below shows the key elements of a digital camera.



Camera lens basics

- The f-number is the focal length divided by the aperture size, also called the f / D ratio. A 50 mm lens with an aperture size D = 25 mm has an f-number

$$f/D = 50 \text{ mm} / 25 \text{ mm} = 2$$

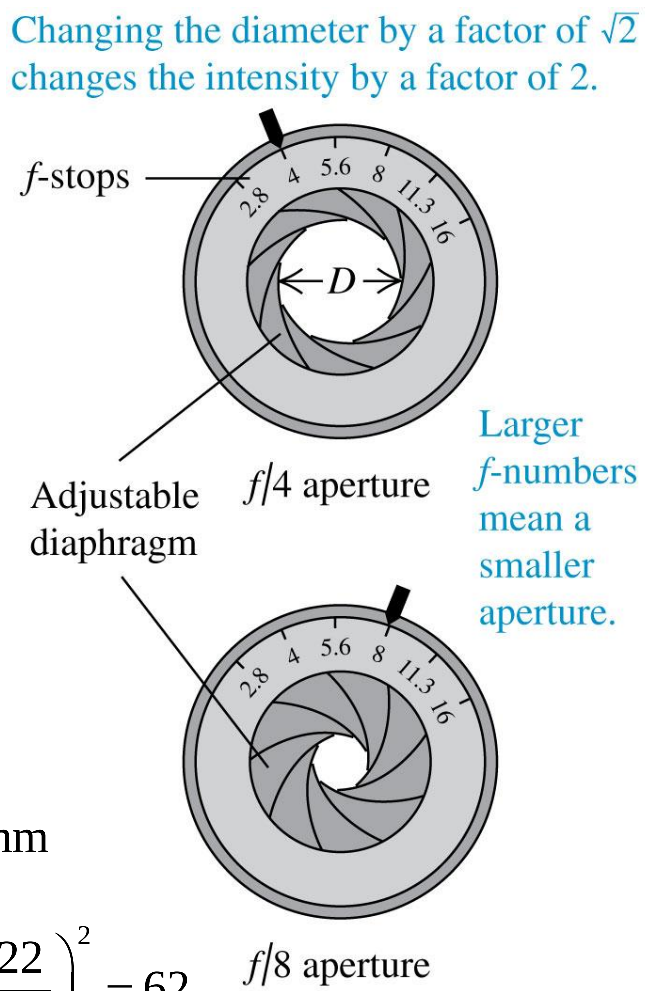
so it would be said to have an f-stop of f/2. Since exposure time depends on the area of the aperature, f-stops changing by square-root of 2 change the exposure time by a factor of 2. Typical f-stops are f/2, f/2.8, f/4, f/5.6, f/8, f/11, f/16.

- Example 34.12: A common telephoto lens for a 35-mm camera has a focal length of 200 mm; its f-stops range from f/2.8 to f/22. (a) What is the range of apertures? (b) What is the corresponding range of intensities on the film?

$$D = \frac{f}{f\text{-number}} = \frac{200 \text{ mm}}{2.8} = 71 \text{ mm and } D = \frac{200 \text{ mm}}{22} = 9.1 \text{ mm}$$

Intensity is proportional to D^2 so the ratio is $\left(\frac{71 \text{ mm}}{9.1 \text{ mm}}\right)^2 = \left(\frac{22}{2.8}\right)^2 = 62$

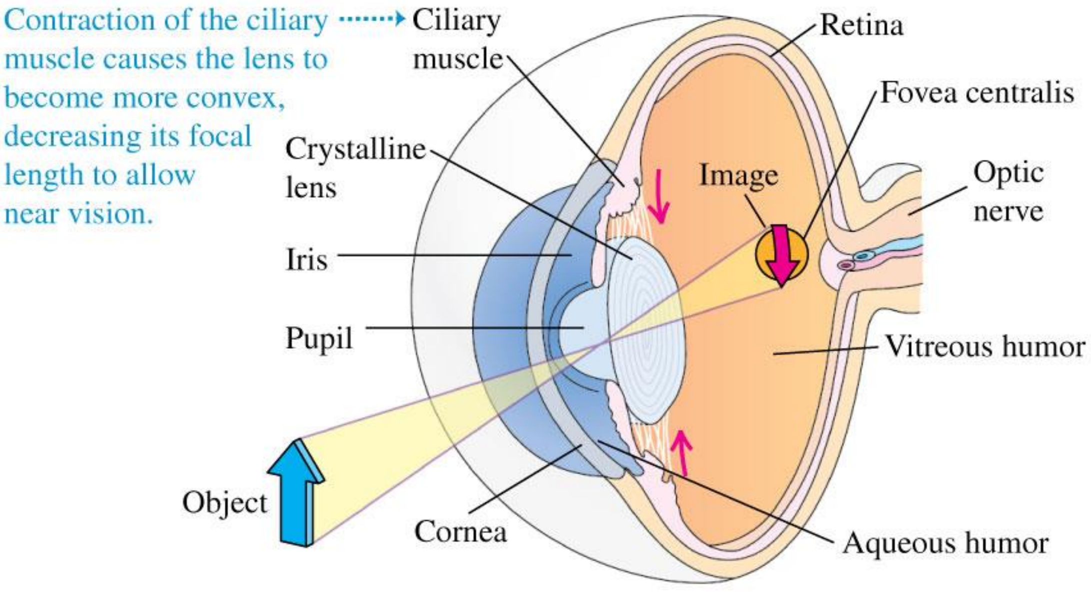
- For an exposure of 1/1000 s at f/2.8, you would have to expose for 62/1000 s ~ 1/16 s at f/22.



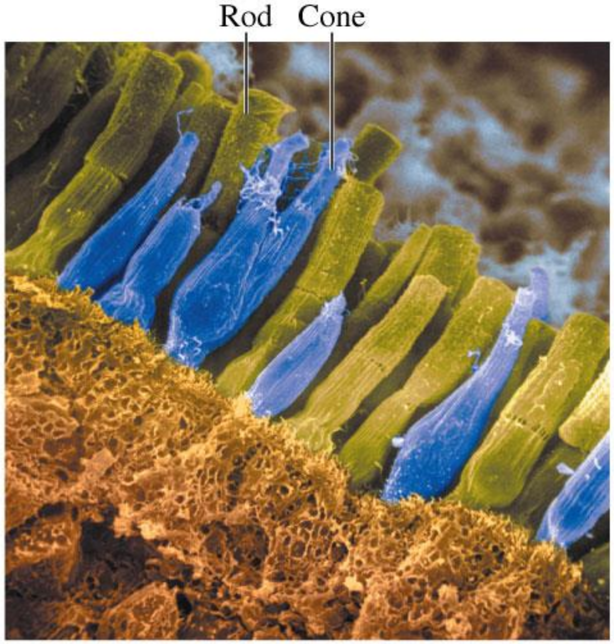
The eye

- The optical behavior of the eye is similar to that of a camera.
- Figure 34.44 below shows the basic structure of the eye.

(a) Diagram of the eye



(b) Scanning electron micrograph showing retinal rods and cones in different colors

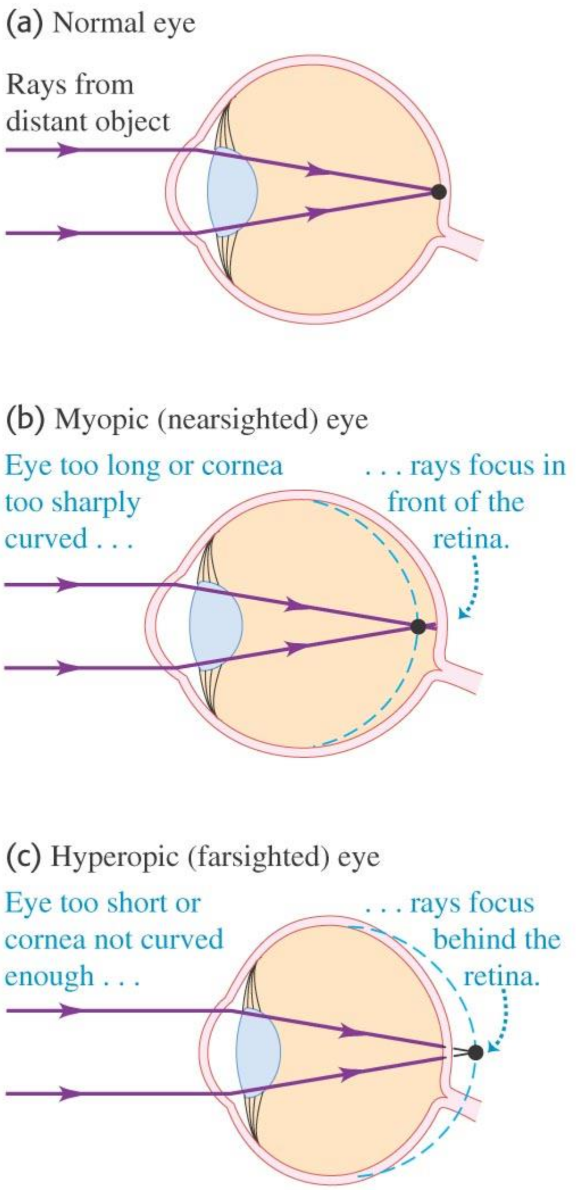


Defects of vision

- The *near point* typically recedes with age, as shown in Table 34.1.
- Figure 34.45 (at right) shows a normal, a myopic, and a hyperopic eye.

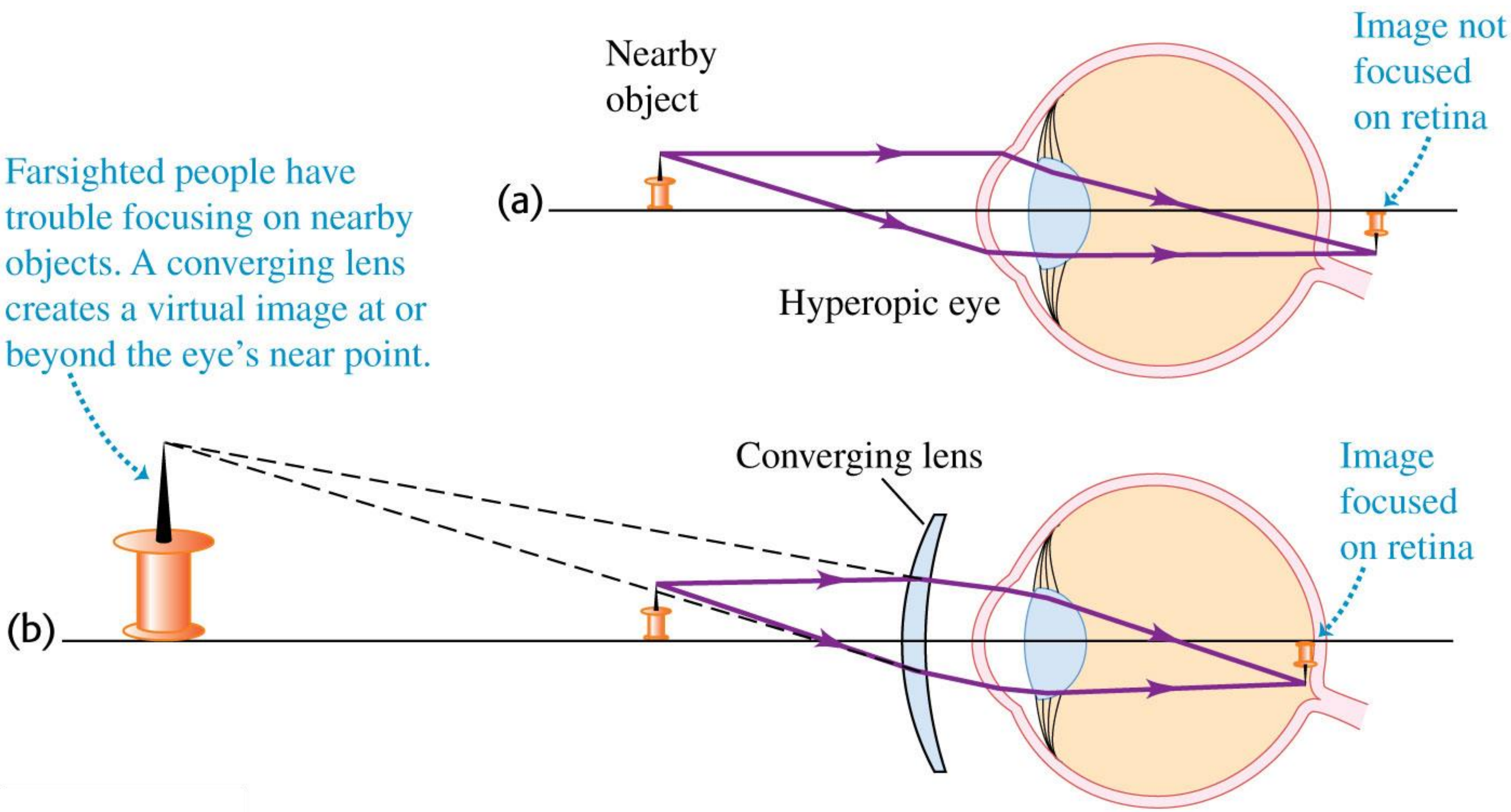
Table 34.1 Receding of Near Point with Age

Age (years)	Near Point (cm)
10	7
20	10
30	14
40	22
50	40
60	200



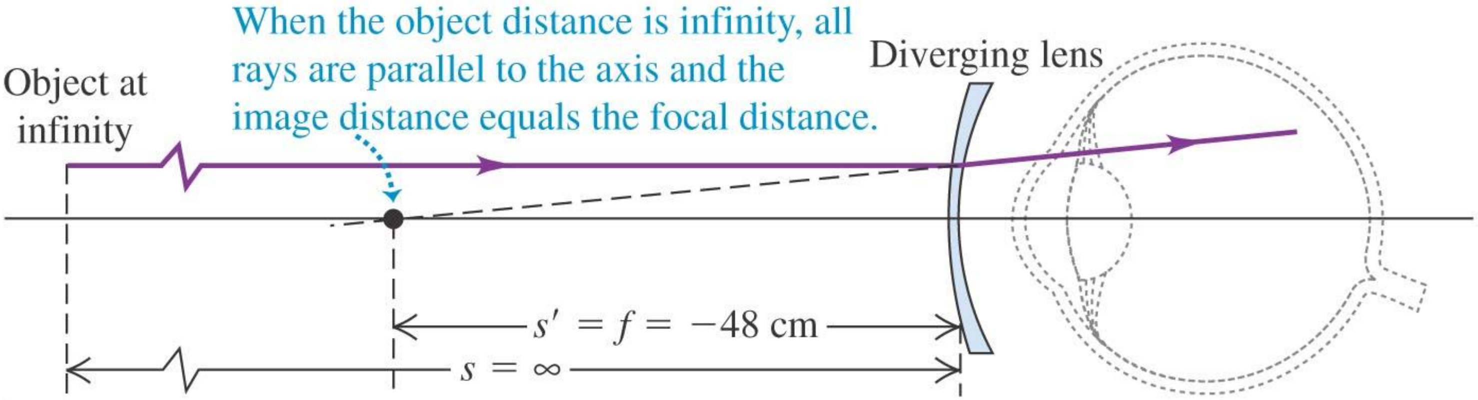
Farsighted correction

- Figure 34.46 below shows how to correct a hyperopic (farsighted) eye using a converging lens.



Correcting for nearsightedness

- The power of an eyeglass lens is defined as $1/f$, and uses the unit of **diopters** in inverse meters. Thus, a lens of focal length 50 cm is 2.0 diopters. A diverging lens with $f = -0.25$ cm is -4.0 diopters.
- Example 34.14: The far point of a certain myopic eye is 50 cm in from of the eye. Find the focal length and power of the eyeglass lens that will permit the wearer to see the object clearly at infinity. Assume the lens is worn 2 cm in front of the eye.

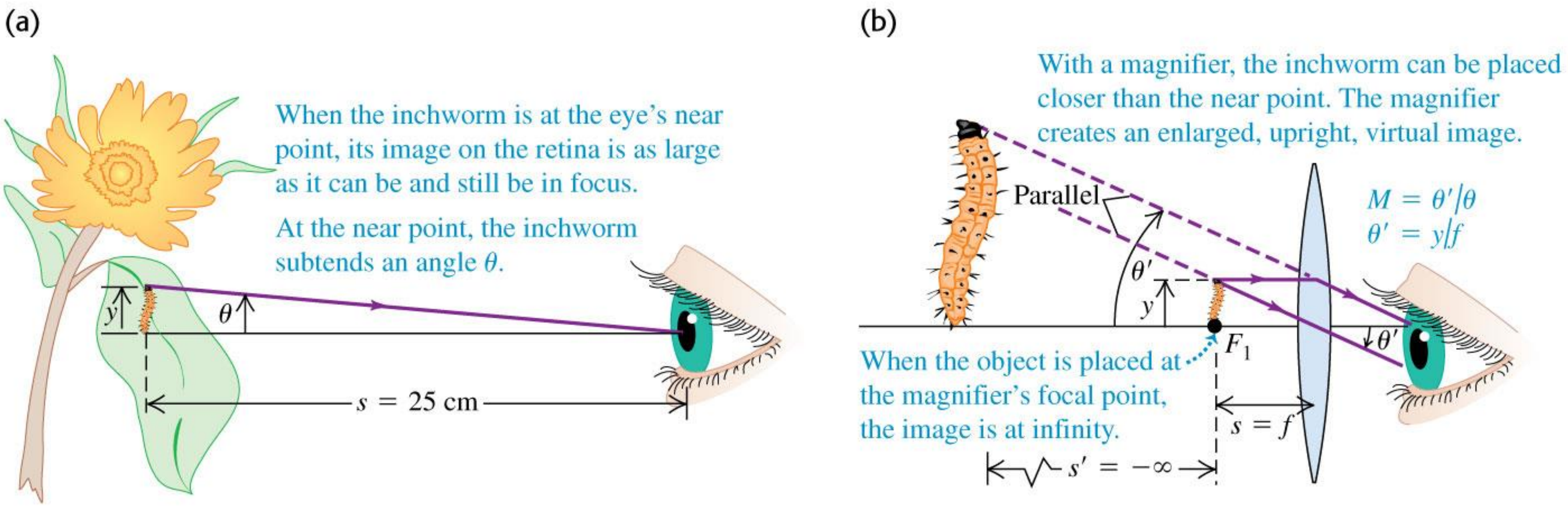


$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-48\text{ cm}} \quad \Rightarrow \quad f = -48\text{ cm}$$

$$\text{power} = \frac{1}{f} = \frac{1}{-48\text{ cm}} = -2.1\text{ diopters}$$

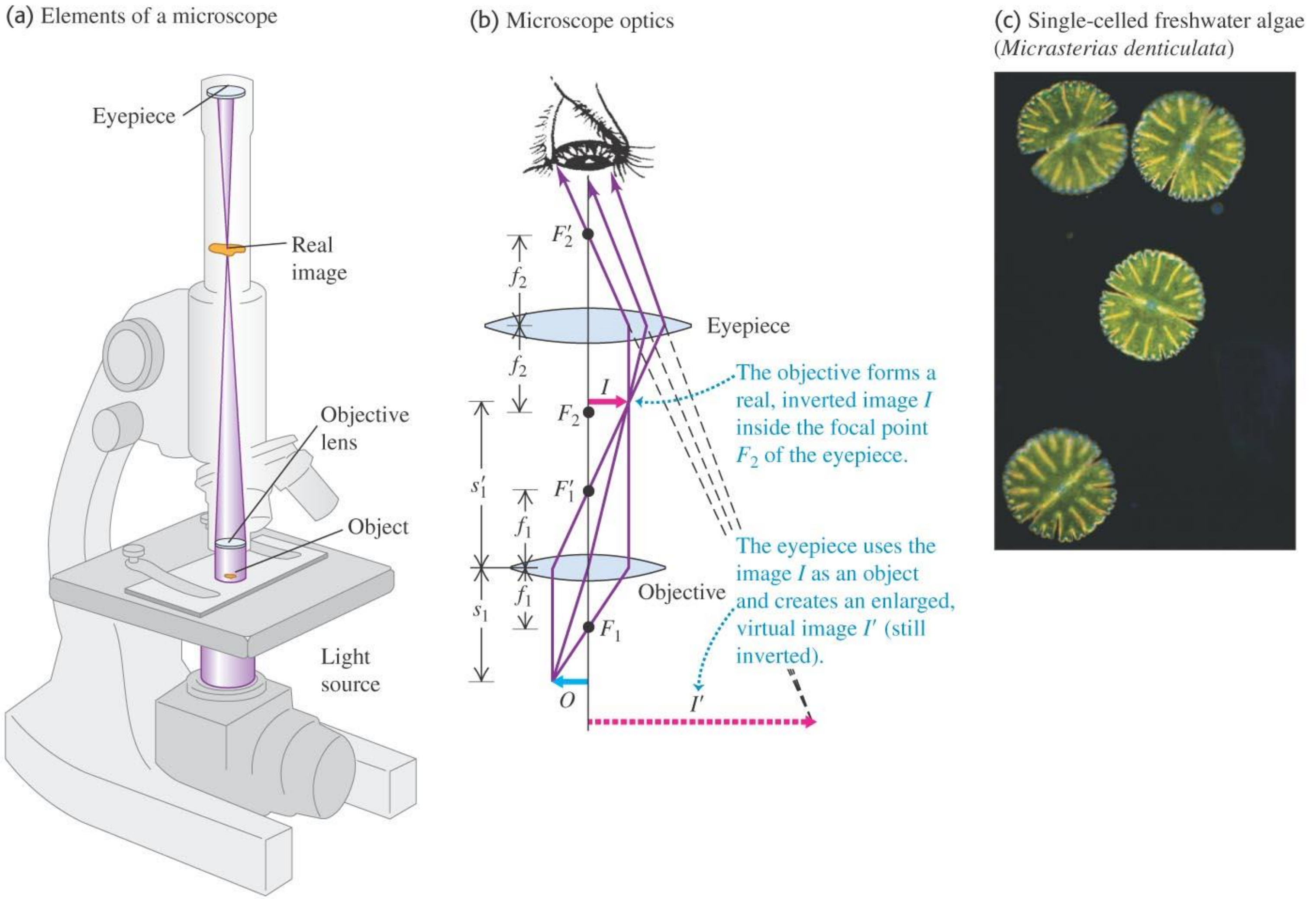
The magnifier

- *Angular magnification* is $M = \theta'/\theta$. See Figure 34.51 below.
- The angular magnification of a simple magnifier is
  $M = \theta'/\theta = (25 \text{ cm})/f$.



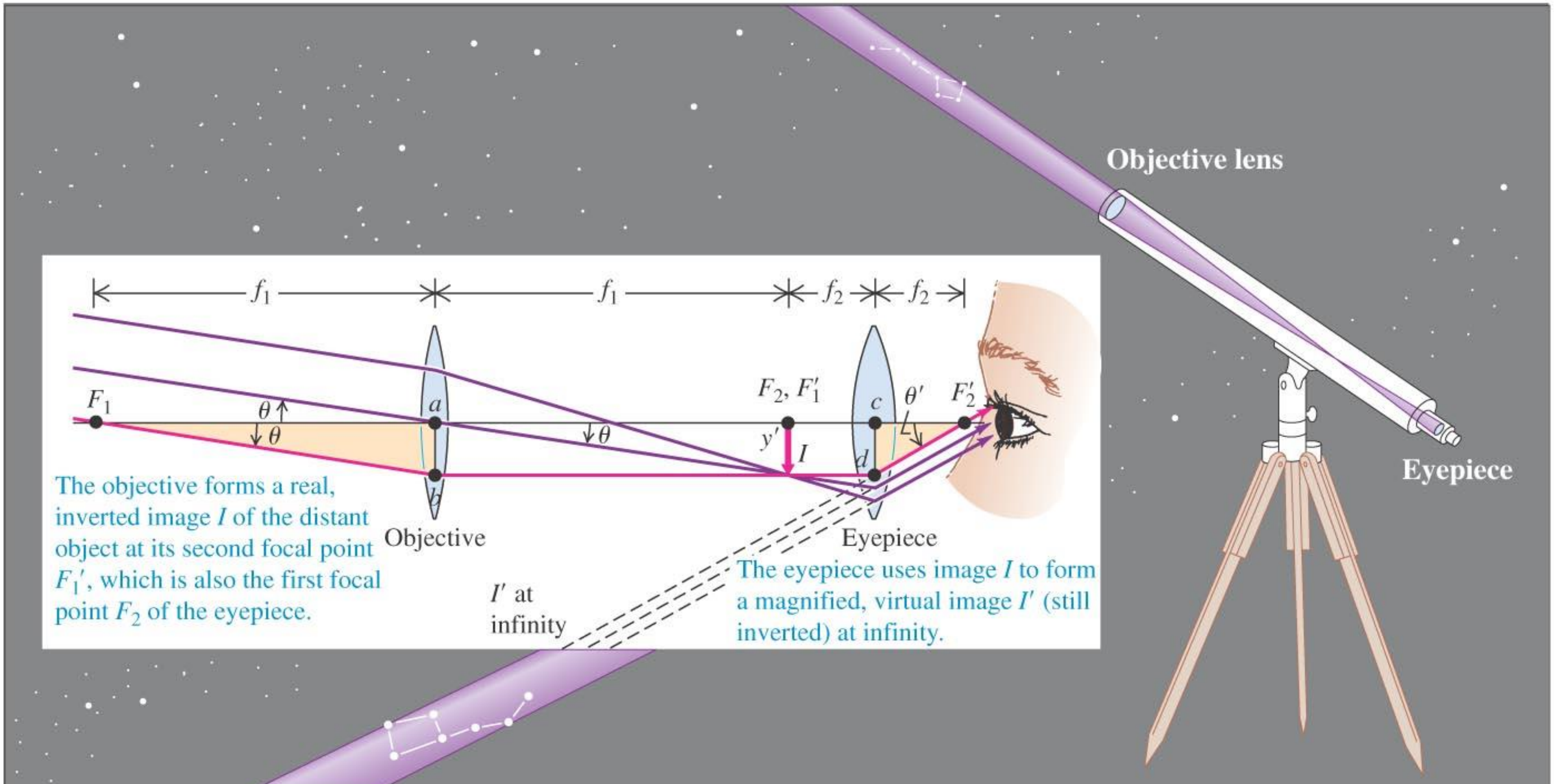
The microscope

- A *compound microscope* consists of an *objective* lens and an *eyepiece*. (See Figure 34.52 below.)



The astronomical telescope

- Figure 34.53 below shows the the optical system of an *astronomical refracting telescope*.



The reflecting telescope

- Figure 34.54 below shows three designs for reflecting telescopes. Part (d) shows the Gemini North telescope, which uses the design in (c) with an objective mirror 8 meters in diameter.