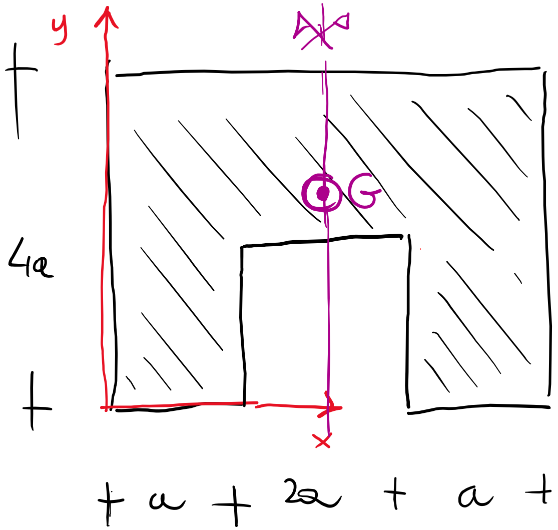
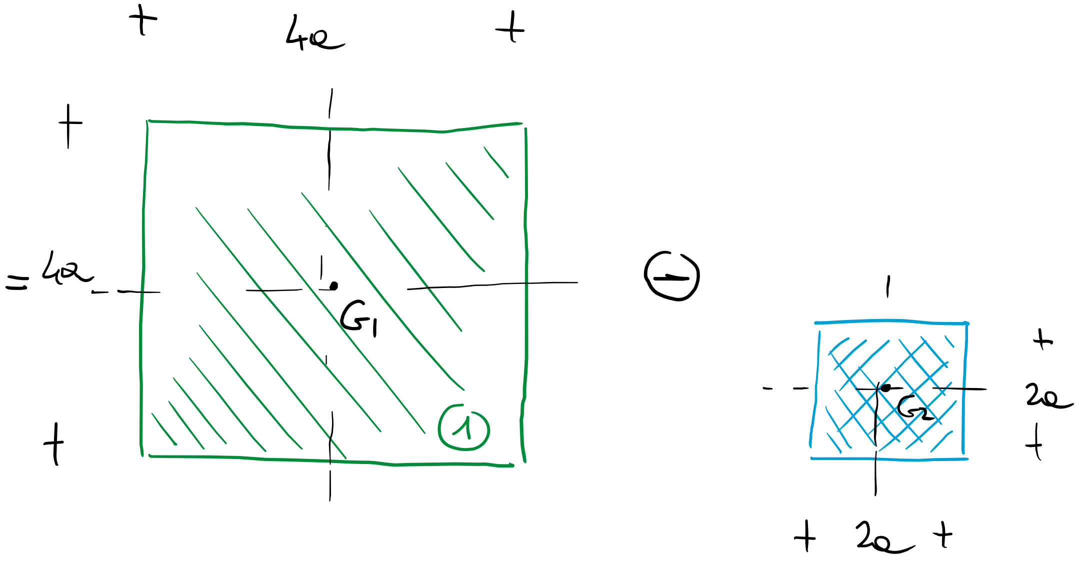
Esercizi
Geometria delle Aree

Marco Rossi

CALCOLO del BARICENTRO





$A_1 = (4a)^2 = 16a^2$, $A_2 = (2a)^2 = 4a^2$

$A = A_1 - A_2 = 12a^2$

$G_1 = (2a, 2a)$

$G_2 = (2a, a)$

COORDINATE del BARICENTRO

$x_G = 2a$ FIGURA DOPPIAMENTE SIMMETRICA

$$y_G = \frac{S_x}{A} = \frac{S_{x_1} - S_{x_2}}{A} = \frac{A_1 y_{G_1} - A_2 y_{G_2}}{A_1 - A_2} = \frac{16a^2 \cdot 2a - 4a^2 \cdot a}{12a^2} = \frac{28}{12} a = \frac{7}{3} a$$

ESERCIZI

RETTANGOLO

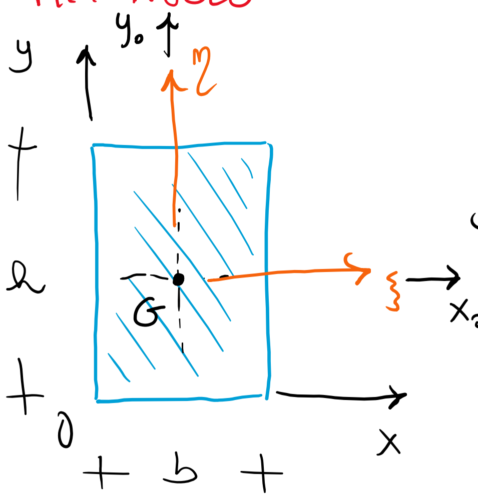


FIGURA DOPPIAMENTE SIMMETRICA

$G = \left(\frac{b}{2}, \frac{h}{2}\right)$ $\qquad A = bh$

$$S_x = \int_A y\,dA = \int_0^h\int_0^b y\,dx\,dy = \int_0^h y\,dy \cdot \int_0^b dx = \frac{bh^2}{2}$$

$S_y = \frac{hb^2}{2}$ → BASTA SCAMBIARE x e y !!

$y_G = \frac{S_x}{A} = \frac{bh^2}{2bh} = \frac{h}{2}$, ok!

$$J_x = \int_A y^2\,dy = \int_0^b\int_0^h y^2\,dx\,dy = \int_0^b dx \cdot \int_0^h y^2\,dy = \frac{bh^3}{3}$$

$J_y = \frac{hb^3}{3}$ $\qquad J_{xy} = \int_0^b x\,dx \cdot \int_0^h y\,dy = \frac{b^2}{2}\cdot\frac{h^2}{2} = \frac{b^2h^2}{4}$

Th del TRASPORTO: $J_x = J_{x_0} + A y_G^2$

$$J_{x_0} = J_x - A y_G^2 = \frac{bh^3}{3} - bh\left(\frac{h}{2}\right)^2 = bh^3\left(\frac{1}{3} - \frac{1}{4}\right) = \boxed{\frac{bh^3}{12}}$$

$$J_{y_0} = J_y - A x_G^2 = \boxed{\frac{hb^3}{12}}$$

$J_{x_0y_0} = J_{xy} - A x_G y_G = \frac{b^2h^2}{4} - bh\frac{b}{2}\cdot\frac{h}{2} = 0$ → IL SISTEMA Ox_0y_0 è CENTRALE D'INERZIA !!!

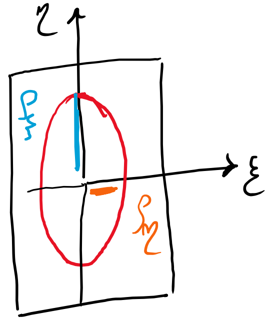
$J_\xi = J_x$, $J_\eta = J_y$ $\quad (h > b)$

$\rho_{x_0} = \sqrt{\frac{J_{x_0}}{A}} = \sqrt{\frac{bh^3}{12bh}} = \frac{h}{\sqrt{12}} \simeq 0{,}289h$, $\quad \rho_{y_0} = \sqrt{\frac{J_{y_0}}{A}} = \frac{b}{\sqrt{12}}$

$\rho_{x_0} \equiv \rho_\xi$, $\rho_\eta = \rho_{y_0}$

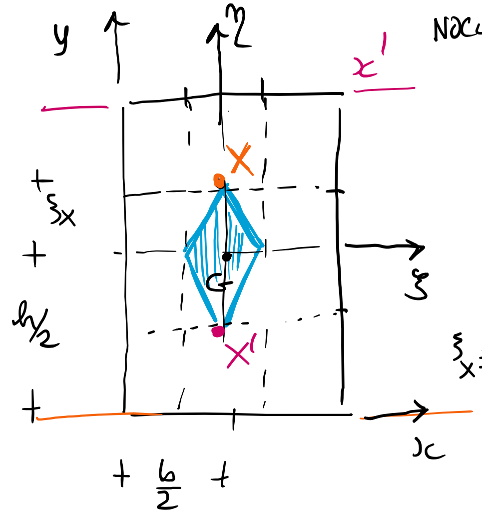
ELLISSE CENTRALE

$\cancel{12}\left(\frac{\xi^2}{b^2} + \frac{\eta^2}{h^2} - \frac{1}{12}\right) = 0$ $\qquad \frac{\xi^2}{b^2} + \frac{\eta^2}{h^2} - \frac{1}{12} = 0$



CONIUGATO

$tg\alpha \cdot tg\beta = -\frac{J_\xi}{J_\eta} = -\frac{bh^3}{12}\cdot\frac{12}{b^3h} = -\frac{h^2}{b^2}$ → $tg\alpha \cdot tg\beta = -\frac{h^2}{b^2}$



NOCCIOLO → TROVO L'ANTIPOLO RISPETTO a x

$\xi_x \cdot \frac{h}{2} = \rho_\xi^2$ (ANTIPOLARITÀ) $\qquad \xi_x = \frac{2}{h}\cdot\frac{h^2}{12} = \frac{h}{6}$

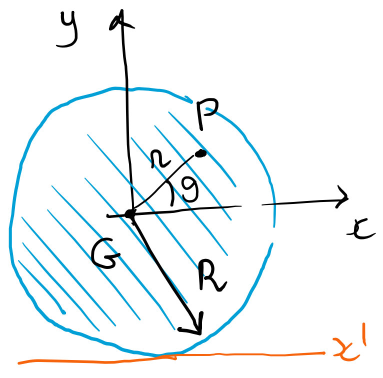
$\xi_{x'} = \frac{h}{6}$

OPPURE CON MODULI di RESISTENZA

$\xi_x \equiv w' = \frac{\rho_\xi^2}{h/2} = \frac{h}{6}$ ok

$$W = A w' = \frac{J_\xi}{h/2} = \frac{bh^3}{12h}\cdot 2 = \frac{bh^2}{6}$$

CERCHIO



PARAMETRIZZAZIONE (COORD. POLARI)

$$\begin{cases} x = r\cos\theta \\ y = r \,\text{sen}\theta \end{cases} \qquad dA = |\det J|\, dr\, d\theta$$

$$\det J = \begin{vmatrix} \cos\theta & -r\,\text{sen}\theta \\ \text{sen}\theta & r\cos\theta \end{vmatrix} = r(\text{sen}^2\theta + \cos^2\theta) = r$$

$$dA = r\, dr\, d\theta$$

$$A = \int_0^{2\pi}\int_0^R r\, dr\, d\theta = \frac{R^2}{2}\cdot 2\pi = \pi R^2 \text{ , TORNA!!}$$

$$G = (0,0)$$

$$J_x = \int_0^{2\pi}\int_0^R r^2\,\text{sen}^2\theta \cdot r\, dr\, d\theta = \int_0^R r^3\, dr \cdot \int_0^{2\pi} \text{sen}^2\theta\, d\theta =$$

$$= \frac{R^4}{4}\cdot\frac{1}{2}\Big[\theta - \text{sen}\theta\cos\theta\Big]_0^{2\pi} = \frac{2\pi R^4}{8} = \frac{\pi R^4}{4}$$

$$\rho_x^2 = \frac{J_x}{A} = \frac{\pi R^4}{4\cdot\pi R^2} = \frac{R^2}{4}$$

$$\rho_x = \frac{R}{2}$$

$$J_{x'} = J_x + A y_G^2 = \frac{\pi R^4}{4} + \pi R^2\cdot R^2 = \frac{5}{4}\pi R^4$$

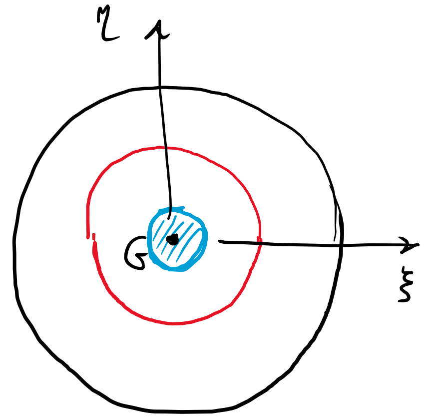
$$J_{xy} = \int_A xy\, dA = \int_0^R\int_0^{2\pi} r^3\,\text{sen}\theta\cos\theta = \frac{R^4}{8}\int_0^{2\pi}\text{sen}(2\theta)\, d\theta = -\frac{R^4}{8}\Big[\cos(2\theta)\Big]_0^{2\pi} = 0$$

IL SISTEMA È CENTRALE D'INERZIA

ELLISSE D'INERZIA

$$\frac{\xi^2}{\rho_\eta^2} + \frac{\eta^2}{\rho_\xi^2} - 1 = 0 \rightarrow \frac{4}{R^2}\left\{\xi^2 + \eta^2 - \left(\frac{R}{2}\right)^2\right\} = 0$$

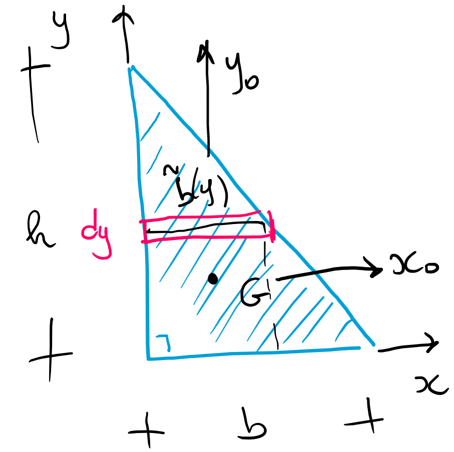
$$\boxed{\xi^2 + \eta^2 = \left(\frac{R}{2}\right)^2} \rightarrow$$ È UNA CIRCONFERENZA DI RAGGIO $R/2$



$$w' = \frac{\rho_x^2}{y'} = \frac{R^2}{4R} = \frac{R}{4} \rightarrow$$ IL NOCCIOLO D'INERZIA È UNA CIRCONFERENZA DI RAGGIO $R/4$

$$W = \frac{J}{y'} = \frac{\pi R^4}{4R} = \frac{\pi R^3}{4}$$

TRIANGOLO



$$\frac{\tilde b(y)}{b} = \frac{h-y}{h} \qquad \tilde b(y) = \frac{b}{h}(h-y)$$

$$A = \frac{bh}{2} \qquad x_G = \frac{S_y}{A}, \quad y_G = \frac{S_x}{A}$$

POSSO PENSARE IL TRIANGOLO DIVISO IN TANTE STRISCE DI SPESSORE dy E SEMPLIFICARE COSÌ IL CALCOLO

OGNI STRISCIA $dS_x = A^s \cdot y_G^s = dy \cdot \tilde b(y) \cdot y = \frac{b}{h}(h-y)\,y\,dy$

$$S_x = \int_0^h dS_x = \int_0^h \frac{b}{h}(hy - y^2)\,dy = \frac{b}{h}\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h =$$

$$= \frac{b}{h}\left(\frac{h^3}{2} - \frac{h^3}{3}\right) = \frac{bh^2}{6} \rightarrow y_G = \frac{S_x}{A} = \frac{bh^2}{6}\cdot\frac{2}{bh} = \frac{h}{3}$$

CALCOLO del MOMENTO 2° ORDINE IN MODO "ESATTO"

$$\frac{b}{x} = \frac{h}{h-y} \rightarrow y = h - \frac{h}{b}x$$

$$J_x = \int_A y^2\,dA = \int_0^b\left(\int_0^{h-\frac{h}{b}x} y^2\,dy\right)dx = \int_0^b \left[\frac{y^3}{3}\right]_0^{h-\frac{h}{b}x} dx =$$

$$= \frac{h^3}{3}\int_0^b \left(1 - \frac{x}{b}\right)^3 dx = \frac{h^3}{3}\int_0^b \left(1 - \frac{x^3}{b^3} - 3\frac{x}{b} + \frac{3x^2}{b^2}\right)dx = (*)$$

$$(*) = \frac{h^3}{3}\left[x - \frac{x^4}{4b^3} - \frac{3}{2}\frac{x^2}{b} + \frac{x^3}{b^2}\right]_0^b = \frac{h^3 b}{3}\left\{1 - \frac{1}{4} - \frac{3}{2} + 1\right\}$$

$$J_x = \frac{bh^3}{12} \rightsquigarrow J_y = \frac{hb^3}{12}$$

MOMENTO CENTRIFUGO INTEGRANDO LE STRISCE.-

$$dJ_{xy} = \cancel{dJ_{x_0y_0}} + A\,y\,\frac{\tilde b(y)}{2} = \tilde b(y)\,dy \cdot y\,\frac{\tilde b(y)}{2} = \frac{1}{2}\,y\,\tilde b^2(y)\,dy$$

$$J_{xy} = \int_0^h dJ_{xy} = \frac{1}{2}\int_0^h \frac{b^2}{h^2}(h-y)^2 y\,dy = \frac{b^2}{2h^2}\int_0^h (h^2y + y^3 - 2hy^2)\,dy =$$

$$= \frac{b^2}{2h^2}\left[\frac{h^2b^2}{2} + \frac{y^4}{4} - \frac{2hy^3}{3}\right]_0^h = \frac{b^2}{2h^2}\cdot\left[\frac{h^4}{2} + \frac{h^4}{4} - \frac{2h^4}{3}\right] = \frac{b^2h^2}{24}$$

TROVIAMO QUELLI BARICENTRALI (che NON sono CENTRALI)

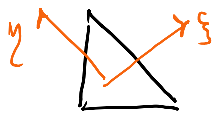
$$J_{x_0} = J_x - Ay_G^2 = \frac{bh^3}{12} - \frac{bh}{2}\cdot\frac{h^2}{9} = \frac{bh^3}{36}, \quad J_{y_0} = \frac{hb^3}{36}$$

$$J_{x_0y_0} = J_{xy} - Ay_Gx_G = \frac{b^2h^2}{24} - \frac{bh}{2}\cdot\frac{bh}{9} = \frac{b^2h^2}{72}$$

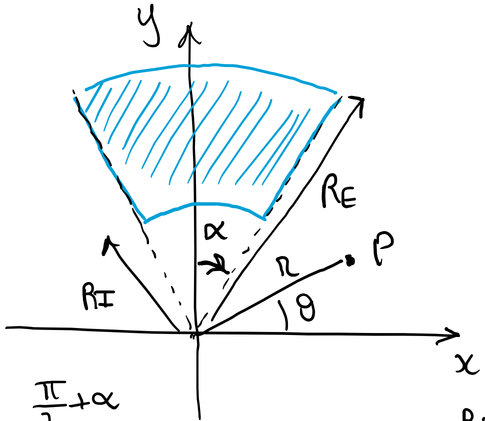
$$tg(2\alpha_0) = -\frac{2J_{x_0y_0}}{J_{x_0} - J_{y_0}} = -2\cdot\frac{b^2h^2}{72}\cdot\frac{1}{\frac{bh^3}{36} - \frac{b^3h}{36}} = -\frac{bh}{h^2 - b^2}$$

$h = b \rightarrow$

$\alpha_0 = 45°$



SETTORE CIRCOLARE



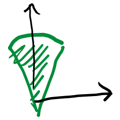
$$\begin{cases} x = r\cos\theta \\ y = r\,\mathrm{sen}\,\theta \end{cases} \qquad dA = r\,dr\,d\theta$$

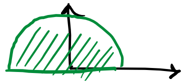
$$A = \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha}\int_{R_I}^{R_E} r\,dr\,d\theta = 2\alpha\cdot\frac{R_E^2-R_I^2}{2} = \alpha\left(R_E^2-R_I^2\right)$$

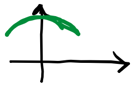
$$S_x = \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha}\int_{R_I}^{R_E} r^2\,\mathrm{sen}\,\theta\,dr\,d\theta = \int_{R_I}^{R_E} r^2\,dr\cdot\int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha}\mathrm{sen}\,\theta\,d\theta = \frac{R_E^3-R_I^3}{3}\cdot\Big[-\cos\theta\Big]_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha} =$$

$$= \frac{R_E^3-R_I^3}{3}\cdot\left\{\cos\left(\frac{\pi}{2}-\alpha\right)-\cos\left(\frac{\pi}{2}+\alpha\right)\right\} = \frac{2}{3}\left(R_E^3-R_I^3\right)\mathrm{sen}\,\alpha$$

$$\boxed{y_G = \frac{S_x}{A} = \frac{2}{3}\cdot\frac{R_E^3-R_I^3}{R_E^2-R_I^2}\cdot\frac{\mathrm{sen}\,\alpha}{\alpha}} \qquad (x_G = 0\text{, simmetria})$$

i] $R_I = 0$  $y_G = \frac{2}{3\alpha}R_E\,\mathrm{sen}\,\alpha$

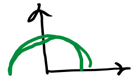
ii] $R_I = 0,\ \alpha = \frac{\pi}{2}$  $y_G = \frac{4R}{3\pi}$

iii] $R_I \to R_E$ 

PROFILO SOTTILE

$$\lim_{R_I\to R_E} y_G = \frac{2}{3\alpha}\,\mathrm{sen}\,\alpha\cdot\frac{-3R_I^2}{2R_I} = R\,\frac{\mathrm{sen}\,\alpha}{\alpha}$$

HOSPITAL

iv] $R_I \to R_E$, $\alpha = \frac{\pi}{2}$ 

$y_G = \frac{2R}{\pi}$

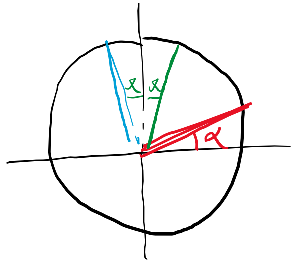
MOMENTO D'INERZIA

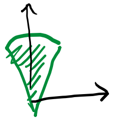
$$J_x = \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha}\mathrm{sen}^2\theta\,d\theta\int_{R_I}^{R_E} r^3\,dr = \frac{R_E^4-R_I^4}{4}\cdot\frac{1}{2}\Big[\theta - \mathrm{sen}\,\theta\cos\theta\Big]_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha} =$$

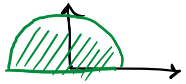
$$= \frac{R_E^4-R_I^4}{8}\left\{2\alpha - \underbrace{\mathrm{sen}\left(\frac{\pi}{2}+\alpha\right)}_{\cos\alpha}\underbrace{\cos\left(\frac{\pi}{2}+\alpha\right)}_{-\mathrm{sen}\,\alpha} + \underbrace{\mathrm{sen}\left(\frac{\pi}{2}-\alpha\right)}_{\cos\alpha}\underbrace{\cos\left(\frac{\pi}{2}-\alpha\right)}_{\mathrm{sen}\,\alpha}\right\} =$$

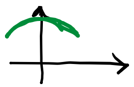
$$= \frac{R_E^4-R_I^4}{8}\left\{2\alpha + 2\,\mathrm{sen}\,\alpha\cos\alpha\right\} =$$

$$\boxed{= \frac{R_E^4-R_I^4}{4}\left(\alpha + \frac{1}{2}\,\mathrm{sen}\,2\alpha\right)}$$



i] $R_1 = 0$ $\quad J_x = \frac{R^4}{4}\left(\alpha + \frac{1}{2}\,\mathrm{sen}\,2\alpha\right)$ 

ii] $R_I = 0,\ \alpha = \frac{\pi}{2}$ $\quad J_x = \frac{R^4}{4}\left(\frac{\pi}{2} + \frac{1}{2}\cancel{\mathrm{sen}\,\pi}\right) = \frac{\pi R^4}{8}$ 

iii] $R_I \to R_E$, $R_E - R_I = \delta$ SPESSORE, $\frac{\delta}{R_E} \to 0$ $\quad R_E = R$ 

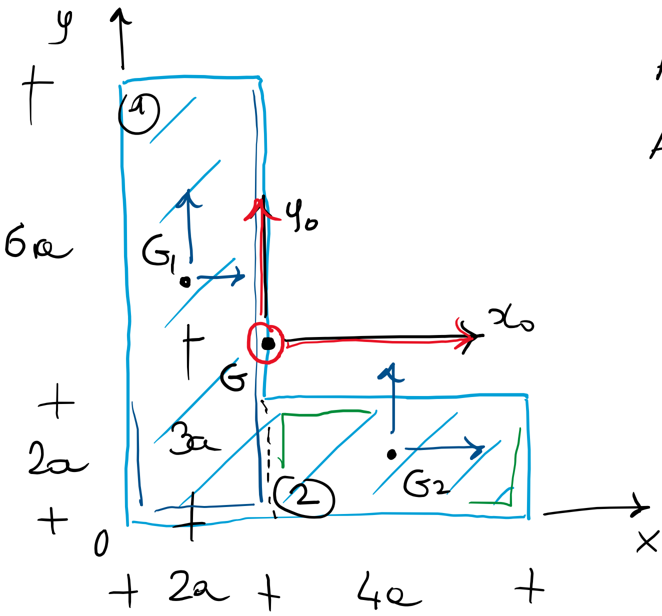
$$R_E^4 - R_I^4 \sim R^4 - (R-\delta)^4 \approx R^4 - R^4\left(1-\frac{\delta}{R}\right)^4 \approx R^4\left[1-\left(1-\frac{\delta}{R}\right)^4\right] \sim$$

$$\sim R^4\left(1+\left(1-\frac{\delta}{R}\right)^2\right)\left(1-\left(1-\frac{\delta}{R}\right)\right)\left(1+\left(1-\frac{\delta}{R}\right)\right) = R^4\,\frac{\delta}{R}\cdot 2\cdot 2 = 4R^3\delta$$

$$\boxed{J_x = R^3\delta\left(\alpha + \frac{1}{2}\,\mathrm{sen}\,2\alpha\right)}$$

iv] $\alpha = \frac{\pi}{2}$, $J_x = R^3\delta\,\frac{\pi}{2}$ $\qquad$ v] $\alpha = \pi$ $\quad J_x = \pi\delta R^3$

SEZIONE a L



AREA:

$$A_1 = 2a \cdot 8a = 16a^2$$

$$A_2 = 4a \cdot 2a = 8a^2$$

$$A = A_1 + A_2 = 24a^2$$

BARICENTRI dei SINGOLI PEZZI di SEZIONE

$$G_1 = (a, 4a)$$

$$G_2 = (4a, a)$$

MOMENTO STATICO della FIGURA, PER TROVARE IL BARICENTRO:

$$S_x = S_{x1} + S_{x2} = A_1 y_{G1} + A_2 y_{G2} = 16a^2 \cdot 4a + 8a^2 \cdot a = 64a^3 + 8a^3 = 72a^2$$

$$S_y = S_{y1} + S_{y2} = A_1 x_{G1} + A_2 x_{G2} = 16a^2 \cdot a + 8a^2 \cdot 4a = 48a^3$$

$$\begin{cases} x_G = \dfrac{S_y}{A} = \dfrac{48a^3}{24a} = 2a \\ y_G = \dfrac{S_x}{A} = \dfrac{72a^3}{24a^2} = 3a \end{cases}$$

$$\boxed{G = (2a, 3a)}$$

MOMENTI D'INERZIA:

1) USO J_x RISPETTO ALL'ASSE "SUL LATO" del RETTANGOLO e POI TRASPORTO NEL SISTEMA BARICENTRICO

$$J_x = \frac{b_1 h_1^3}{3} + \frac{b_2 h_2^3}{3} = \frac{2a \cdot (8a)^3}{3} + \frac{4a \cdot (2a)^3}{3} = \frac{32 \cdot 33}{3} a^4 = 352a^4$$

$$J_{x_0} = J_x - A y_G^2 = 352a^4 - 24a^2 \cdot (3a)^2 = 136a^4$$

2) CALCOLO I MOMENTI DATI NEL SISTEMA CENTRALE D'INERZIA DELLE SINGOLE PARTI (PIÙ FACILI DA RICORDARE) E POI TRASPORTO

$$J_{y1} = \frac{h_1 b_1^3}{12} = \frac{8a \cdot (2a)^3}{12} = \frac{16}{3} a^4, \quad J_{y2} = \frac{h_2 b_2^3}{12} = \frac{2a \cdot (4a)^3}{12} = \frac{32}{3} a^4$$

$$J_{y01} = J_{y1} + A_1 x_{G1}^2 = \frac{16}{3} a^4 + 16a^2 \cdot a^2 = \frac{64}{3} a^4$$

$$J_{y02} = J_{y2} + A_2 x_{G2}^2 = \frac{32}{3} a^4 + 8a^2 \cdot 4a^2 = \frac{128}{3} a^4$$

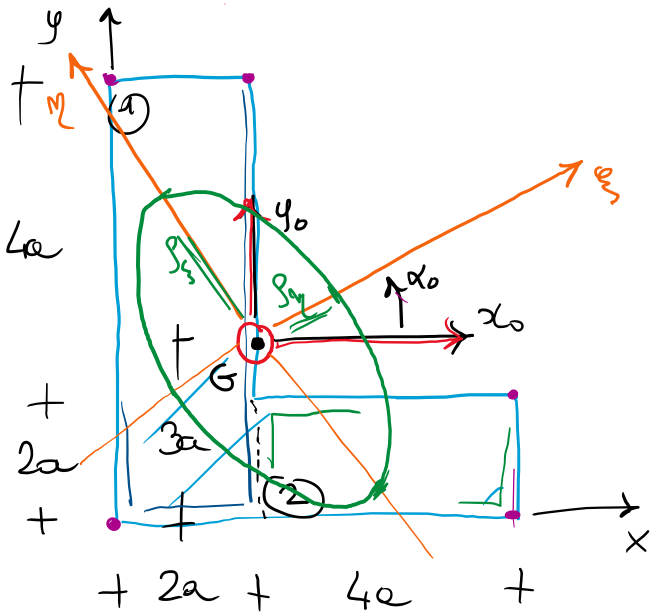
$$J_{y_0} = J_{y01} + J_{y02} = 64a^4$$

MOMENTO CENTRIFUGO (USO 2° METODO, $J_{x_1 y_1} = J_{x_2 y_2} = 0$)

$$J^{(1)}_{x_0 y_0} = \cancel{J_{x_1 y_1}} + A_1 x_{G1} y_{G1} = A_1 x_{G1} y_{G1}$$

$$J_{x_0 y_0} = 16a^2 \cdot (-a) \cdot a + 8a^2 \cdot 2a \cdot (-2a) = -48a^4$$

SEZIONE a L



TROVO SISTEMA CENTRALE D'INERZIA:

$$tg(2\alpha_0) = -\frac{2J_{x_0y_0}}{J_{x_0}-J_{y_0}} = \frac{2\cdot 48\,a^4}{(136-64)a^4} = \frac{4}{3}$$

$$\alpha_0 = \frac{1}{2}\,arctg\left(\frac{4}{3}\right)$$

$$\boxed{\alpha_0 = 0.4636\ldots \simeq 26.5°}$$

CALCOLO I MOMENTI D'INERZIA NEL SISTEMA $G\xi\eta$

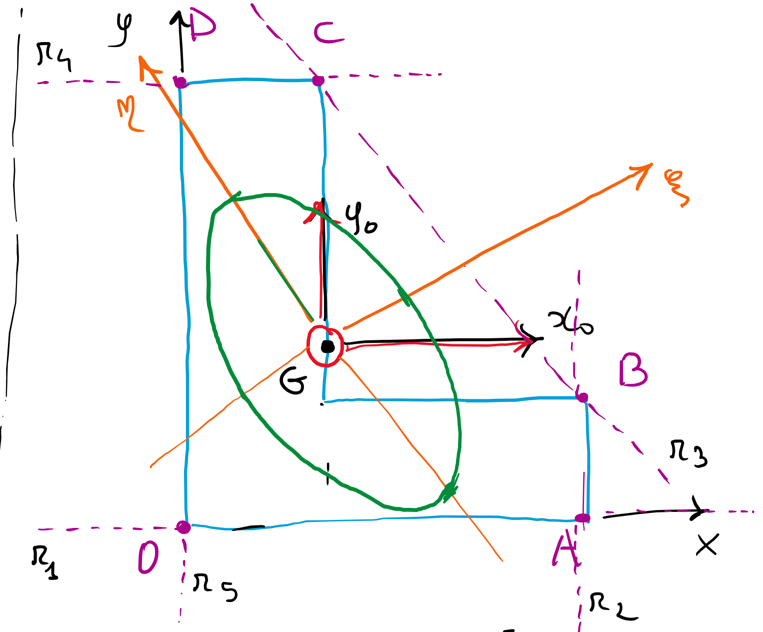
$$\left.\begin{matrix} J_\xi \\ J_\eta \end{matrix}\right\} = \frac{J_{x_0}+J_{y_0}}{2} \pm \sqrt{\left(\frac{J_{x_0}-J_{y_0}}{2}\right)^2 + J_{x_0y_0}^2} = \left\{\begin{matrix} 160a^4 \\ 40a^4 \end{matrix}\right.$$

RAGGI D'INERZIA

$$\rho_\xi = \sqrt{\frac{J_\xi}{A}} = \sqrt{\frac{160a^4}{24a^2}} \simeq 2.58a \qquad \rho_\eta = \sqrt{\frac{J_\eta}{A}} = \sqrt{\frac{40a^4}{24a^2}} \simeq 1.29a$$

L'ELLISSE D'INERZIA HA EQUAZIONE

$$\frac{\xi^2}{\rho_\eta^2} + \frac{\eta^2}{\rho_\xi^2} - 1 = 0$$



NEL SISTEMA Gx_0y_0

$O = (-2a, -3a)$

$A = (4a, -3a)$

$B = (4a, -a)$

$C = (0, 5a)$

$D = (-2a, 5a)$

PER TRACCIARE IL NOCCIOLO SI DEVONO SCRIVERE LE EQUAZIONI DELLE RADENTI in $G\xi\eta$

$$\begin{Bmatrix} \xi_i \\ \eta_i \end{Bmatrix} = \underline{R} \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix}, \quad \underline{R} = \begin{bmatrix} \cos\alpha_0 & \sin\alpha_0 \\ -\sin\alpha_0 & \cos\alpha_0 \end{bmatrix} \rightarrow$$

DA QUI SI OTTENGONO LE COORDINATE DEI PUNTI NEL SISTEMA ξ-η

LE EQ. delle RADENTI SONO

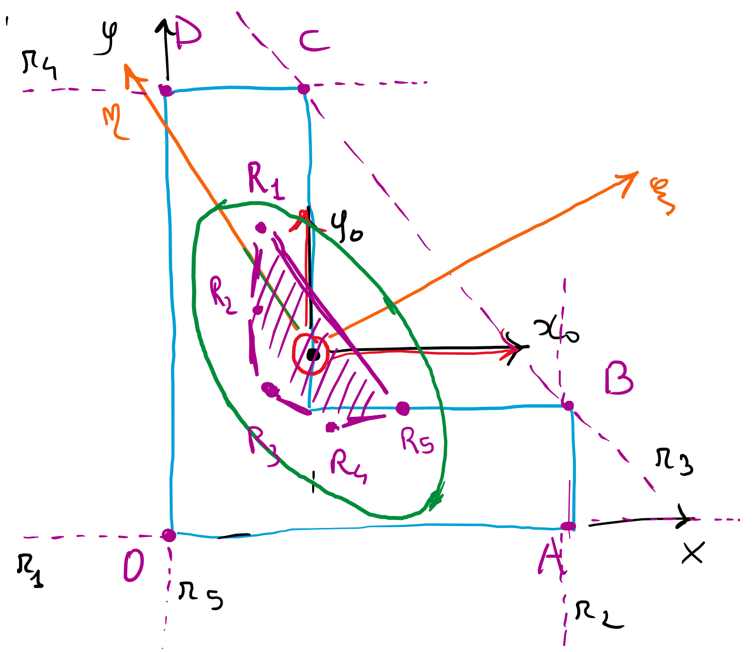
$$\frac{\eta-\eta_i}{\eta_j-\eta_i} = \frac{\xi-\xi_i}{\xi_j-\xi_i} \rightarrow \begin{cases} \bar{\xi}_K = \dfrac{\xi_i\eta_j - \xi_j\eta_i}{\eta_j-\eta_i} \\ \bar{\eta}_K = -\dfrac{\xi_i\eta_j - \xi_j\eta_i}{\xi_j-\xi_i} \end{cases}$$

PUNTI INTERSEZIONE ASSI ξ-η

DATI I PUNTI D'INTERSEZIONE CON GLI ASSI, LE COORDINATE DEI CENTRI RELATIVI SONO

$$\xi_{R,K} = -\frac{\rho_\eta^2}{\bar{\xi}_K}, \quad \eta_{R,K} = -\frac{\rho_\xi^2}{\bar{\eta}_K} \rightarrow$$

DA QUI LE COORDINATE DEGLI ANTIPOLI



$$\bar{\xi}_K = \frac{\xi_i \eta_j - \xi_j \eta_i}{\eta_j - \eta_i} \qquad \bar{\eta}_K = -\frac{\xi_i \eta_j - \xi_j \eta_i}{\xi_j - \xi_i}$$

	$\bar{\xi}_K$	$\bar{\eta}_K$
r_1	-6.708203 a	-3.354101 a
r_2	4.472135 a	-8.944271 a
r_3	2.795084 a	22.360679 a
r_4	11.180339 a	5.590169 a
r_5	-2.236067 a	4.472135 a

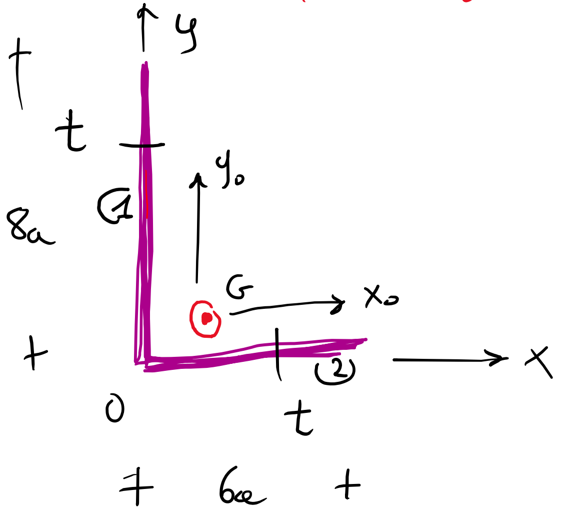
$$\xi_{R,K} = -\frac{\rho_\eta^2}{\bar{\xi}_K} \qquad \eta_{R,K} = -\frac{\rho_\xi^2}{\bar{\eta}_K}$$

	ξ_R	η_R
r_1	0.248452 a	1.98762 a
r_2	-0.372678 a	0.745356 a
r_3	- 0.596285 a	-0.298142 a
r_4	- 0.149071 a	-1.19257 a
r_5	0.745356 a	-1.49071 a

ORA SI PUÒ TORNARE INDIETRO $\begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} = \underline{\underline{R}}^T \begin{Bmatrix} \xi \\ \eta \end{Bmatrix}$

	x_{0R}	y_{0R}
R_1	-0.666667 a	1.88889 a
R_2	-0.666667 a	0.5 a
R_3	-0.4 a	-0.533333 a
R_4	0.4 a	-1.13333 a
R_5	1.33333 a	- a

SEZIONE a L - PROFILO SOTTILE



$$A = 6at + 8at = 14at$$

$$S_x = 8at \cdot 4a + \cancel{6at \cdot 0} = 32a^2t$$

$$S_y = 6at \cdot 3a + \cancel{8at \cdot 0} = 18a^2t$$

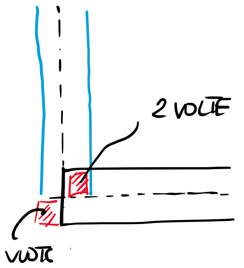
$$x_G = \frac{32a^2t}{14at} = \frac{16}{7}a$$

$$y_G = \frac{18a^2t}{14at} = \frac{9}{7}a$$

$t \ll 6a \rightarrow$ PROFILO SOTTILE

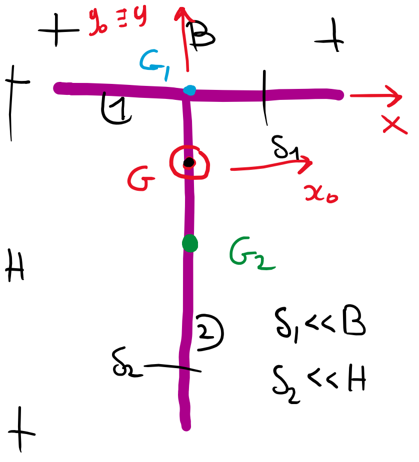
MOMENTO D'INERZIA

$$J_x = \underbrace{\frac{t(8a)^3}{12}}_{\text{MOMENTO PROPRIO PEZZO (1)}} + \underbrace{8at \cdot \left(4a - \frac{9}{7}a\right)^2}_{\text{MOM. TRASPORTO del PEZZO (1)}} + \underbrace{\cancel{\frac{6a \cdot t^3}{12}}}_{\text{MOM. PROPRIO PEZZO (2)}} + \underbrace{6at \cdot \left(\frac{9}{7}a\right)^2}_{\text{MOM. TRASPORTO PEZZO (2)}}$$



IN GENERE QUESTA PARTE SI TRASCURA !!!

$t \ll a \Rightarrow t^3 \to 0 \rightsquigarrow$ IL MOMENTO PROPRIO ATTORNO ALL'ASSE DEBOLE SI TRASCURA!!



$\delta_1 \ll B$

$\delta_2 \ll H$

$A_1 = B\delta_1$

$A_2 = H\delta_2$

$$A = B\delta_1 + H\delta_2 .$$

$G_1 = (0, 0)$

$G_2 = \left(0, -\frac{H}{2}\right)$

$x_G = 0$

$$y_G = \frac{S_x}{A} = \frac{0 + A_2 \cdot y_{G2}}{A_1 + A_2} = \frac{-\frac{H}{2} \cdot H\delta_2}{B\delta_1 + H\delta_2}$$

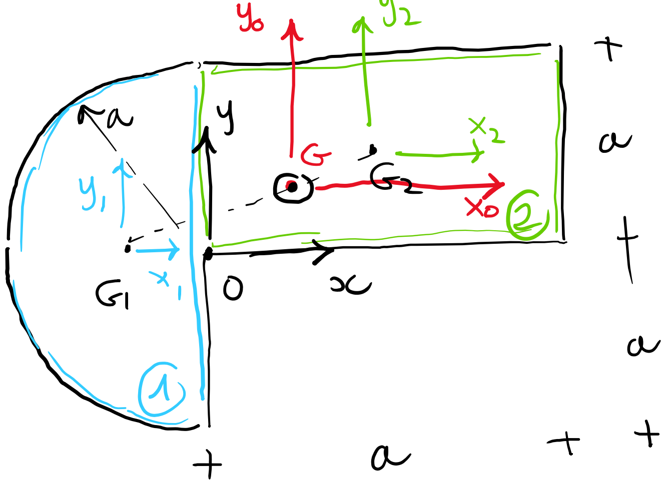
$$G = \left(0, -\frac{\delta_2 H^2}{2(B\delta_1 + H\delta_2)}\right)$$

MOMENTI D'INERZIA

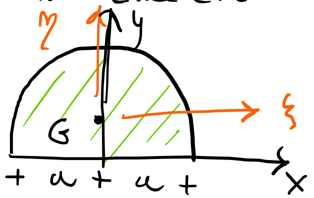
$$J_{x_0} = \underbrace{\frac{B\delta_1^3}{12}}_{\lll A_1 y_G^2 \text{ TRASCURABILE}} + A_1 y_G^2 + \frac{\delta_2 H^3}{12} + A_2\left(\frac{H}{2} - y_G\right)^2$$

$$J_{y_0} = \frac{\delta_1 B^3}{12} + \underbrace{\frac{H\delta_2^3}{12}}_{\ll \frac{\delta_1 B^3}{12}} = \frac{\delta_1 B^3}{12}$$

SEZIONE COMPOSTA



RICHIAMO SEMICERCHIO



$$y_G = \frac{4a}{3\pi}, \quad J_\xi = J_y = J_\eta = \frac{\pi a^4}{8}$$

$$A_1 = \frac{\pi a^2}{2}, \quad G_1 = \left(-\frac{4a}{3\pi}, 0\right)$$

$$A_2 = 2a^2, \quad G_2 = \left(a, \frac{a}{2}\right)$$

$$A = A_1 + A_2 = 2a^2 + \frac{\pi a^2}{2} = \frac{4+\pi}{2} a^2$$

BARICENTRO

$$x_G = \frac{S_y}{A} = \frac{A_1 x_{G1} + A_2 x_{G2}}{A} = \frac{2}{(4+\pi)a^2} \left\{ \frac{\pi a^2}{2} \left(-\frac{4a}{3\pi}\right) + 2a^2 \cdot a \right\} =$$

$$= \frac{8}{3(4+\pi)} a \simeq 0.373a$$

$$y_G = \frac{S_x}{A} = \frac{A_1 y_{G1} + A_2 y_{G2}}{A} = \frac{2}{(4+\pi)a^2} \cdot \left\{ 0 + 2a^2 \cdot \frac{a}{2} \right\} = \frac{2a}{4+\pi} \simeq 0.28a$$

MOMENTI D'INERZIA

METODO 1] $J_{x_0} = \underbrace{J^{(1)}_{x_1} + A_1 d_1^2 + J^{(2)}_{x_2} + A_2 d_2^2}_{\text{MOMENTI SINGOLE PARTI}}$ (Th HUYGENS)

METODO 2] $J_{x_0} = J_x - A y_G^2$

$\rightarrow$ MOMENTO COMPLESSIVO RISPETTO a Oxy

- PER CALCOLARE J_x e J_y USO METODO 2:

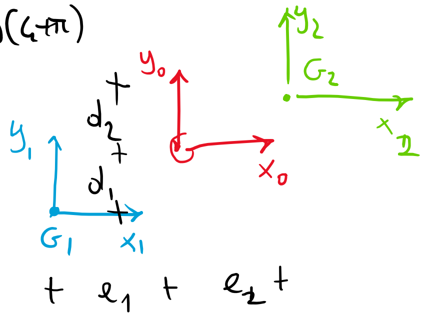
$$J_x = J_x^{(1)} + J_x^{(2)} = \frac{\pi a^4}{8} + \frac{2a \cdot a^3}{3} = \frac{3\pi + 16}{24} a^4$$

$$J_y = J_y^{(1)} + J_y^{(2)} = \frac{\pi a^4}{8} + \frac{a \cdot (2a)^3}{3} = \frac{3\pi + 64}{24} a^4$$

$$J_{x_0} = J_x - A y_G^2 = \frac{3\pi + 16}{24} a^4 - \frac{(4+\pi)a^2}{2} \cdot \frac{4a^2}{(4+\pi)^2} \simeq 0.779a^4$$

$$J_{y_0} = J_y - A x_G^2 = \frac{3\pi + 64}{24} a^4 - \frac{(4+\pi)a^2}{2} \cdot \frac{64a^2}{9(4+\pi)^2} \simeq 2.562a^4$$

- PER CALCOLARE J_{xy} USO METODO 1:



$$J_{x_0 y_0} = \underbrace{J^{(1)}_{x_1 y_1}}_{=0} + A_1 e_1 d_1 + \underbrace{J^{(2)}_{x_2 y_2}}_{=0} + A_2 e_2 d_2$$

$$e_1 = \frac{4a}{3\pi} + x_G \simeq 0.798a \qquad e_2 = -(a - x_G) \simeq 0.627a$$

$$d_1 = y_G = +0.28a \qquad d_2 = -\left(\frac{a}{2} - y_G\right) \simeq -0.220a$$

$$J_{x_0 y_0} = A_1 e_1 d_1 + A_2 e_2 d_2 = \frac{4+3\pi}{3(4+\pi)} a^4 \simeq 0.626a^4$$

SISTEMA PRINCIPALE

$$tg(2\alpha_0) = -\frac{2J_{x_0y_0}}{J_{x_0}-J_{y_0}} \rightarrow \boxed{\alpha_0 = 17.54^\circ}$$

$$\left.\begin{matrix} J_\xi \\ J_\eta \end{matrix}\right\} = \left(\frac{J_{x_0}+J_{y_0}}{2}\right) \pm \sqrt{\left(\frac{J_{x_0}-J_{y_0}}{2}\right)^2 + J_{x_0y_0}^2} = \left\{\begin{matrix} 2.759a^4 \\ 0.581a^4 \end{matrix}\right.$$

$$\rho_\xi = \sqrt{\frac{J_\xi}{A}} = 0.87a$$

$$\rho_\eta = \sqrt{\frac{J_\eta}{A}} = 0.403a$$

$J_{x_0} < J_{y_0} \Rightarrow y \rightarrow \xi$