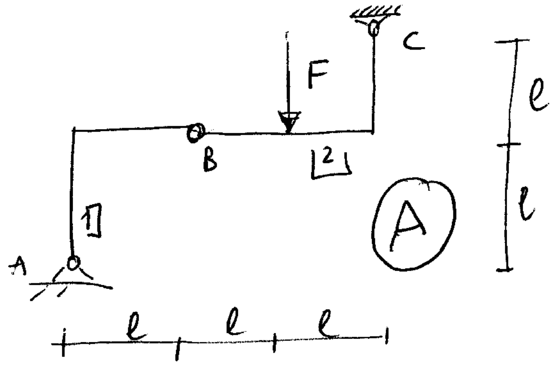
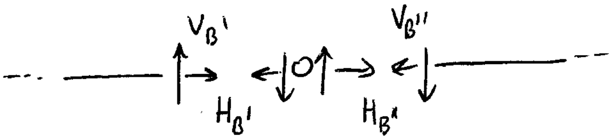
NOTE SULLE CERNIERE CARICATE CON FORZE CONCENTRATE



In questo caso le incognite statiche sono 6 : H_A, V_A, H_C, V_C V_B, H_B .

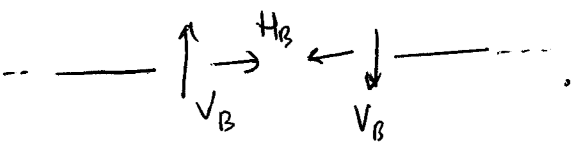
Le equaz. a disposizione sono 6 : 3 per il corpo [1], 3 per il corpo [2] .

In questo schema si da per scontato che la cerniera B sia in equilibrio :

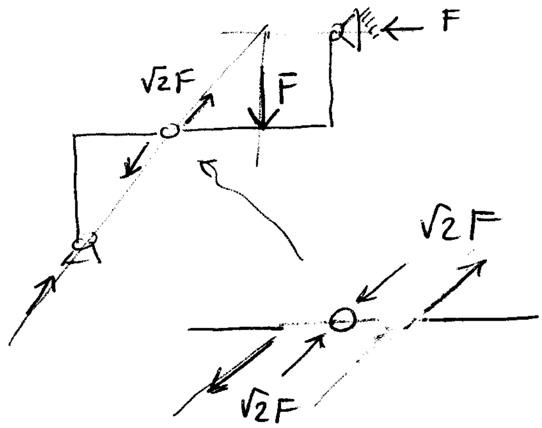


l'eq. della cerniera :

$H_{B''} = H_{B'}$; $V_{B''} = V_{B'}$ da cui



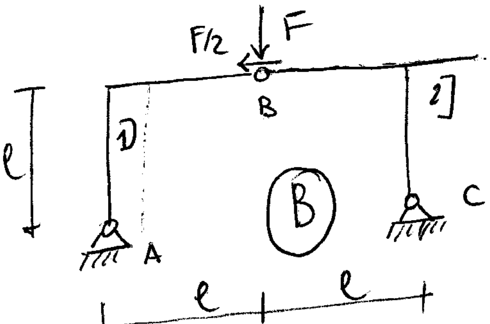
Soluz. esercizio precedente

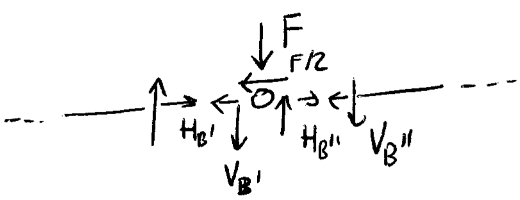


Quando la cerniera è caricata

$H_{B'} \neq H_{B''}$; $V_{B'} \neq V_{B''}$: le incognite del problema diventano 8 :

$V_A, H_A, V_C, H_C, H_{B'}, V_{B'}, H_{B''}, V_{B''}$





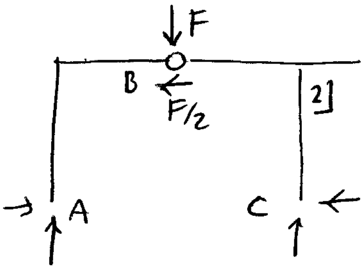
⟹

Le equazioni a disposizione sono però 8:

3 C.R. $\boxed{1}$, 3 C.R $\boxed{2}$, 2 equilibrio cerniera:

$\overset{+}{\rightarrow}$: $H_{B'} + \frac{F}{2} - H_{B''} = 0$; $+\uparrow$: $-V_{B'} - F + V_{B''} = 0$

Per risolvere l'esercizio si possono scrivere 3 eq. eq. globale + 1 ausiliaria per ottenere V_A, H_A, V_C, H_C:



$\overset{+}{\rightarrow}$: $H_A - F/2 - H_C = 0$

$+\uparrow$: $V_A + V_C - F = 0$

$A\circlearrowleft$: $V_C\, 2\cancel{\ell} + \frac{F}{2}\cancel{\ell} - F\cancel{\ell} = 0$

$M_B^{\boxed{2}}\circlearrowleft$: $V_C\,\ell - H_C\,\ell = 0$

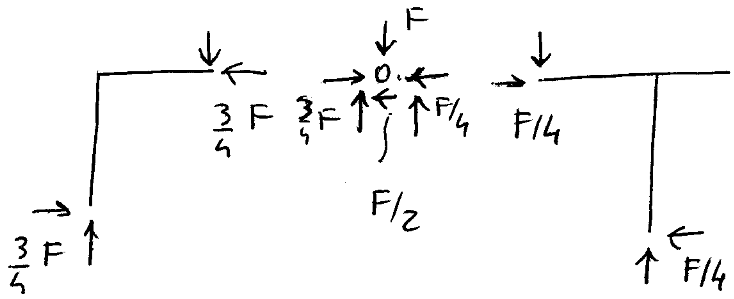
$H_A = 3/4\, F$

$V_A = F - V_C = \frac{3}{4} F$

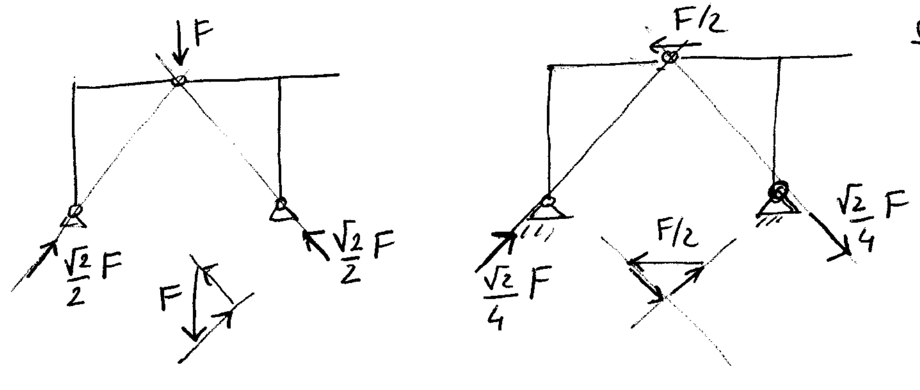
$V_C = \frac{F}{4}$

$H_C = \frac{F}{4}$

Poi si equilibrano i corpi $\boxed{1}$ e $\boxed{2}$ e si verifica che la cerniera sia anch'essa in equilibrio:

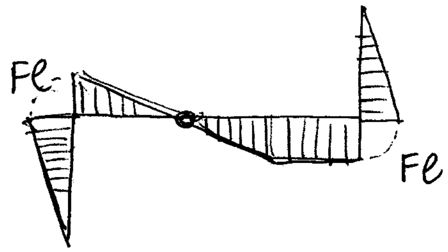


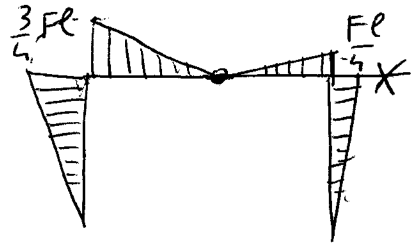
Graficamente:



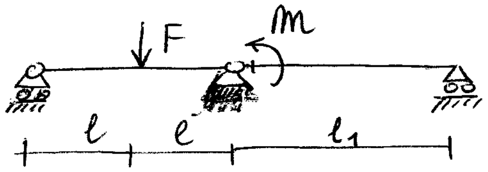
La somma dei 2 schemi corrisponde al calcolo analitico.

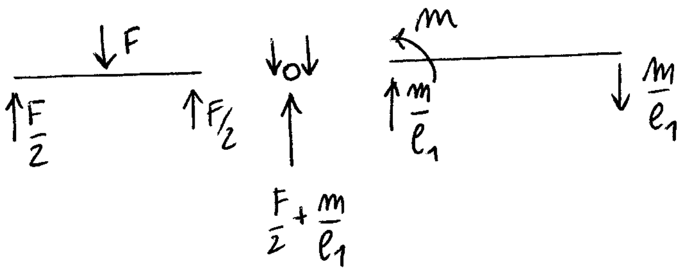
Diagrammi dei momenti delle strutture (A) e (B)

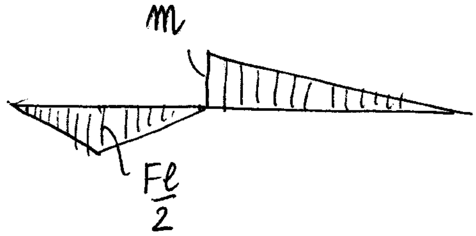




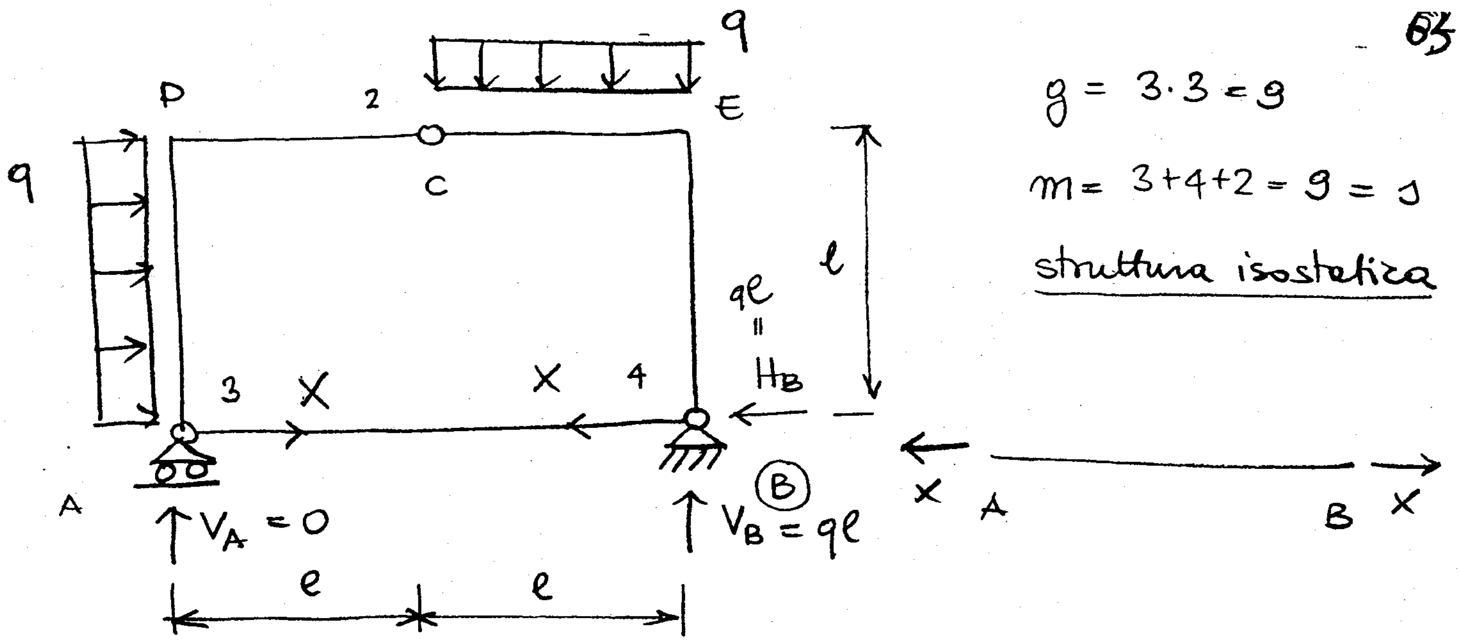
Altro esercizio semplificativo: trave gerber con vincolo interno/esterno:







(M)



$g = 3 \cdot 3 = 9$

$m = 3+4+2 = 9 = g$

struttura isostatica

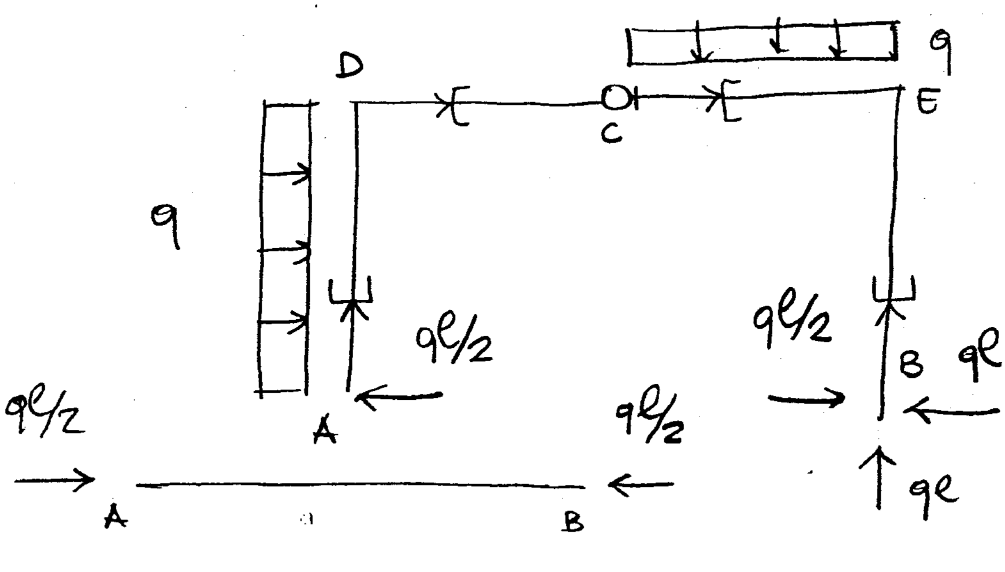
Troviamo le reazioni vincolari

$$\begin{cases} -H_B + ql = 0 & H_B = ql \\ V_A + V_B - ql = 0 & V_B = ql \\ -V_A 2l - \frac{ql^2}{2} + \frac{ql^2}{2} = 0 \Rightarrow & V_A = 0 \end{cases}$$

Apriamo la struttura, sostituendo al tratto AB l'azione interna incognita X. Per determinare X scriviamo un equazione ausiliaria per il tratto CA, ovvero $M_{CA} = 0$.

$$Xl + \frac{ql^2}{2} = 0 \Rightarrow X = -\frac{ql}{2}$$

(verso opposto a quello ipotizzato)



AB $(0 \leq x_3 \leq 2\ell)$

$$\begin{cases} N = -\frac{q\ell}{2} \\ T = M = 0 \end{cases}$$

AD $(0 \leq x_3 \leq \ell)$

$$\begin{cases} N = 0 \\ T = \frac{q\ell}{2} - q x_3 \\ M = \frac{q\ell}{2} x_3 - \frac{q x_3^2}{2} \end{cases}$$

DC $(0 \leq x_3 \leq \ell)$

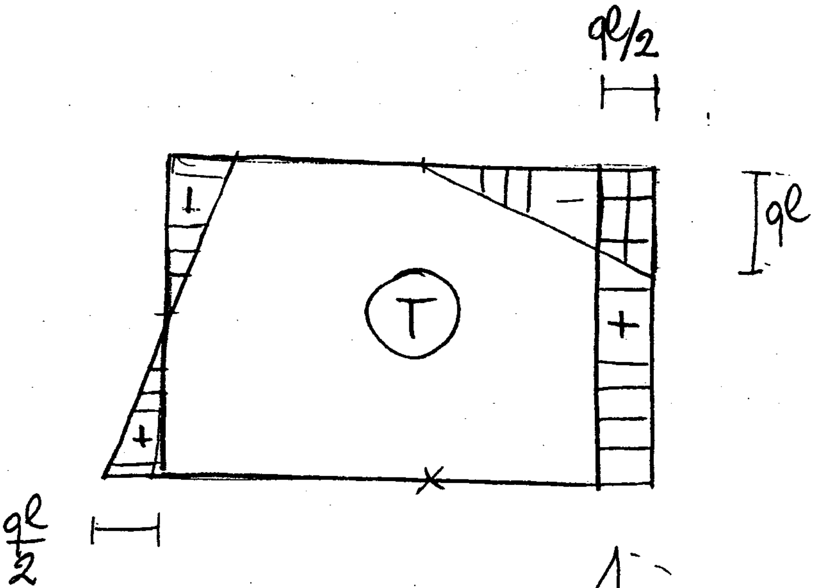
$$\begin{cases} N = -\frac{q\ell}{2} \\ T = 0 \\ M = \frac{q\ell}{2}\ell - q\ell\frac{\ell}{2} = 0 \end{cases}$$

CE $(0 \leq x_3 \leq \ell)$

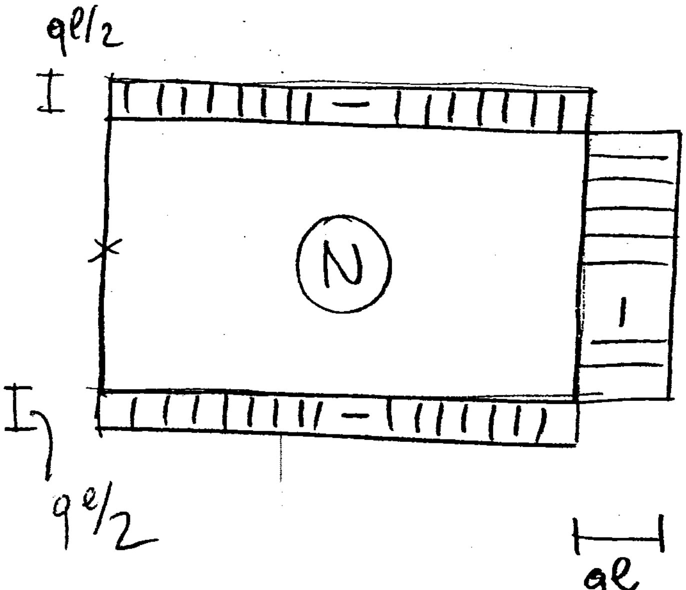
$$\begin{cases} N = -\frac{q\ell}{2} \\ T = -q x_3 \\ M = -q\frac{x_3^2}{2} \end{cases}$$

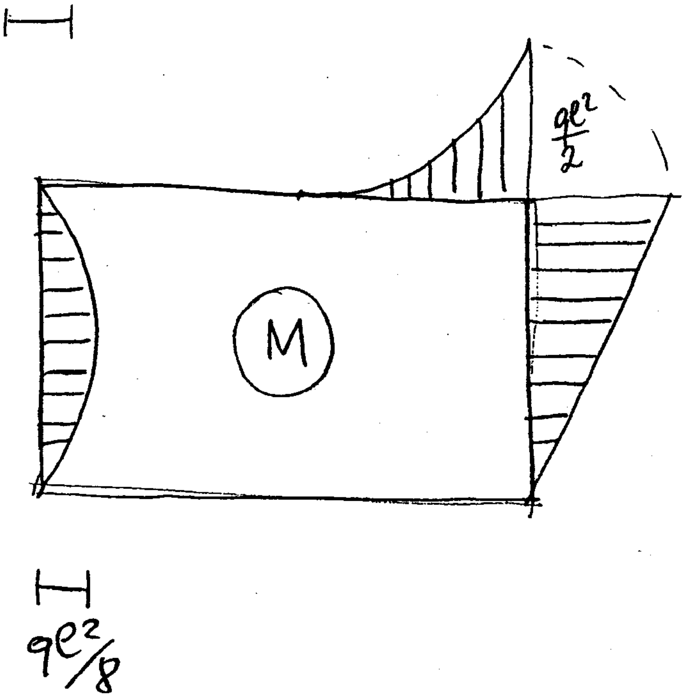
BE $(0 \leq x_3 \leq \ell)$

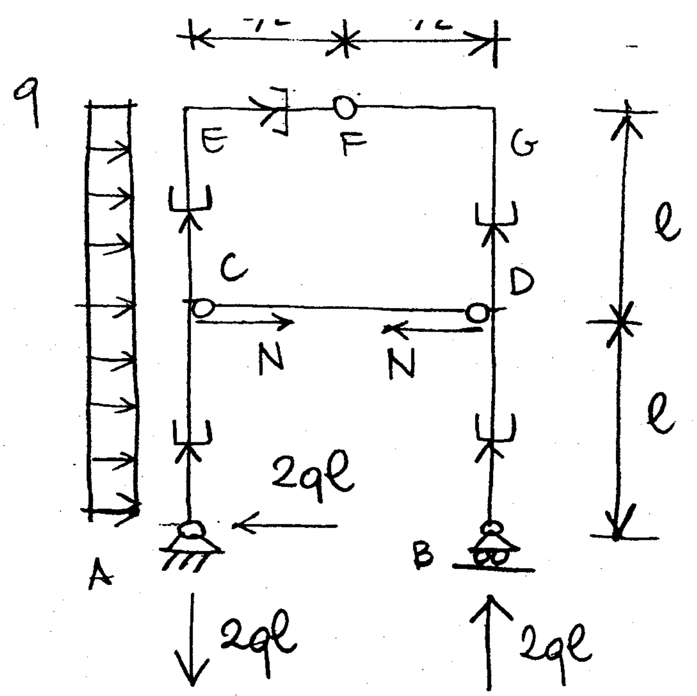
$$\begin{cases} N = -q\ell \\ T = q\ell/2 \\ M = \frac{q\ell}{2} x \end{cases}$$

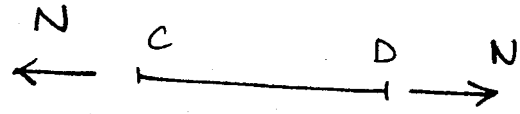


diagrammi :









Per aprire la struttura è conveniente sostituire la biella con la forza N da essa trasmessa alla struttura (1 sola incognita)

e.e.a:

$$M_{FB} = 2q\frac{\ell^{\cancel{2}}}{2} - N\cancel{\ell} = 0 \qquad N = q\ell$$

<u>AC</u> $(0 \le x_3 \le \ell)$:

$$\begin{cases} N = 2q\ell \\ T = 2q\ell - qx_3 \\ M = 2q\ell x_3 - \frac{qx_3^2}{2} \end{cases}$$

<u>CE</u> $(0 \le x_3 \le \ell)$

$$\begin{cases} N = 2q\ell \\ T = -qx_3 \\ M = 2q\ell\left(\ell + \frac{x_3}{2}\right) - \cancel{q\ell x_3} - \frac{q(\ell+x_3)}{2} \end{cases}$$

$$= \frac{3}{2}q\ell^2 - \frac{qx_3^2}{2}$$

<u>EG</u> $(0 \le x_3 \le \ell)$:

$$\begin{cases} N = -q\ell \\ T = -2q\ell \\ M = 2q\ell(\ell - x_3) - q\ell^2 \\ \quad = q\ell^2 - 2q\ell x_3 \end{cases}$$

<u>DG</u> $(0 \le x_3 \le \ell)$

$$\begin{cases} N = -2q\ell \\ T = q\ell \\ M = q\ell x_3 \end{cases}$$

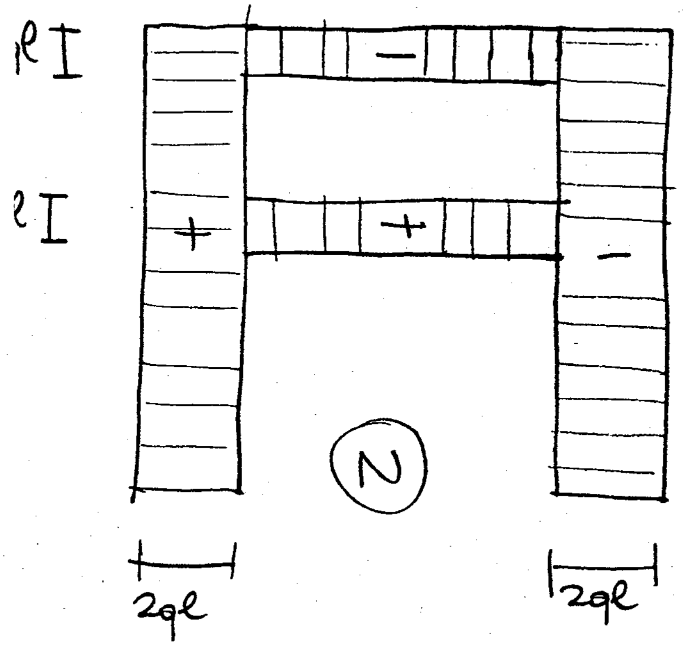
<u>BD</u>

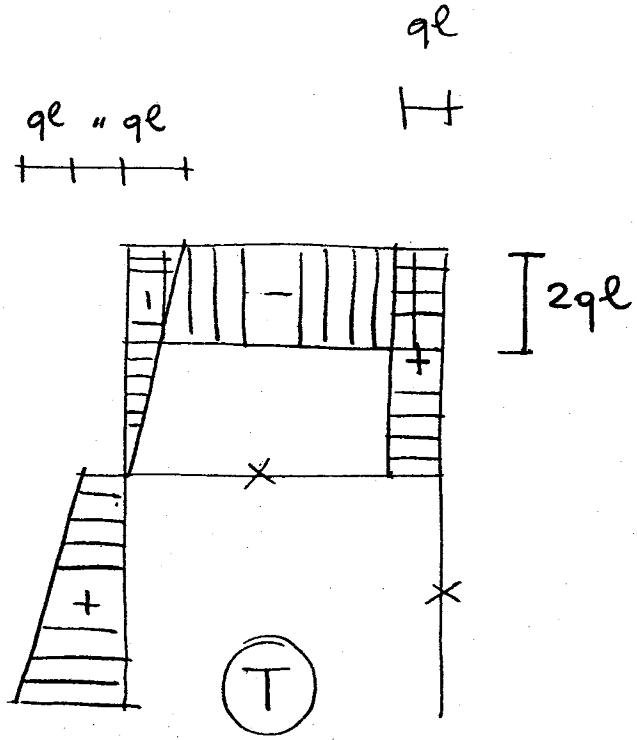
$$\begin{cases} N = -2q\ell \\ M = T = 0 \end{cases}$$

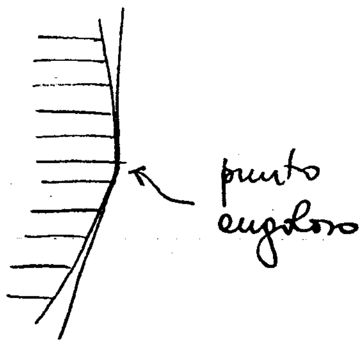
<u>CD</u>

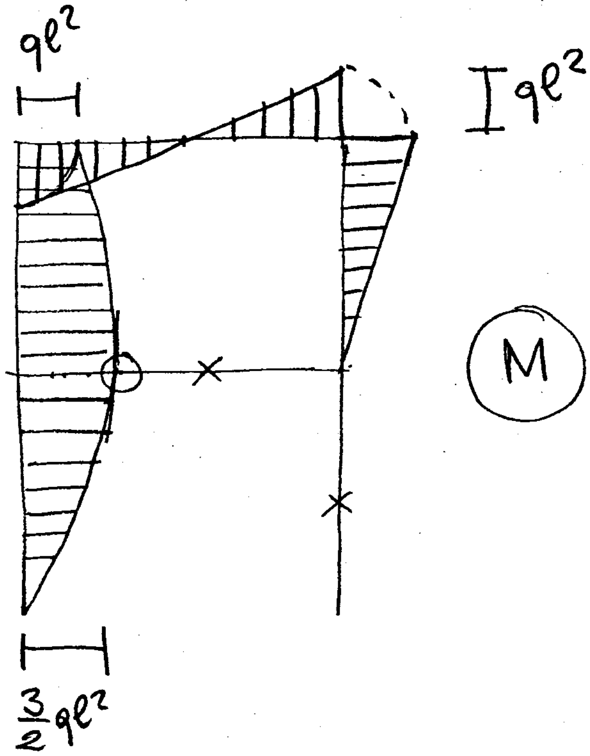
$$\begin{cases} N = q\ell \\ M = T = 0 \end{cases}$$

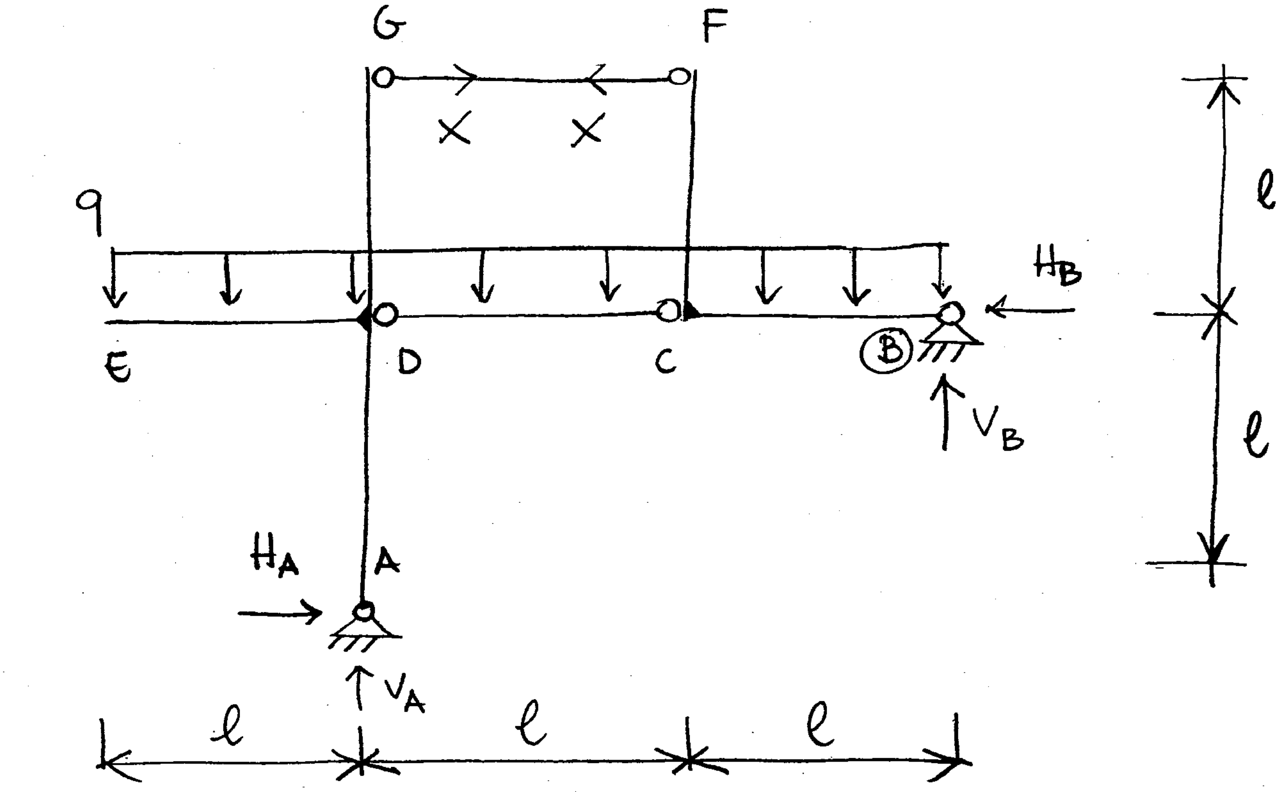
Diagrammi











e e q:

$$\begin{cases} H_A - H_B = 0 \\ V_A + V_B = 3ql \\ -V_A 2l + H_A l + \frac{9}{2} ql^2 = 0 \end{cases}$$

3 eqn nelle 4 incognite: V_A V_B H_A e H_B

e.e.a: introduco l'incognita X e scrivo 2 eea:

$$M_{CFB} = Xl + V_B l - \frac{ql^2}{2} = 0$$

$$M_{DFB} = Xl + V_B 2l - 2ql^2 = 0$$

da cui ottengo:

$$V_B = \frac{3}{2} ql \qquad X = -ql$$

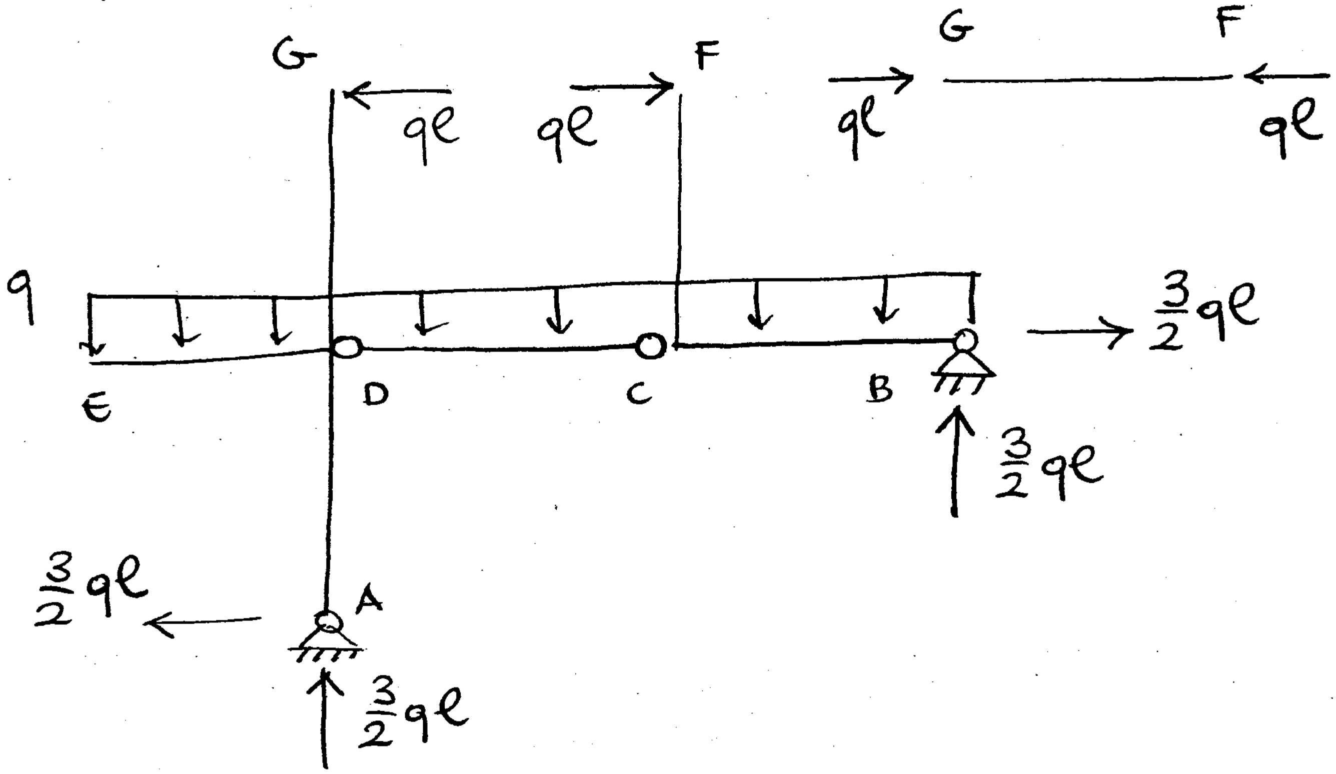
(verso contrario a quello ipotizzato)

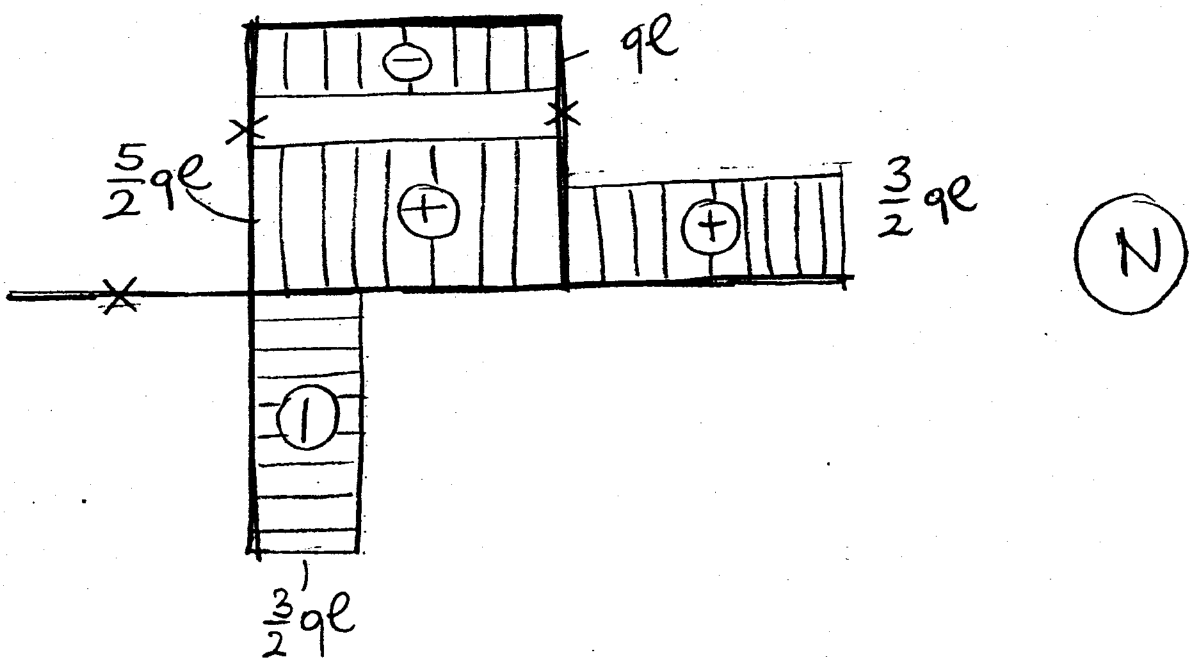
sostituendo V_B nelle e.e.g. si ottiene:

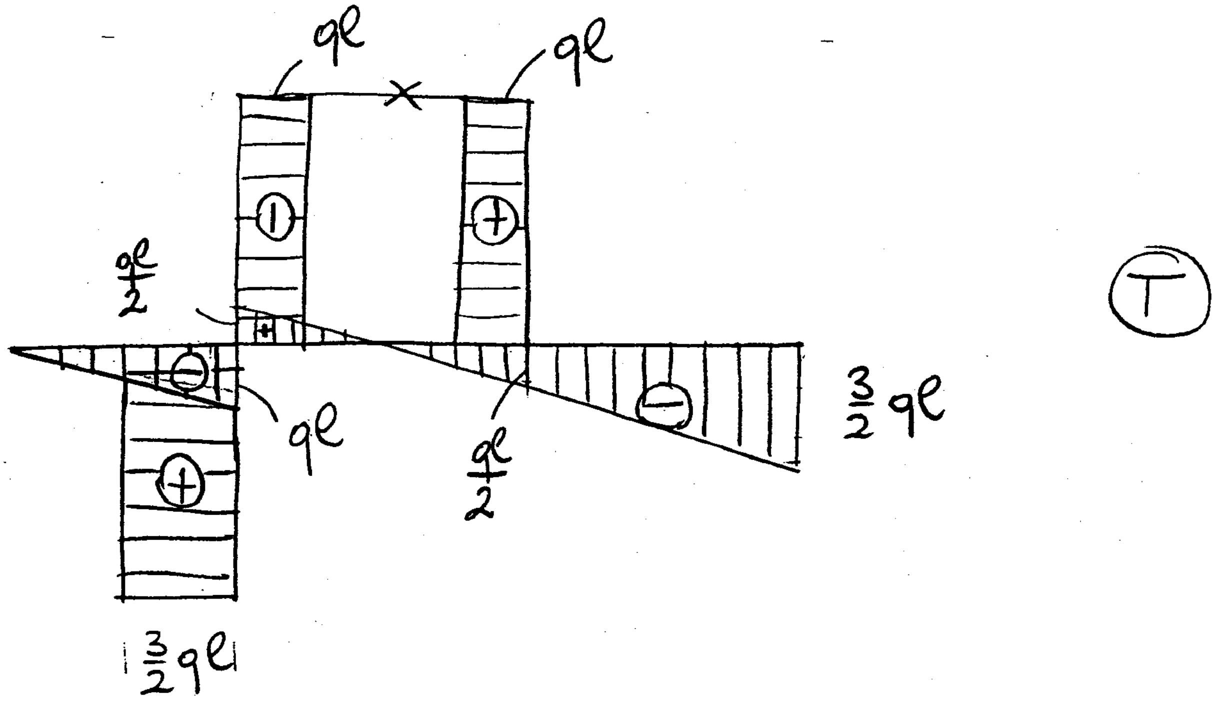
$$V_A = \frac{3}{2} q\ell$$

$$H_A = -\frac{3}{2} q\ell = H_B$$

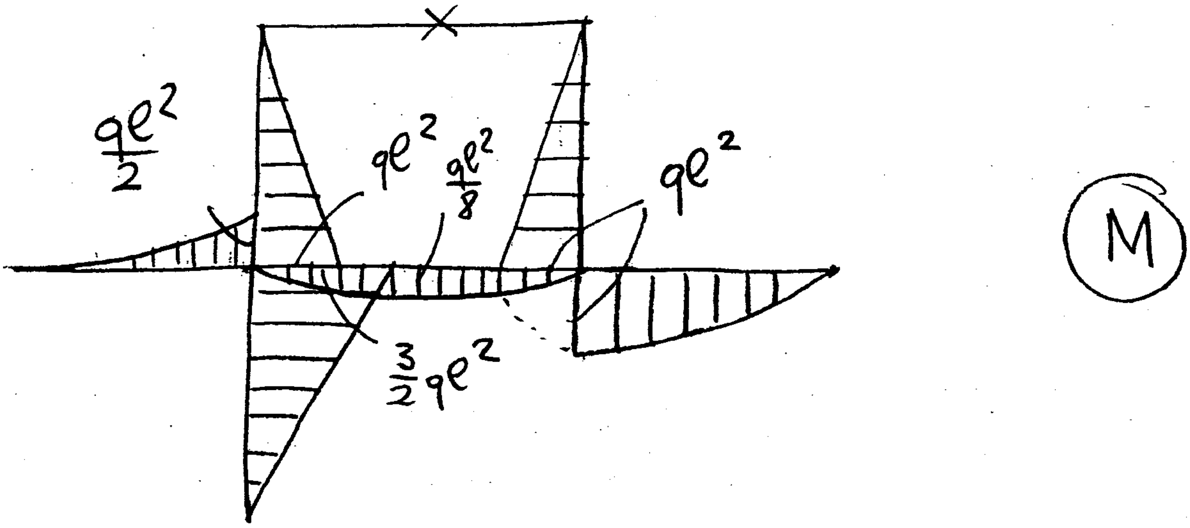
Pertanto:







ql
ql
$\frac{ql}{2}$
ql
$\frac{ql}{2}$
$\frac{3}{2}ql$
$\frac{3}{2}ql$
T



$\frac{ql^2}{2}$
ql^2
$\frac{ql^2}{8}$
ql^2
$\frac{3}{2}ql^2$
M

Le reazioni vincolari di tale struttura sono già state calcolate nell'esercizio 2 del paragrafo 1.5 e valgono:

$$V_A = \frac{pR}{4}, \quad V_B = \frac{3pR}{4}, \quad H = \frac{pR}{4}. \tag{a}$$

Assunta a qualificare la posizione della generica sezione del semiarco AC l'ascissa angolare ω ed operando con le forze a sinistra (fig. 55) si ha:

$$\begin{aligned} N(\omega) &= -H \sin\omega - V_A \cos\omega = -\frac{pR}{4}(\sin\omega + \cos\omega) \\ T(\omega) &= -H \cos\omega + V_A \sin\omega = \frac{pR}{4}(\sin\omega - \cos\omega) \\ M(\omega) &= -HR \sin\omega + V_A R(1 - \cos\omega) = \\ &= \frac{pR^2}{4}(1 - \sin\omega - \cos\omega). \end{aligned} \tag{b}$$

Le (b) sono valide in tutto l'intervallo:

$$0 \le \omega \le \frac{\pi}{2}. \tag{c}$$

Per determinare le caratteristiche nell'intervallo:

$$\frac{\pi}{2} \le \omega \le \pi \tag{d}$$

conviene operare con le forze a destra (fig. 56) assumendo la variabile ausiliaria:

$$\vartheta = \pi - \omega. \tag{e}$$

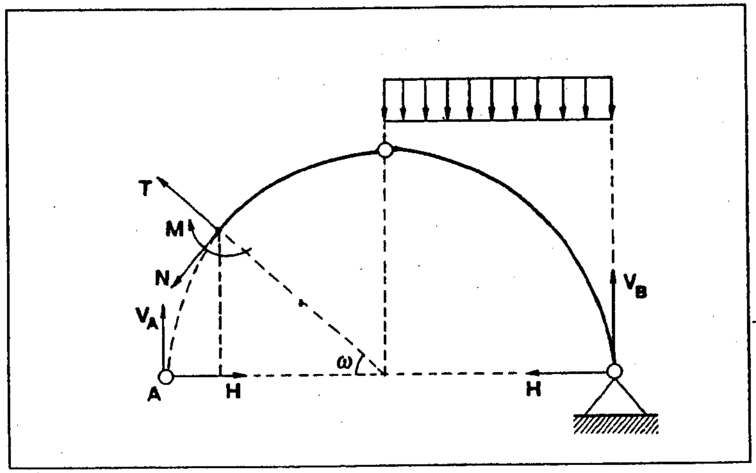


fig. 55

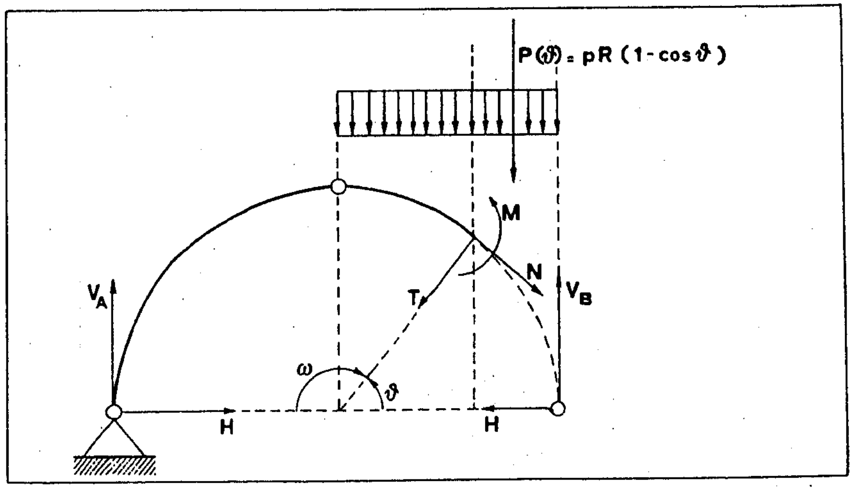


fig. 56

In questo caso fra le forze agenti lungo l'arco sotteso dall'angolo ϑ bisognerà annoverare, oltre alle reazioni vincolari della cerniera B, il carico:

(f) $$P(\vartheta) = pR(1 - \cos\vartheta)$$

che compete al segmento di arco anzidetto.

Si ha così:

(g)
$$\begin{aligned}
N(\vartheta) &= -H \sin\vartheta - V\cos\vartheta + P(\vartheta)\cos\vartheta = \\
&= -\frac{pR}{4}(\sin\vartheta - \cos\vartheta + 4\cos^2\vartheta) \\
T(\vartheta) &= H\cos\vartheta - V\sin\vartheta + P(\vartheta)\sin\vartheta = \\
&= \frac{pR}{4}(\sin\vartheta + \cos\vartheta + 4\sin\vartheta\cos\vartheta) \\
M(\vartheta) &= -HR\sin\vartheta + VR(1-\cos\vartheta) + \\
&- P(\vartheta)\frac{R(1-\cos\vartheta)}{2} = \\
&= \frac{pR^2}{4}(1 - \sin\vartheta + \cos\vartheta - 2\cos^2\vartheta).
\end{aligned}$$

Riscrivendo le (g) in conformità con la (e) si ha in definitiva:

(h)
$$\begin{aligned}
N(\omega) &= -\frac{pR}{4}(\sin\omega + \cos\omega + 4\cos^2\omega) \\
T(\omega) &= \frac{pR}{4}(\sin\omega - \cos\omega + 4\sin\omega\cos\omega) \\
M(\omega) &= \frac{pR^2}{4}(1 - \sin\omega - \cos\omega - 2\cos^2\omega).
\end{aligned}$$

Le (h), valide in tutto l'intervallo (d), consentono, insieme alle (b) di valutare le caratteristiche della sollecitazione in una qualunque sezione della struttura in esame.

1.10. IL CALCOLO DELLE AZIONI INTERNE NELLE TRAVATURE RETICOLARI PIANE

Un cenno a parte meritano le travature reticolari che, com'è noto, sono strutture variamente vincolate esternamente costituite da elementi rettilinei o curvilinei (*aste*) articolate alle estremità in punti detti *nodi* nei quali si ritengono applicate le forze esterne (fig. 57).

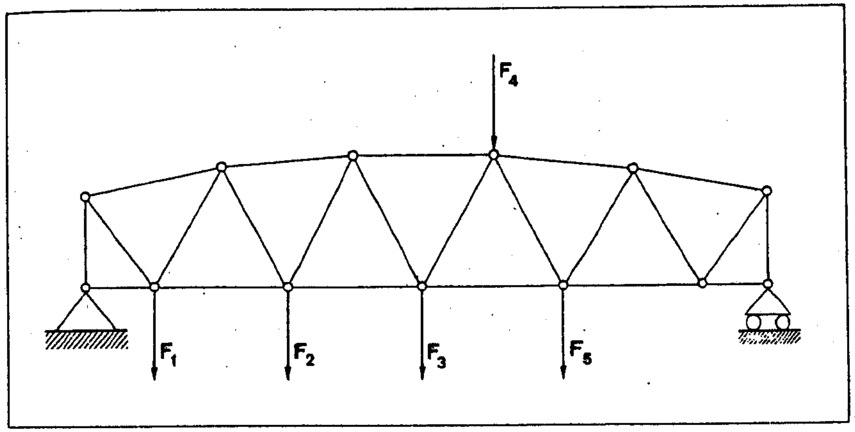


fig. 57

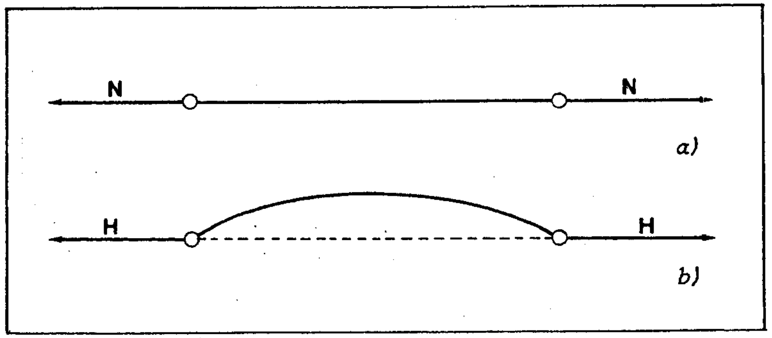


fig. 58