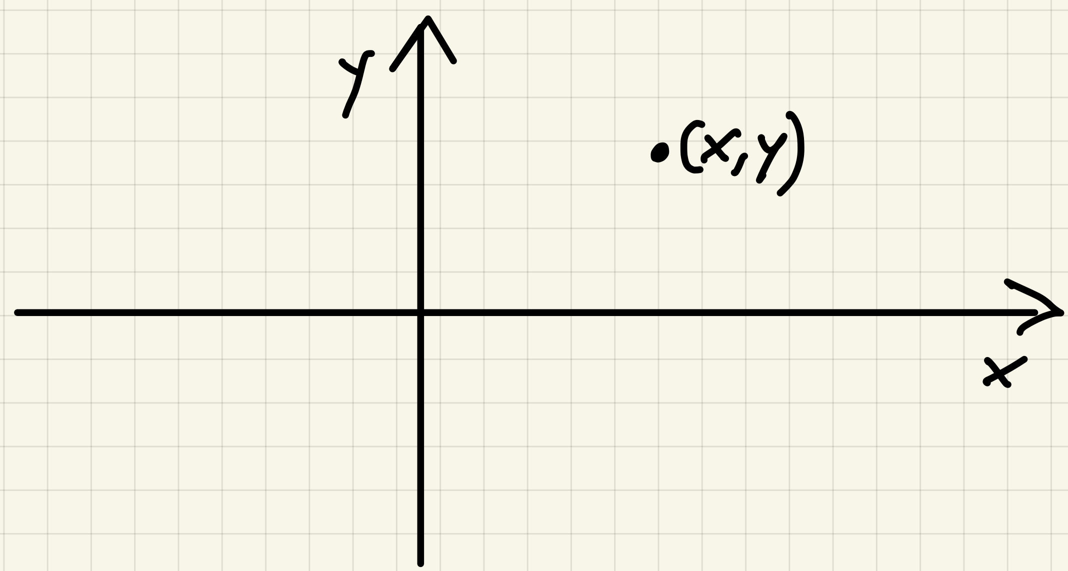
7 ottobre

Mercoledì 8 – 11 Aula magna Economia

19 – 20 – 21 lezioni An Mat cancellate

Numeri Complessi $\mathbb{C} = \mathbb{R}^2 =$

$$= \{ (x,y) : x \in \mathbb{R},\ y \in \mathbb{R} \}$$



$$(x,y) + (u,v) = (x+u, y+v)$$

$$c\,(x,y) = (cx, cy)$$

$$(x,y)\,(u,v) = (xu - yv, xv + yu)$$

$$\mathbb{R} \ni x \longrightarrow (x,0) \in \mathbb{C}$$

$$\mathbb{R} \ni x + u \longrightarrow (x+u, 0) = (x,0) + (u,0)$$

$$\mathbb{R} \ni x u \longrightarrow (xu, 0) = (x,0)(u,0)$$

$$(x,0)(u,v) = (xu, xv) = x(u,v)$$

$$(x,y) = (x,0) + (0,y) = x(1,0) + y(0,1)$$

$$= (x,0) + y(0,1)$$

I numeri

$(x,0)$ li identifichiamo con x

$$i := (0,1)$$

$$(x,y) = x + yi = x + iy$$

$$i^2 = i\, i = (0,1)(0,1) =$$

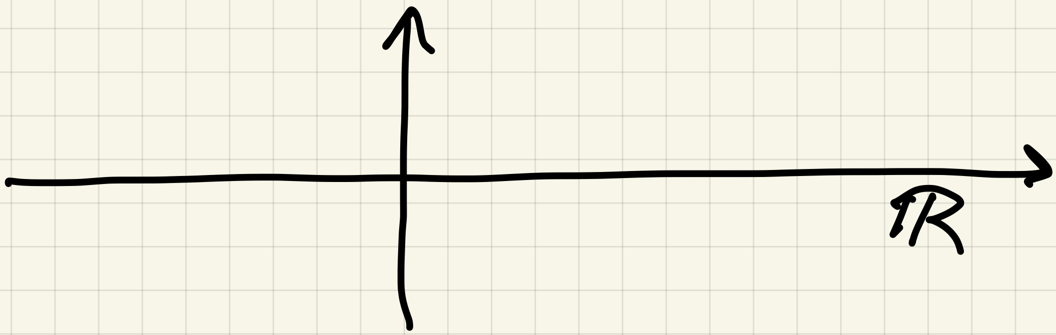
$$(x,y)(u,v) = (xu - yv, xv + yu)$$

$$= (-1, 0) = -1$$

$$i^2 = -1$$

i è una soluzione dell'equazione $z^2+1=0$

Tutti sappiamo che $x^2+1=0$ non ha soluzioni in $\mathbb{R}$



$z^2+1=0$ ha 2 soluzioni

$z=\pm i$

$$(-z)^2 = ((-1)\cdot z)^2 = (-1)^2 z^2$$
$$= z^2$$

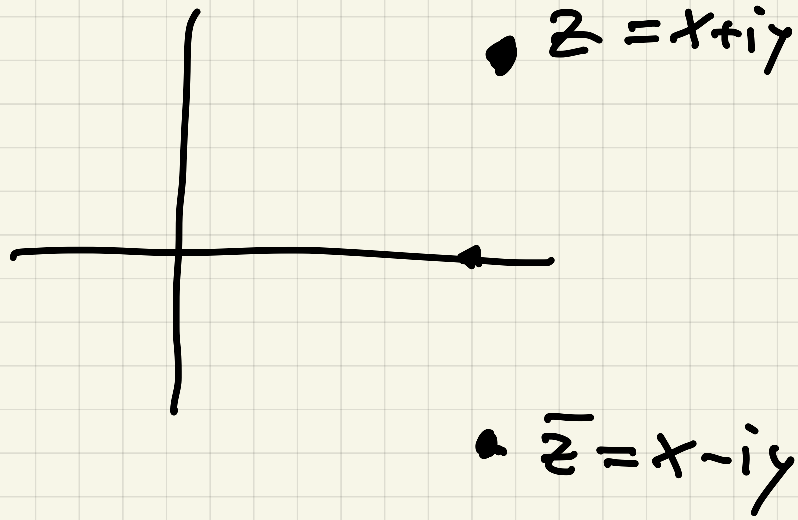
$$z^{27}+z^{23}+z^2+z^{28}+1=0$$

$z = x + iy$ $\qquad\qquad$ $z = (x, y)$

$x = \operatorname{Re} z$

$y = \operatorname{Im} z$

$|z| = \sqrt{x^2 + y^2}$



$z = x + iy$

$\bar{z} = x - iy$

$z = \bar{z} \iff z \in \mathbb{R}$

$\bar{z} = -z \iff z = iy$ con $y \in \mathbb{R}$

Golamir

$z = x+iy$

$\bar{z} = x-iy$

$\overline{z+w} = \bar{z} + \bar{w}$

$\overline{zw} = \bar{z}\,\bar{w}$

$i = (0,1)$

$z\bar{z} = x^2 + y^2 =: |z|^2$ $\qquad$ $z = x+iy = (x,y)$

$(x,y) = x\overbrace{(1,0)} + y\overbrace{(0,1)}^{i} = x + iy$

$$z w = (x+iy)(u+iv) =$$

$$= xu + ixv + iyu + i^2 yv \qquad i^2 = -1$$

$$= xu - yv + i(xv + yu)$$

$$(x,y)(u,v) = (xu - yv,\ xv + yu)$$

<u>Lemma</u> Se $z \neq 0$ esiste un unico numero che denoto con $\frac{1}{z}$ t.c.

$$z \, \frac{1}{z} = \frac{1}{z} \, z = 1$$

$$\frac{1}{z} = \frac{1}{|z|^2} \, \bar{z} = \frac{\bar{z}}{|z|^2} \qquad z \neq 0 \Downarrow |z| > 0$$

$$z \frac{\bar{z}}{|z|^2} = z \bar{z} \frac{1}{|z|^2} = |z|^2 \frac{1}{|z|^2} = 1$$

$$\frac{3 + i2}{1 + i3} = x + iy$$

$$(3+i2)\frac{1}{1+i3} =$$

$$= (3+i2)\frac{1-i3}{1^2+3^2} =$$

$$= (3+i2)(1-i3)\frac{1}{10}$$

$$= (3+i2(-i)3+i2-i9)\frac{1}{10}$$

$$= \frac{9-7i}{10}$$

$i(-i)=1$

$(-1)i^2=$

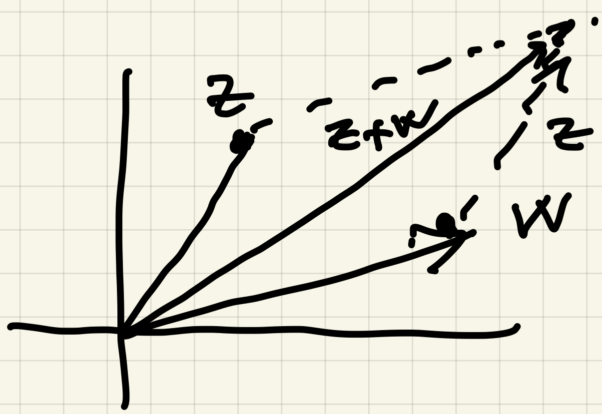
$(-1)^2=1$

$$\frac{1}{i} = -i$$

$$\frac{1}{i} = \frac{-i}{|i|^2} = \frac{-i}{1} = -i$$

$$|z+w| \le |z|+|w|$$

disuguaglian-za triangolare



$$|z\,w| = |z|\,|w|$$

$$\left|\frac{z}{w}\right| = \frac{|z|}{|w}$$