Image Processing for Physicists

Prof. Pierre Thibault
pthibault@units.it

Image manipulation in the spatial domain

Overview

- coordinate transformations

  – translation, rotation, shear, ... *geometry*

- intensity transformations

  – normalization, gamma, thresholding, ... *acting on the pixel values*

- image analysis using morphological operations

  – dilation, erosion, opening, closing, ...

- image segmentation

  – by morphology, intensity, region, ...

  operations on binary images ("masks")

General image transformations

- coordinate transformations $\rightarrow$ *change pixel positions*
  $\hookrightarrow$ *implies interpolation*
  most of the time

- intensity transformations $\rightarrow$ *change pixel values*

- pixel-wise transformations

- neighborhood transformations } *morphology*
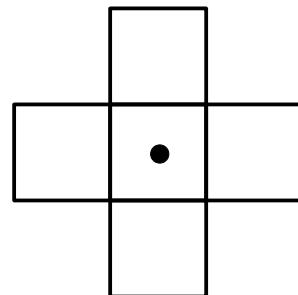

4-neighborhood   8-neighborhood

Image manipulations in the spatial domain
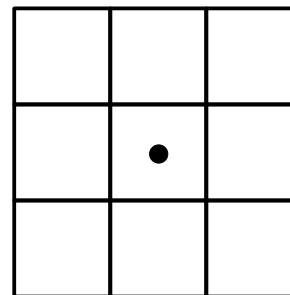
General image transformations

- images as an array

$$M \times N \; (\times \; \text{channels})$$

M

N

- sub array operations

x_1 x_2

y_1

y_2

region of interest

ROI
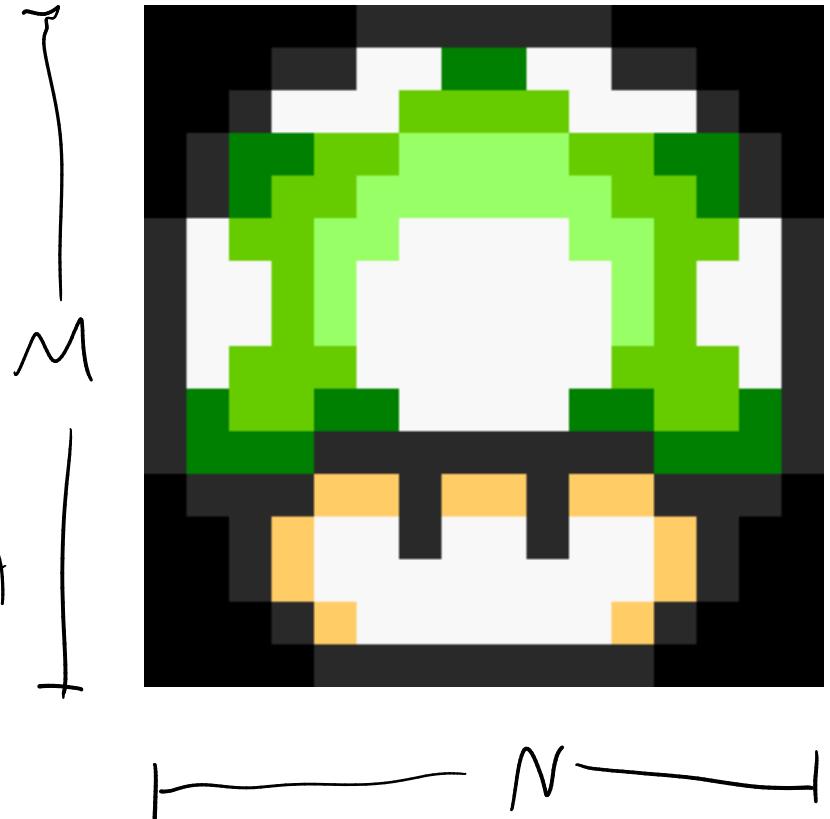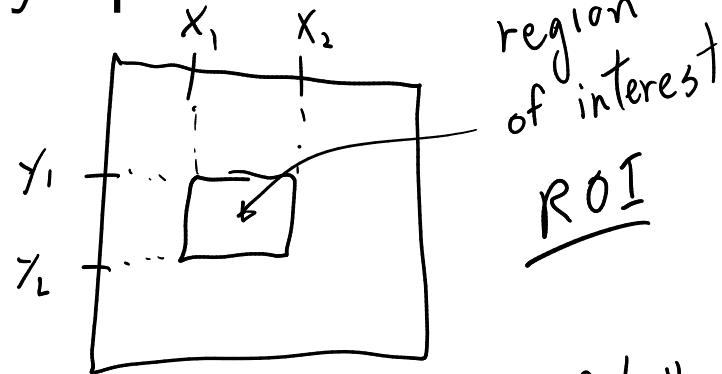
- line extractions = "line profile"

s

intensity

s
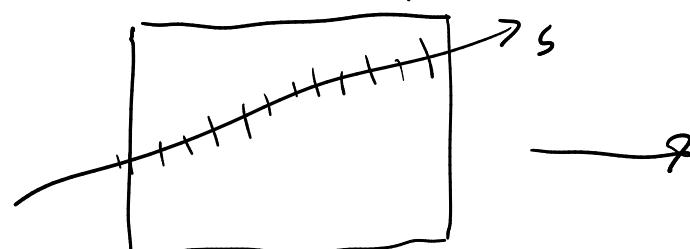
General image transformations

- element wise addition

$$I = I_1 + I_2$$

$$I(i,j) = I_1(i,j) + I_2(i,j) \quad \forall_{i,j}$$

$$\hookrightarrow \text{also subtraction}$$

- element wise multiplication

$$I = I_1 \times I_2$$

$$I(i,j) = I_1(i,j) \times I_2(i,j)$$

$$\hookrightarrow \text{also division}$$

Image manipulations in the spatial domain

Image Subtraction Example

- Digital Subtraction Angiography

- Xray images before/after contrast agent

blood vessels
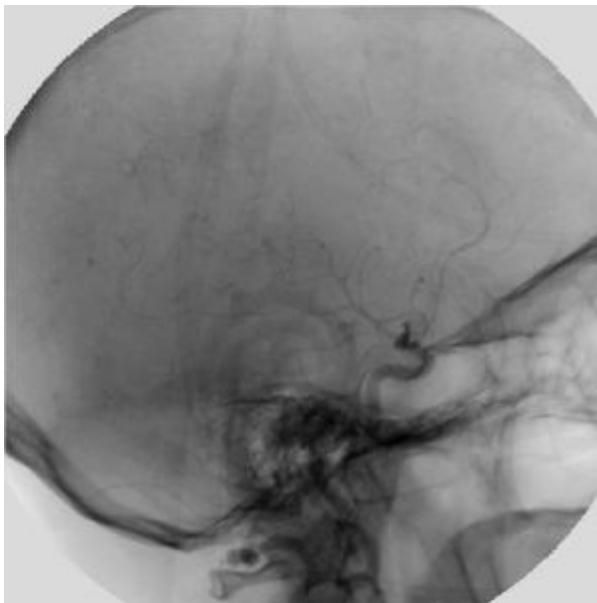
Live or contrast image

Mask image

DSA image

motion artifacts

Source: Gonzales, Digital Image Processing

Image manipulations in the spatial domain
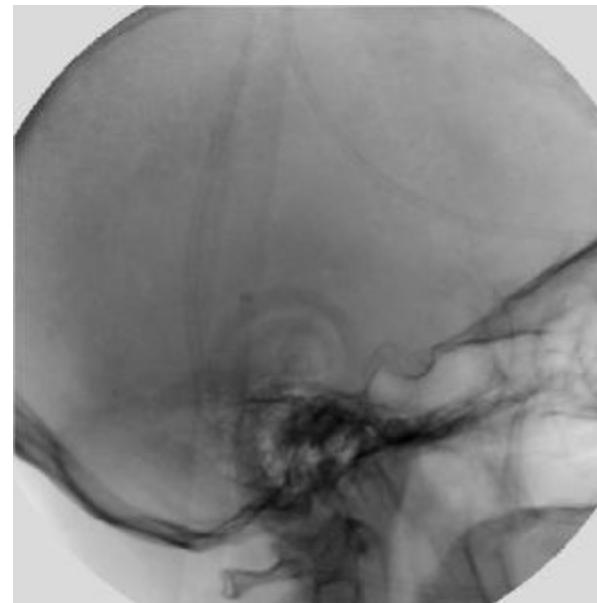
Image Addition Example

- Add multiple noisy images of same object

- (More on noise in later lectures)
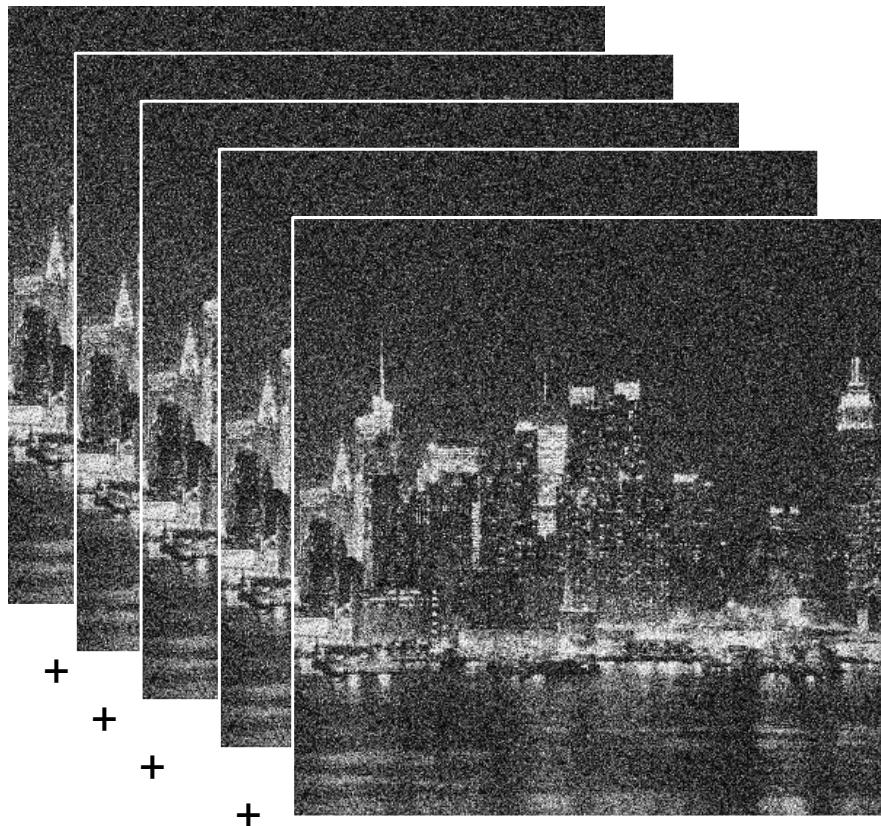
a possible way to improve signal-to-noise ratio

=

Image Multiplication Example

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Another common example: flat correction

1) image without subject, flat illumination "flat field" I_o

2) image with subject I_s

→ I_s/I_o might reduce effects of inhomogeneity in the illumination

Source: Gonzales, Digital Image Processing

Image manipulations in the spatial domain

Affine transformations

- identity

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
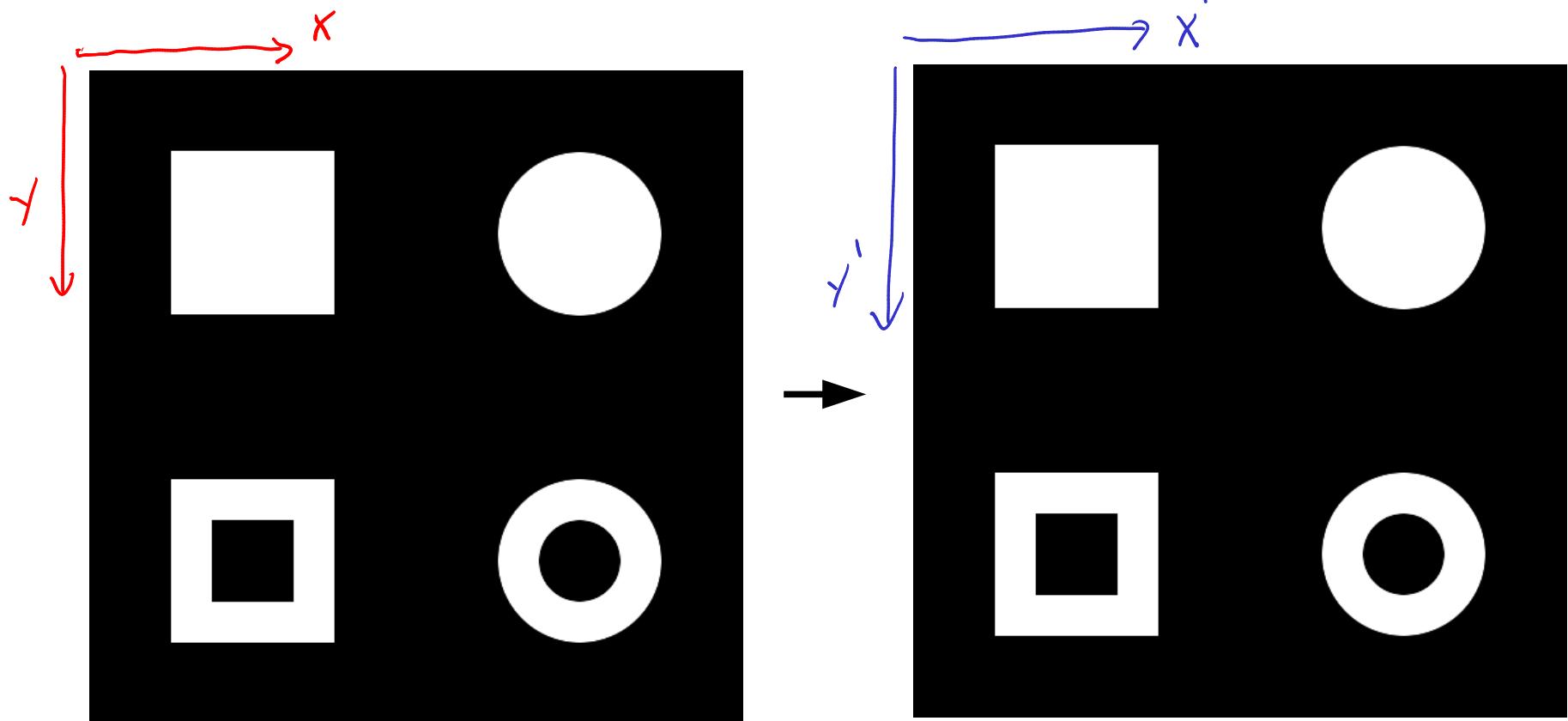
Affine transformations

- translation

linear transformation

translation vector ("offset")

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$
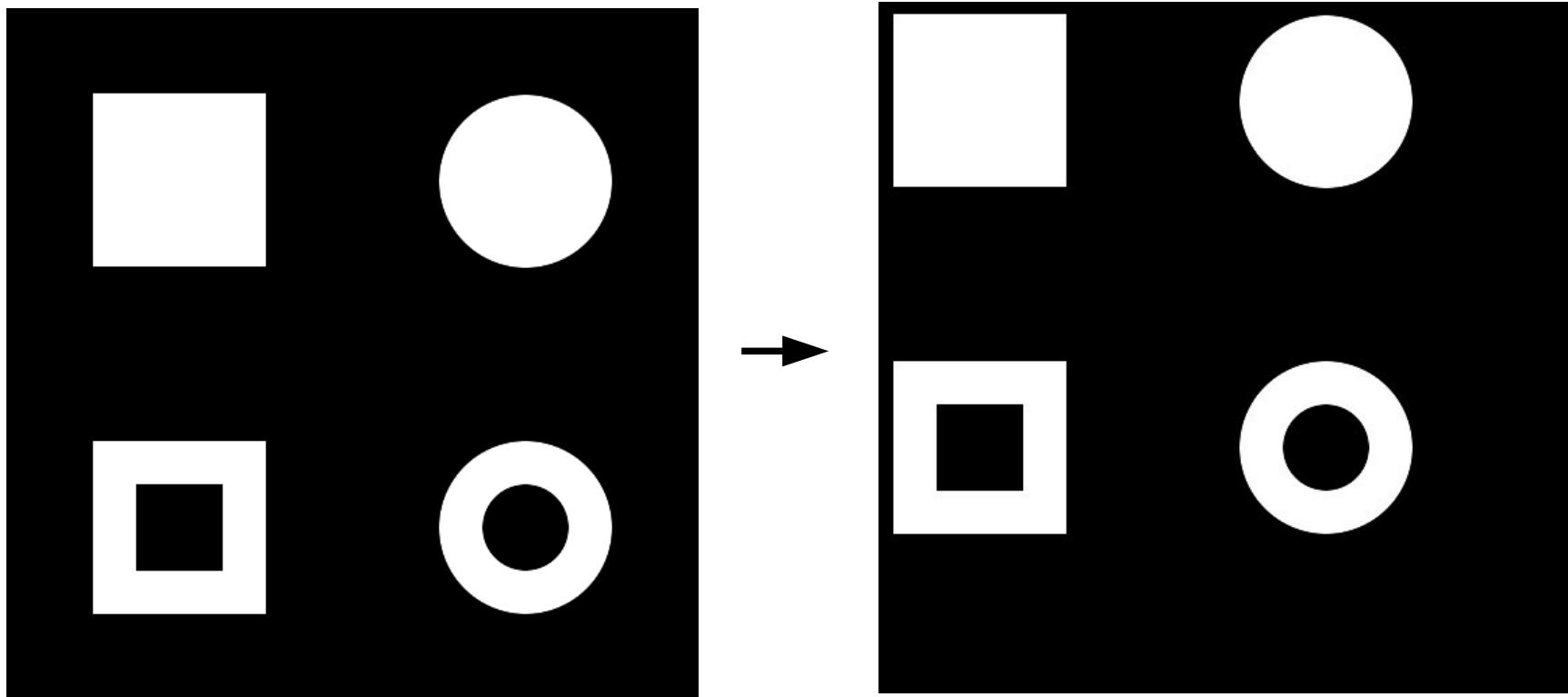
Affine transformations

- scaling

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x' = a x$$
$$y' = b y$$

$$a = b = \tfrac{1}{2}$$

Image manipulations in the spatial domain
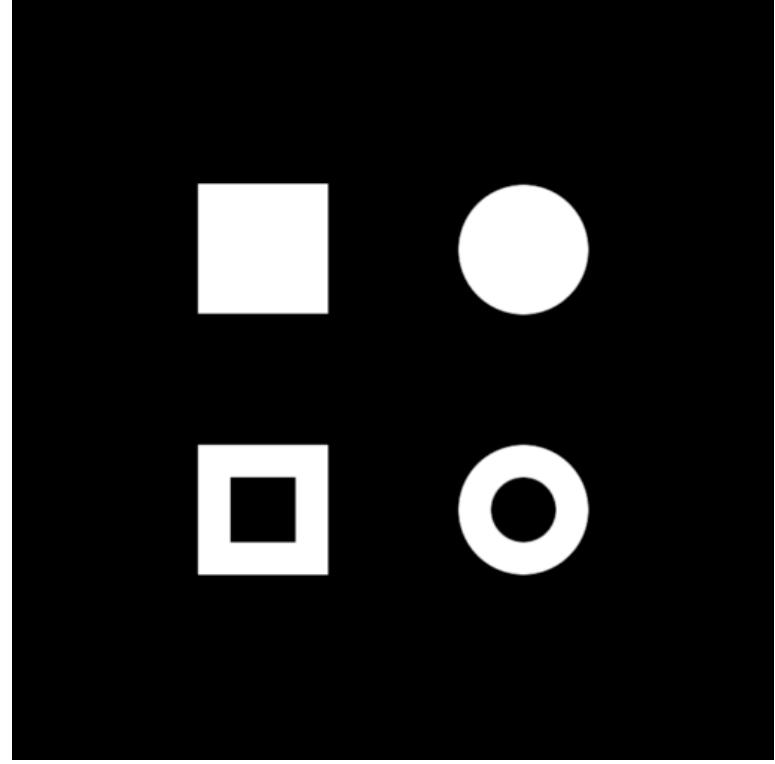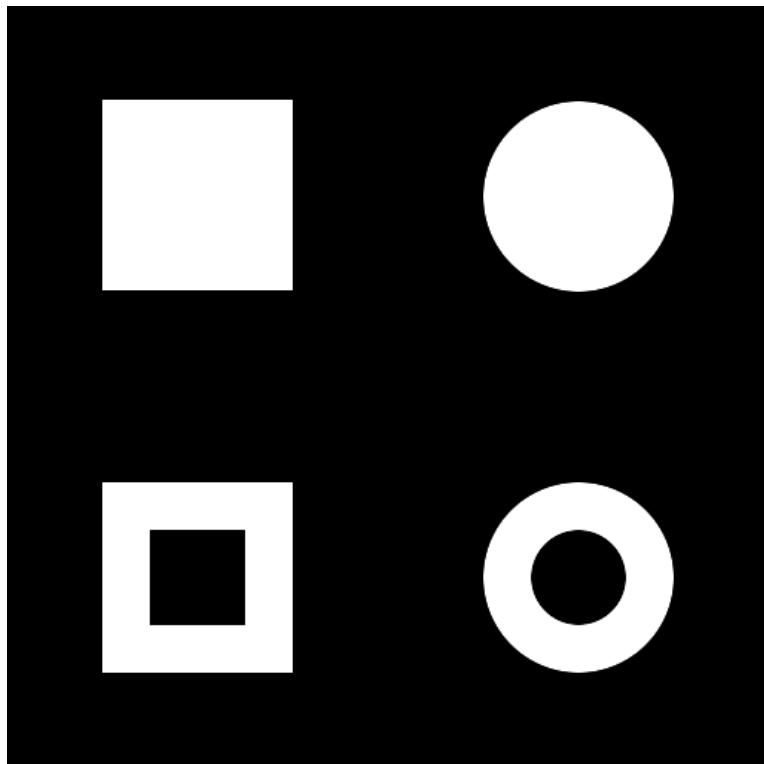
Affine transformations

- reflections

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$x' = y$

$y' = -y$
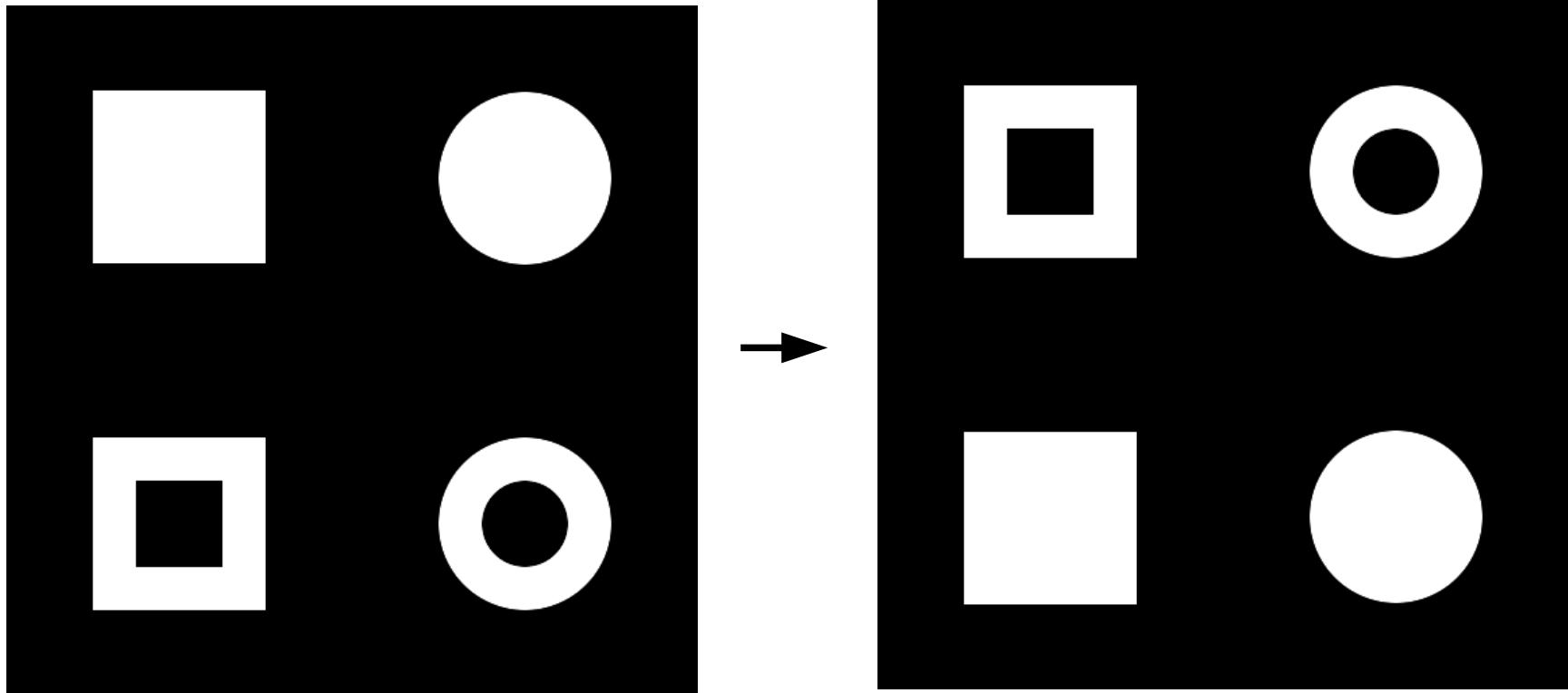
Affine transformations

- rotation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Image manipulations in the spatial domain
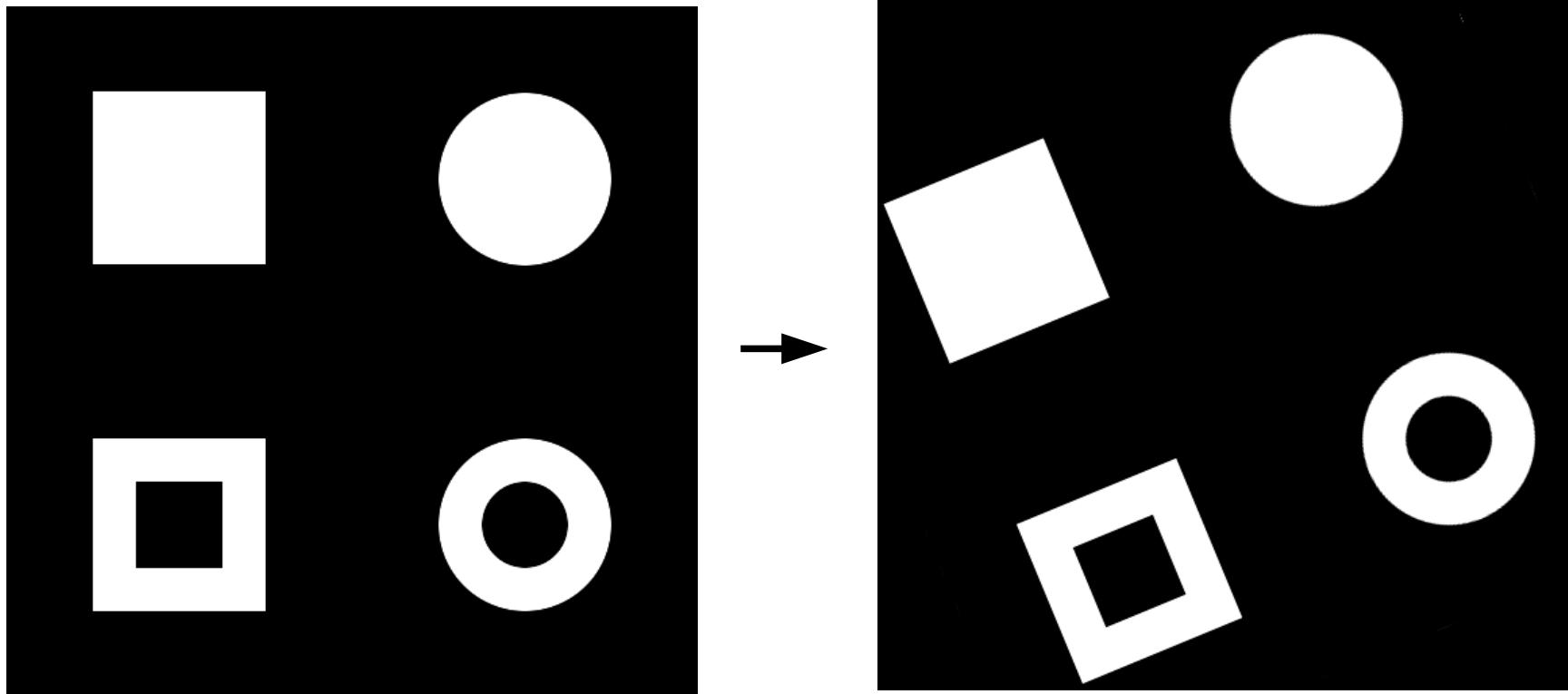
Affine transformations

- shear

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Image manipulations in the spatial domain
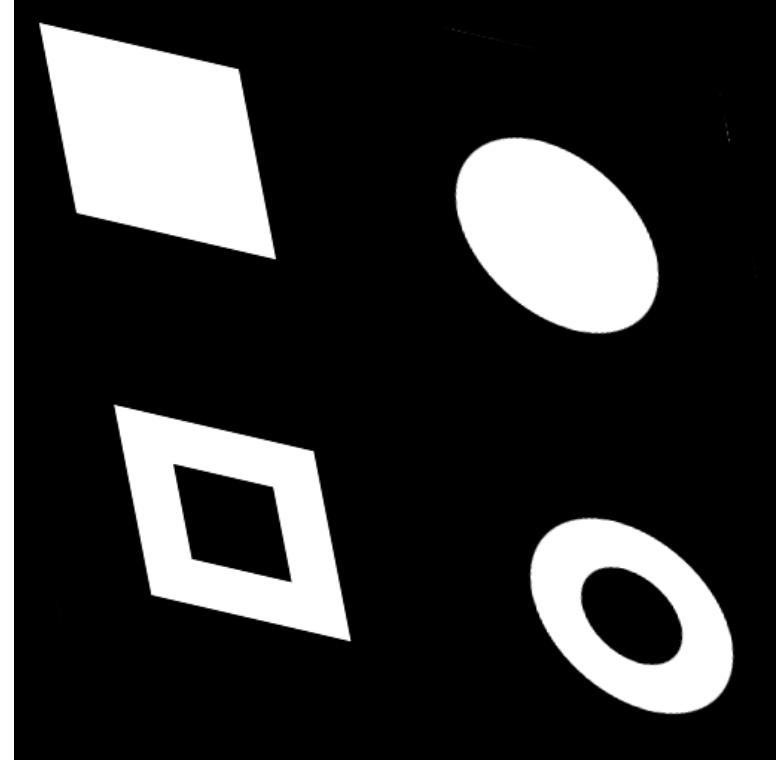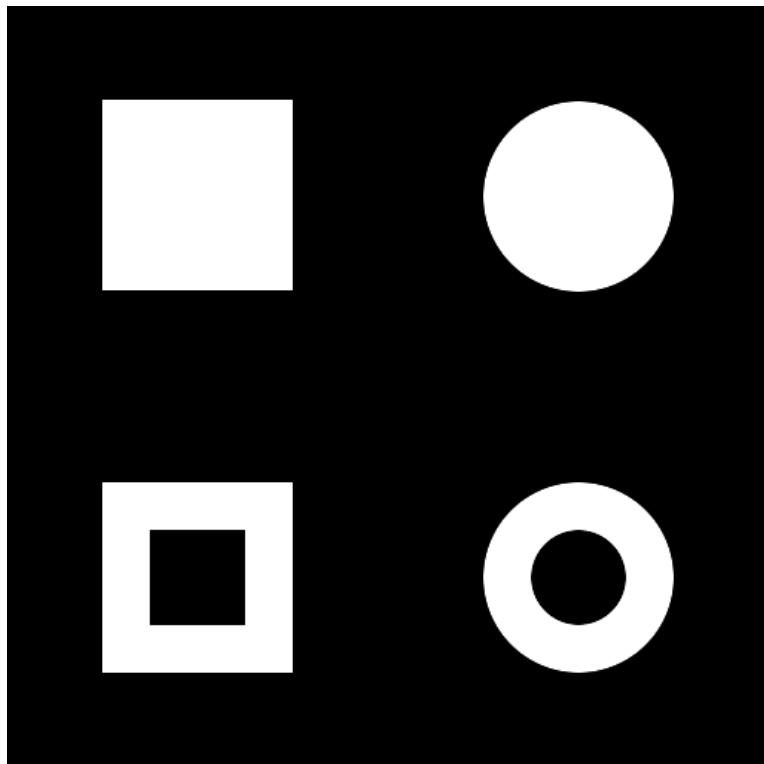
Nonlinear coordinate transformation

- pincushion and barrel distortion
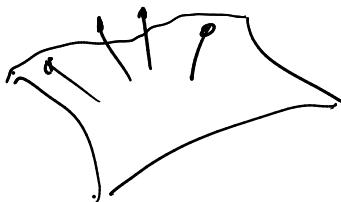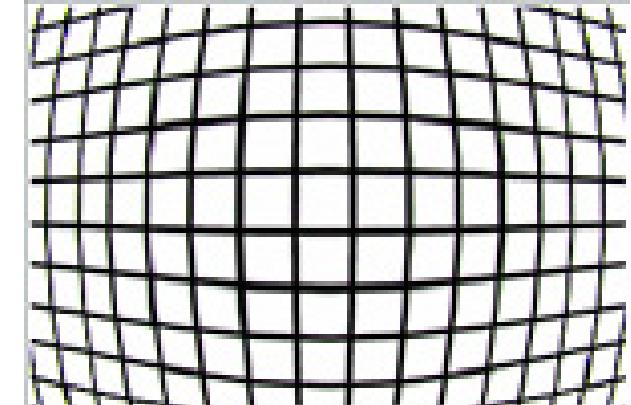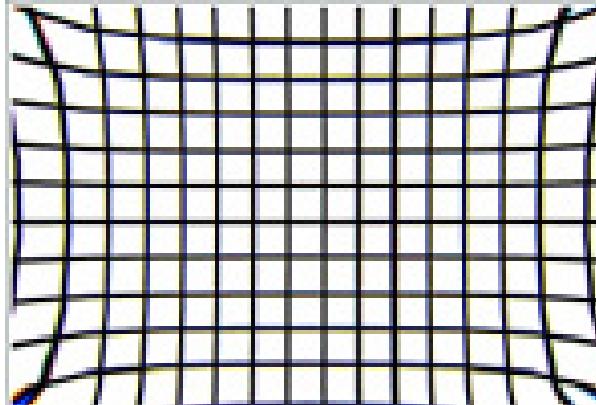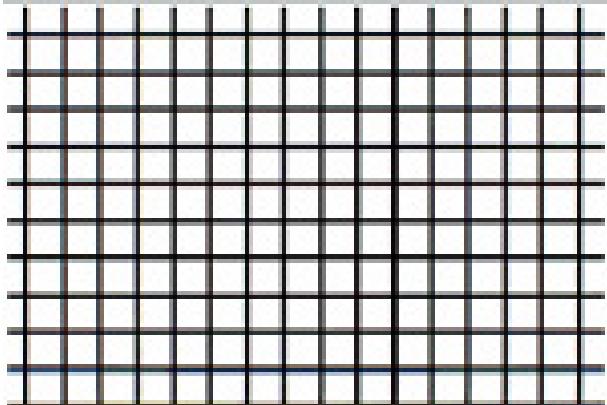
- mapping depends on radial distance from centre

$$x' = x_0 + ax + by + cx^2 + dxy + ey^2 + \ldots$$

$$y' = y_0 + fx + gy + hx^2 + ixy + jy^2 + \ldots$$

Image manipulations in the spatial domain

Intensity mapping

output values

255

intensity of each pixel

0

255

input values
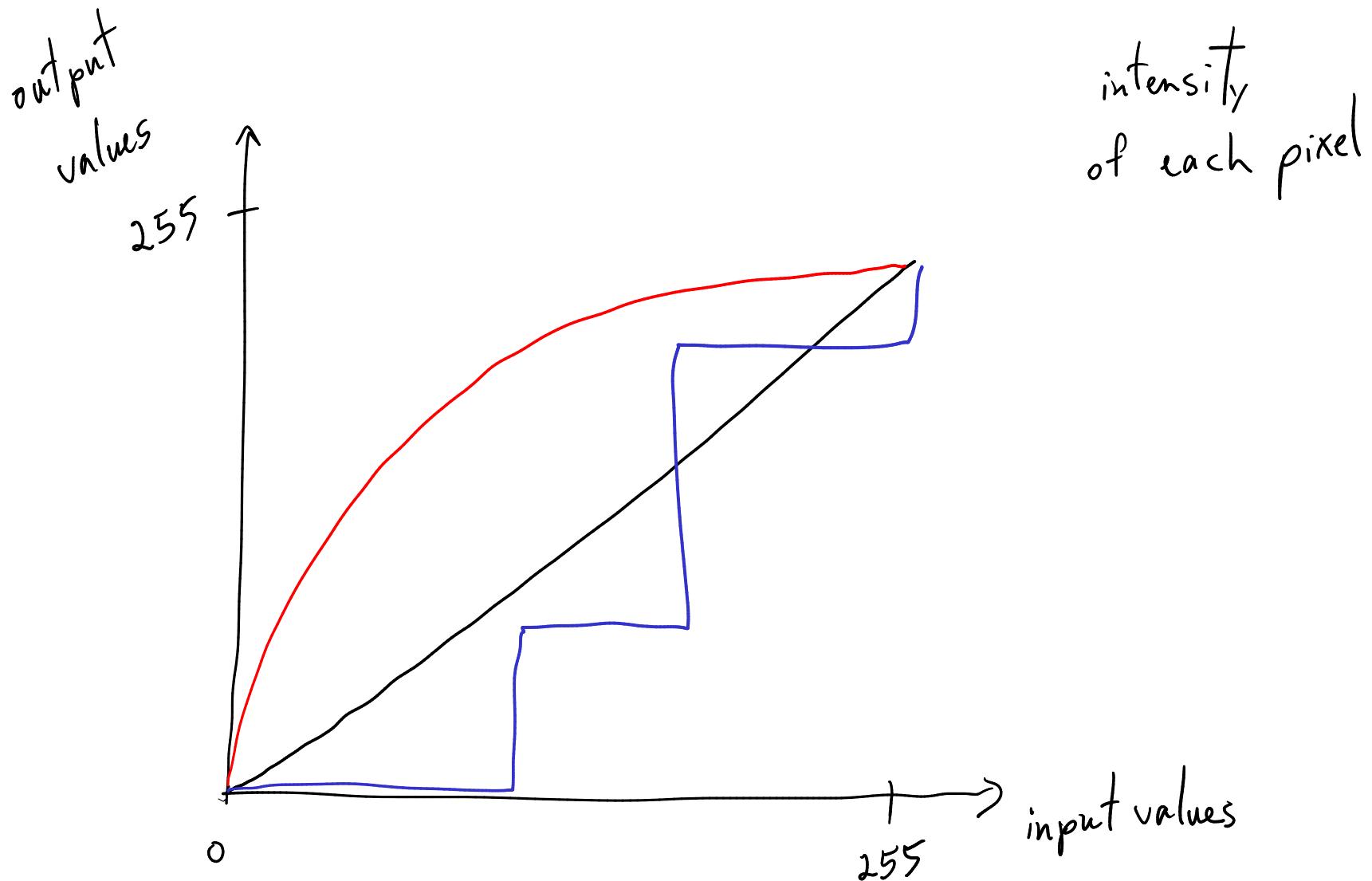
Intensity mappings

Identity

original   remapped

Image manipulations in the spatial domain

Intensity mappings

Brightness

overflow = saturation

output = input + B

output intensity

input intensity

original   remapped

Image manipulations in the spatial domain
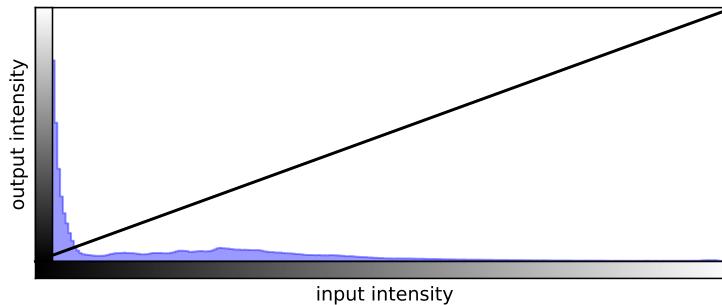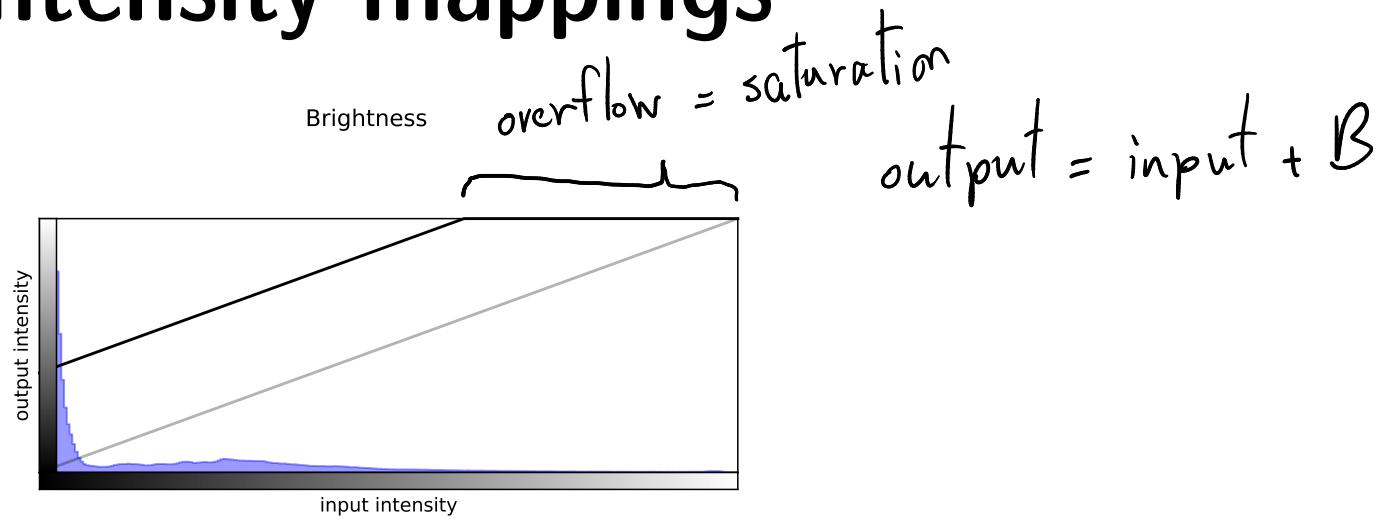
Intensity mappings

Contrast
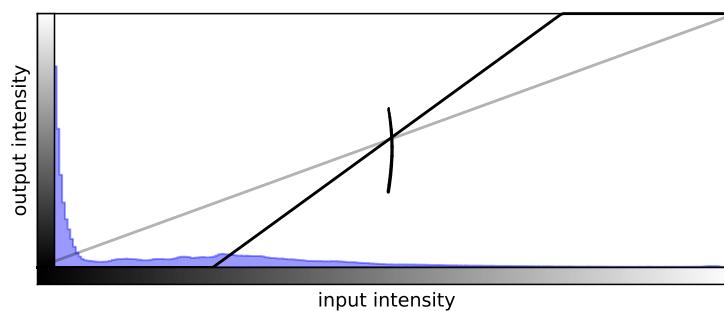
$$\text{output} = C\,(\text{input} - 128) + 128$$

original   remapped

Image manipulations in the spatial domain

Intensity mappings

Gamma

$\gamma < 1$

$\gamma > 1$

output intensity

input intensity

$$\text{output} = 255 \left(\frac{\text{input}}{255} \right)^{\gamma}$$

original

remapped

Image manipulations in the spatial domain
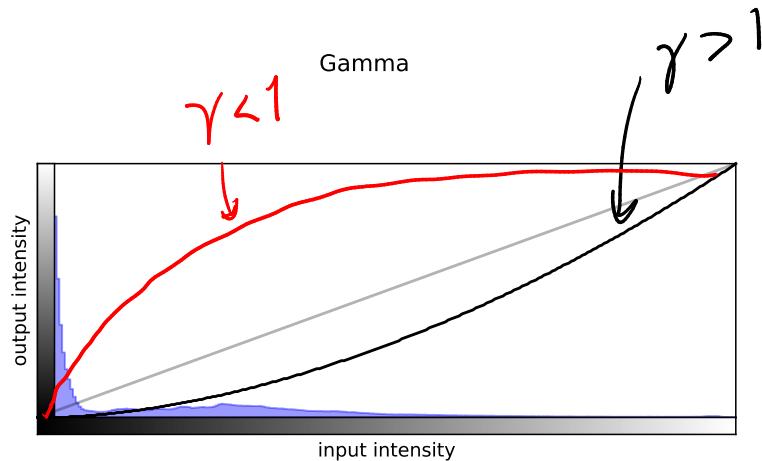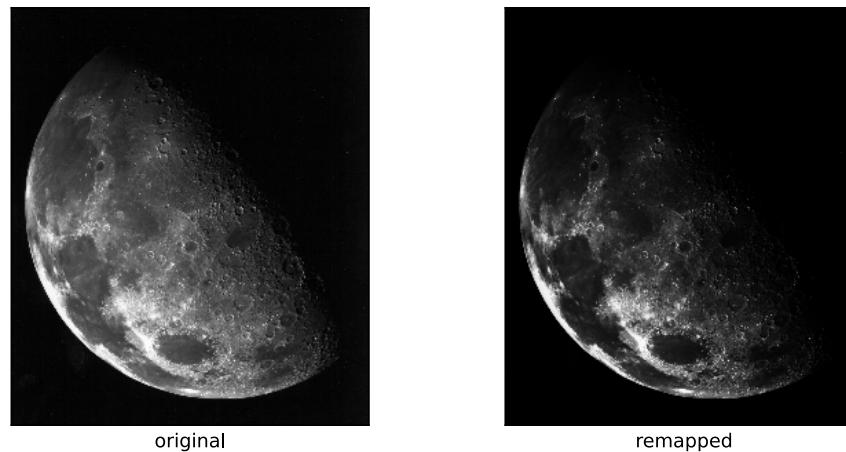
Intensity mappings

Inversion

output =

255 - input

original   remapped

Image manipulations in the spatial domain

Intensity mappings

Threshold
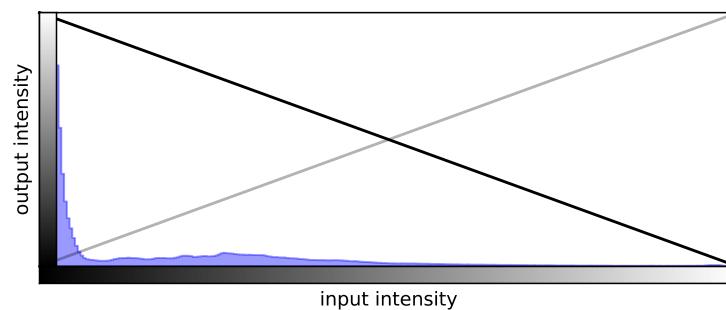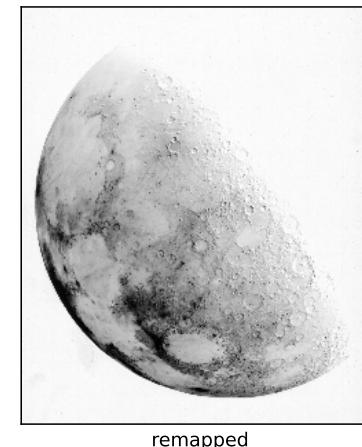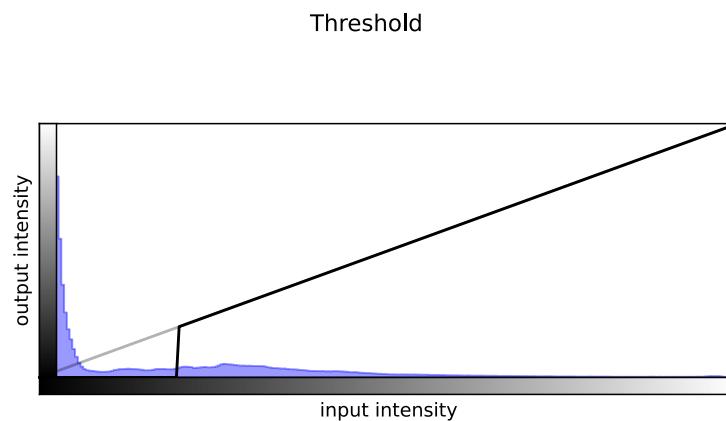
$$\text{output} = \begin{cases} 0 & \text{if input} < A \\ \text{input} & \text{otherwise} \end{cases}$$

original

remapped

Image manipulations in the spatial domain

Intensity mappings

Binary threshold
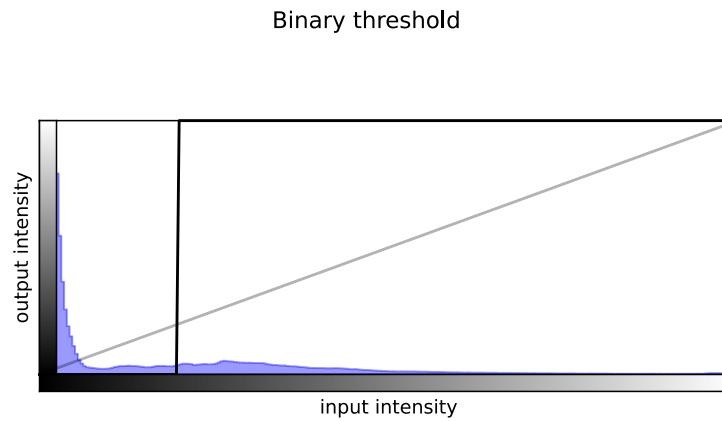
$$\text{output} = \begin{cases} 0 \text{ if input} < A \\ 255 \text{ if input} \geq A \end{cases}$$

original   remapped

Image manipulations in the spatial domain
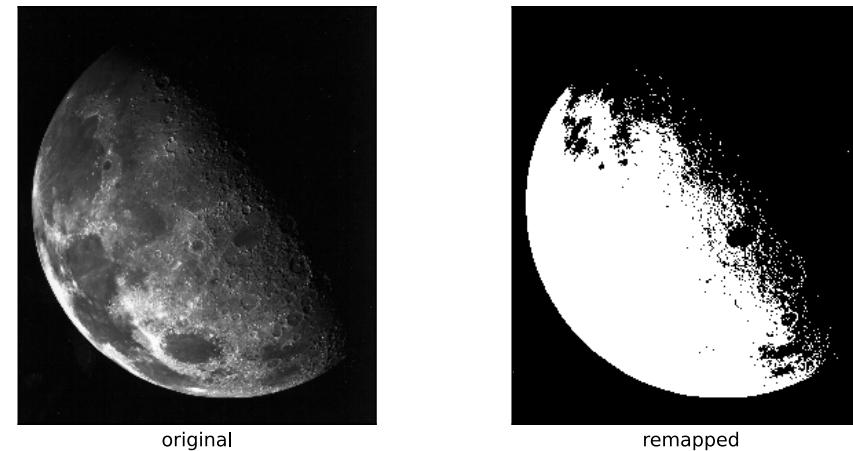
Intensity mappings
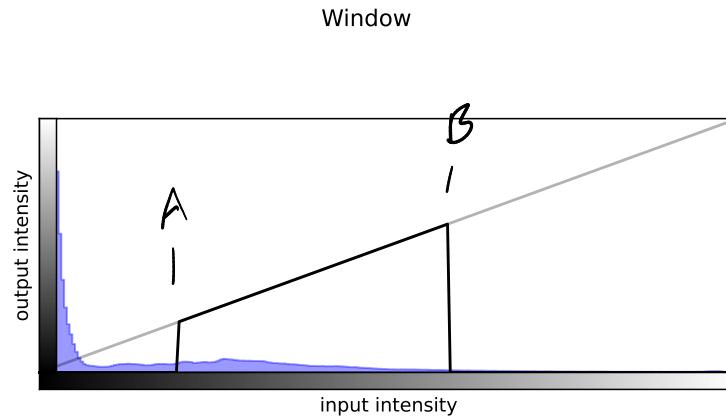
Window

$$output = \begin{cases} 0 & \text{if input} < A \\ 0 & \text{if input} > B \\ input & \text{otherwise} \end{cases}$$

original

remapped

Image manipulations in the spatial domain
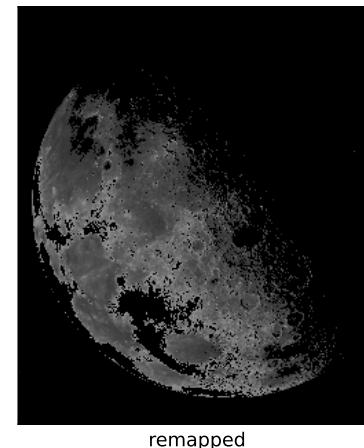
Intensity mappings

Binary window

original   remapped

Image manipulations in the spatial domain
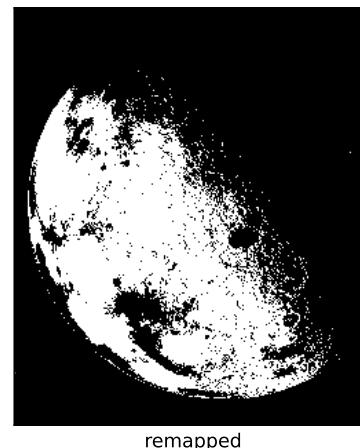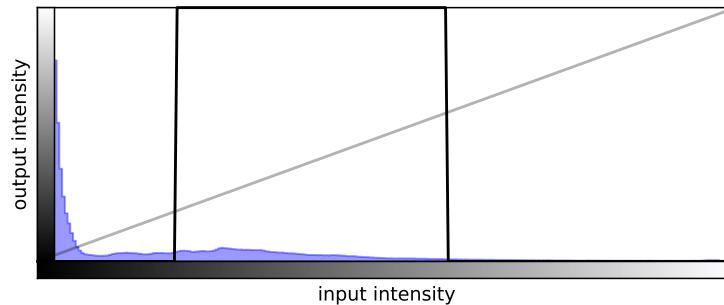
Intensity mappings

Posterization

original   remapped

Image manipulations in the spatial domain
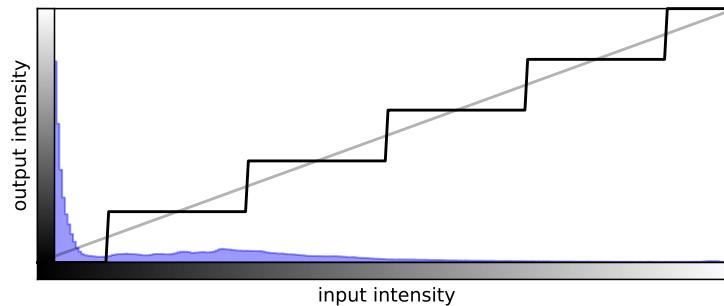
Morphological operations

- analyze morphology of image structures

    – based on set theory and topology

- extract image information

    – shape

    – size

    – connectivity

    – number

    – boundary

- mostly on binary images

Set operations

binary mask
A

True *False*
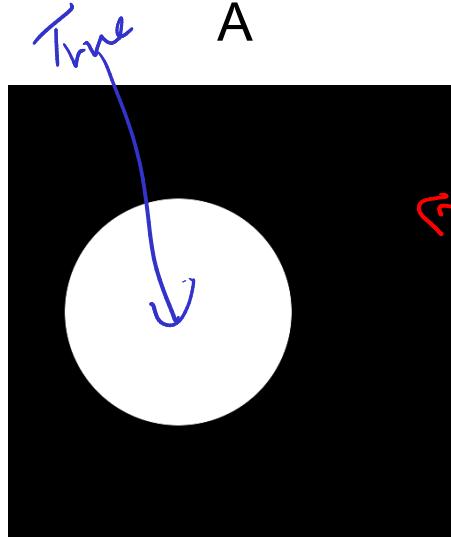
complement (background)
A^c
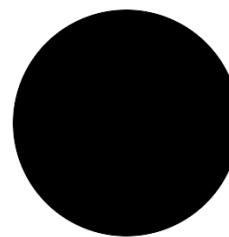
union
A ∪ B

"or"

difference
A \ B

"and not"

intersection
A ∩ B

"and"
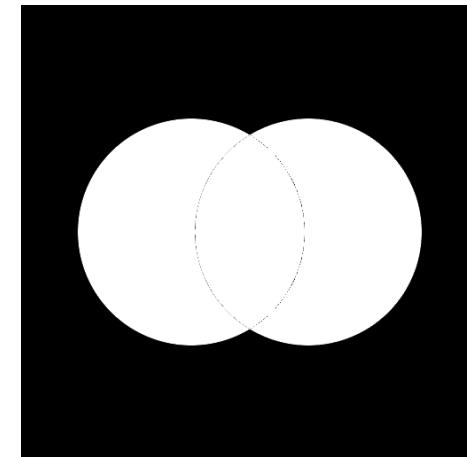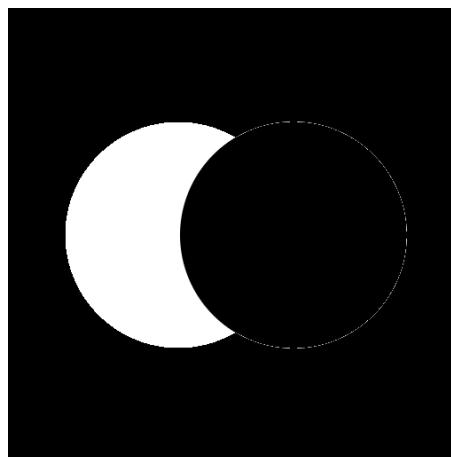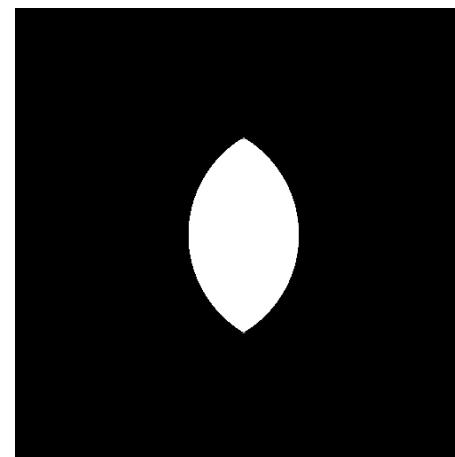
symmetric difference
A △ B

"xor"

Image manipulations in the spatial domain
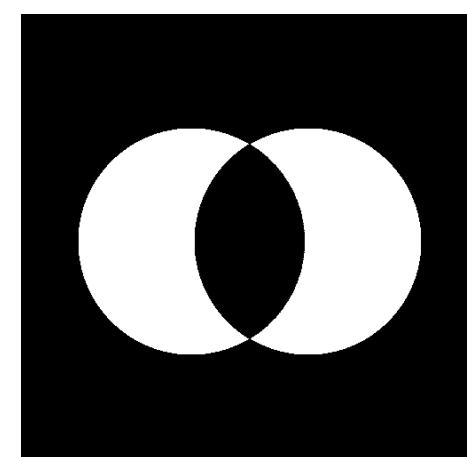
Structuring elements

- small bit mask to probe the image

- scan origin of SE over image

- check overlap between SE and image

- set pixel(s) to zero (or one)
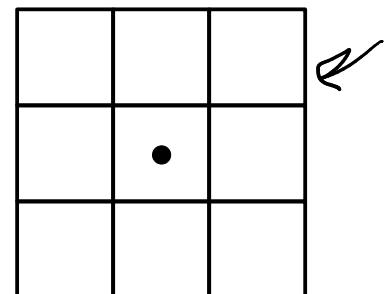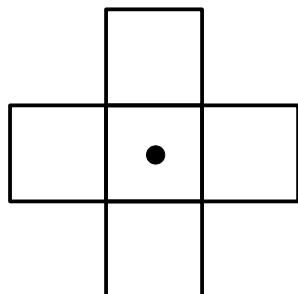
"structuring elements"

"footprint"
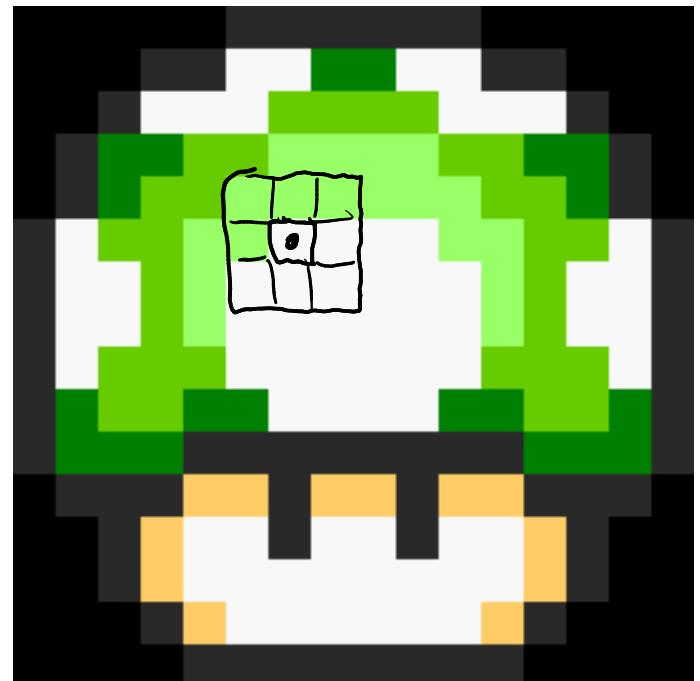
Basic operations

- Dilation: expand region
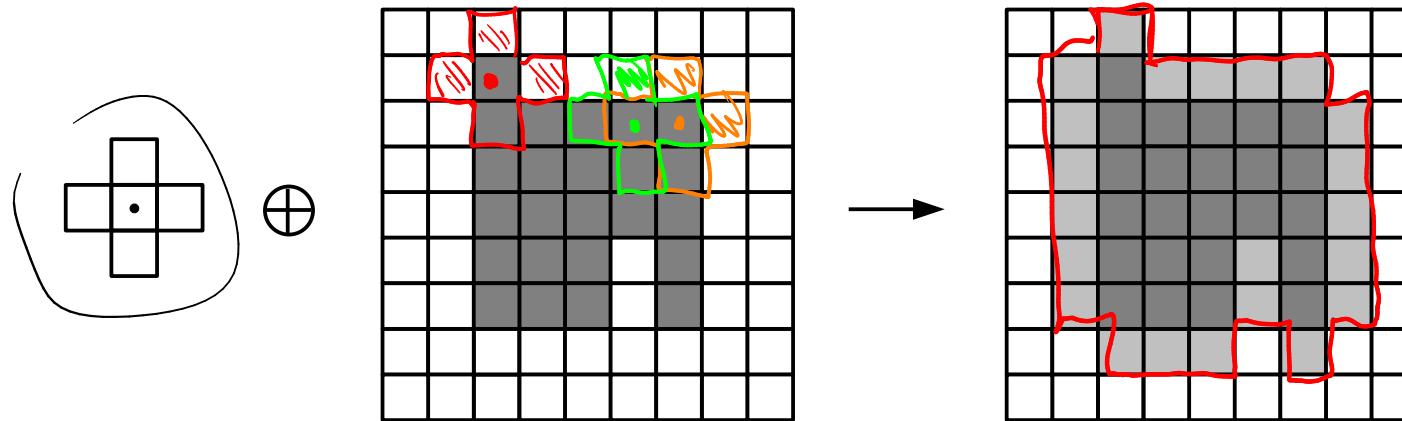
- Erosion: shrink region
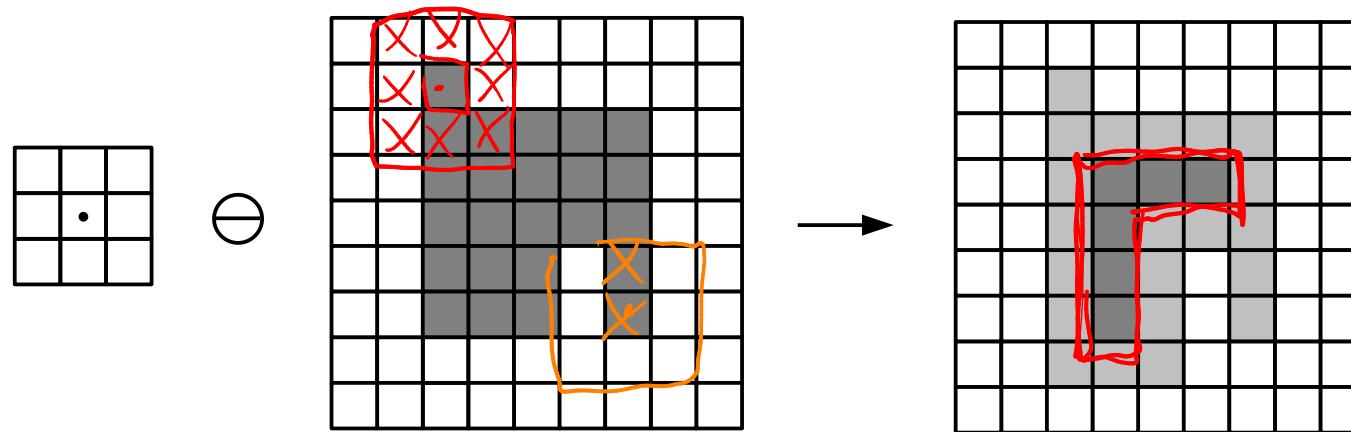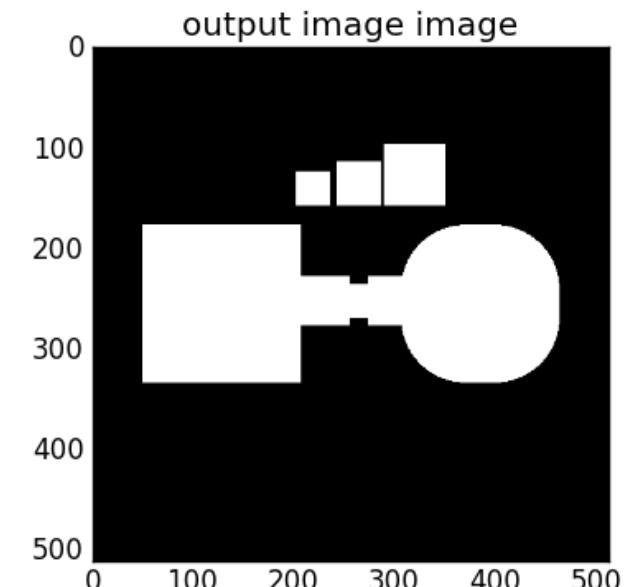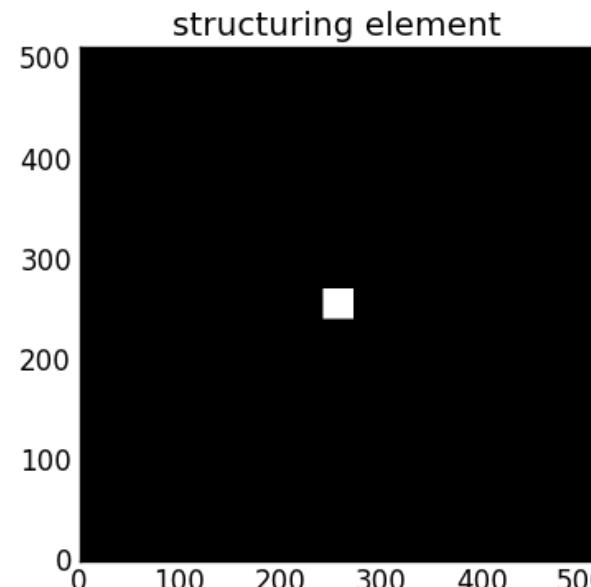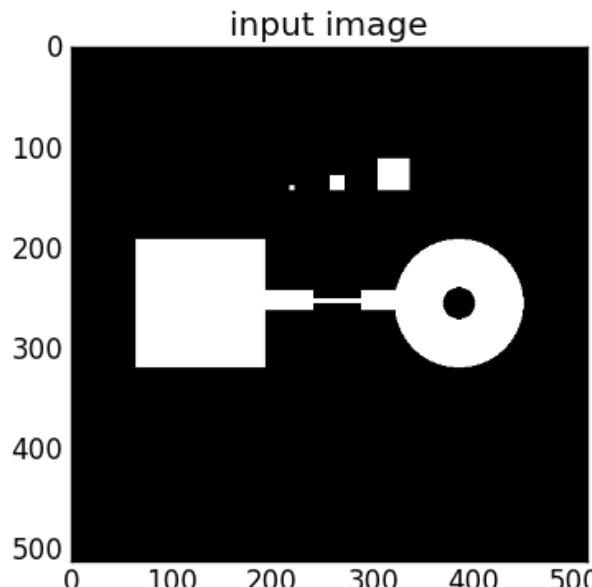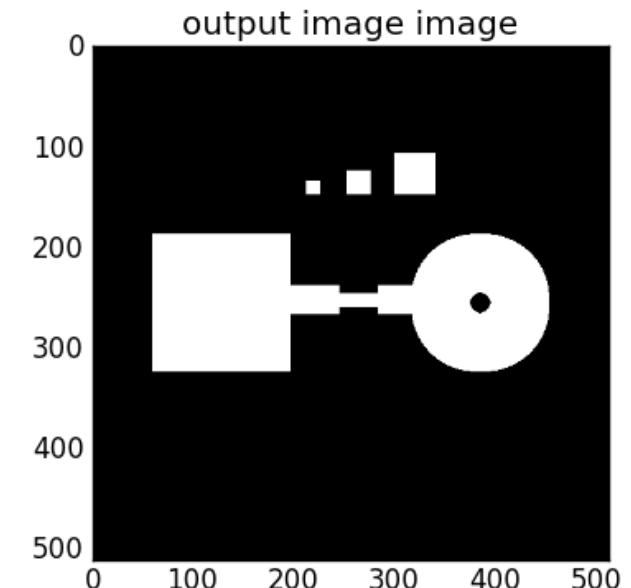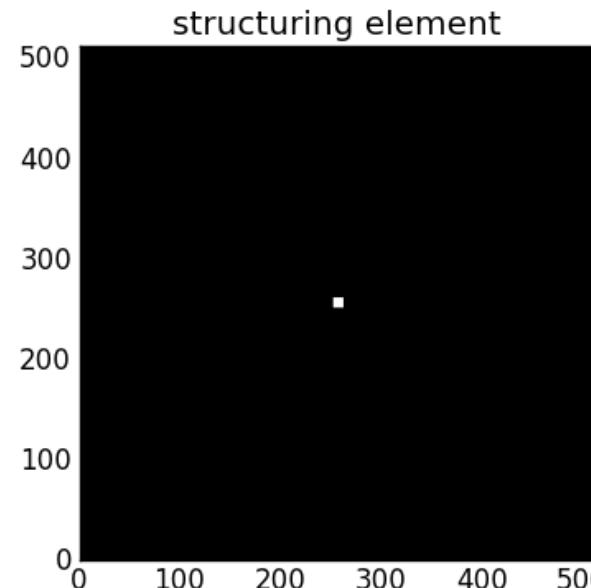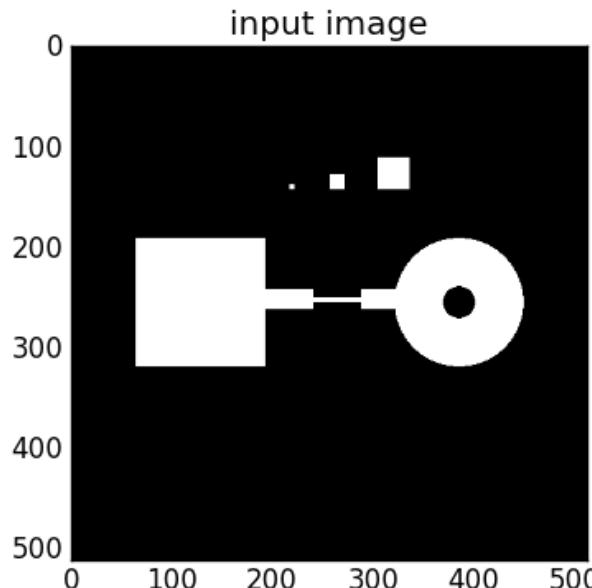

Image manipulations in the spatial domain

# Morphological operations

- dilation

Morphological operations
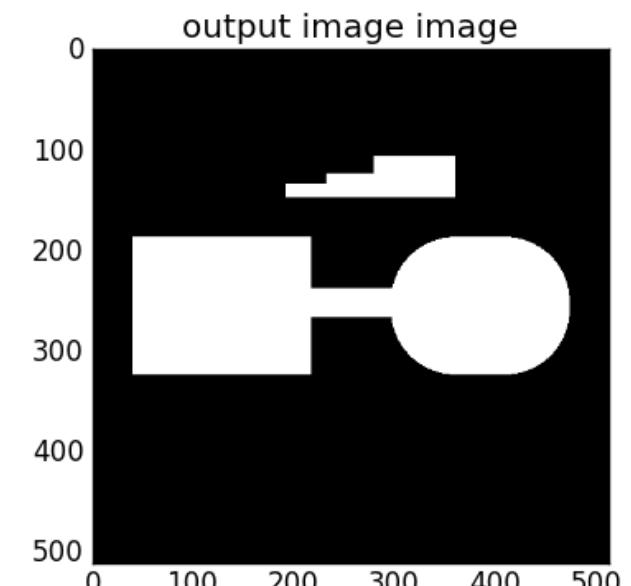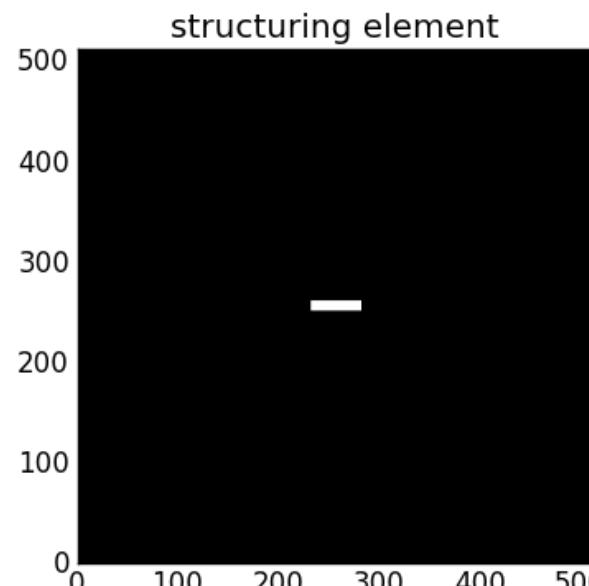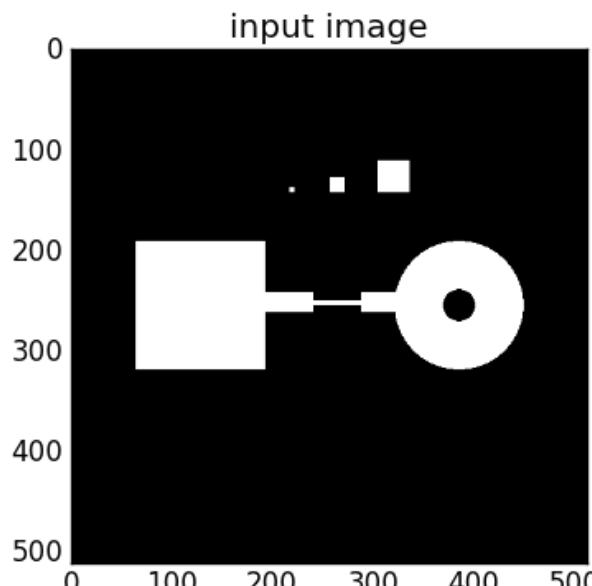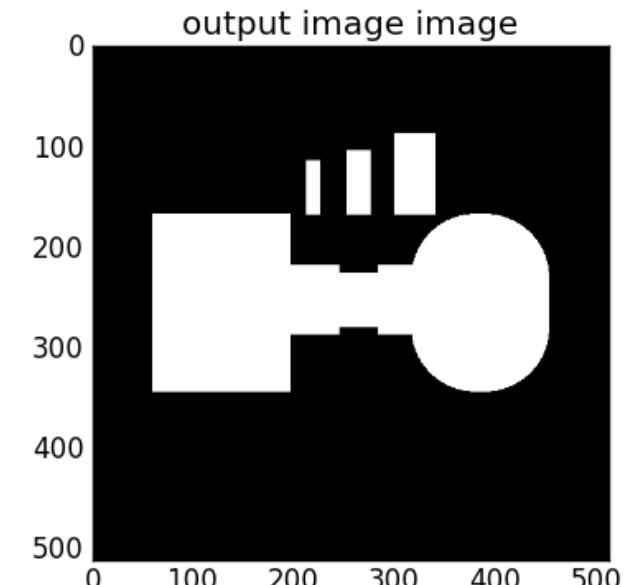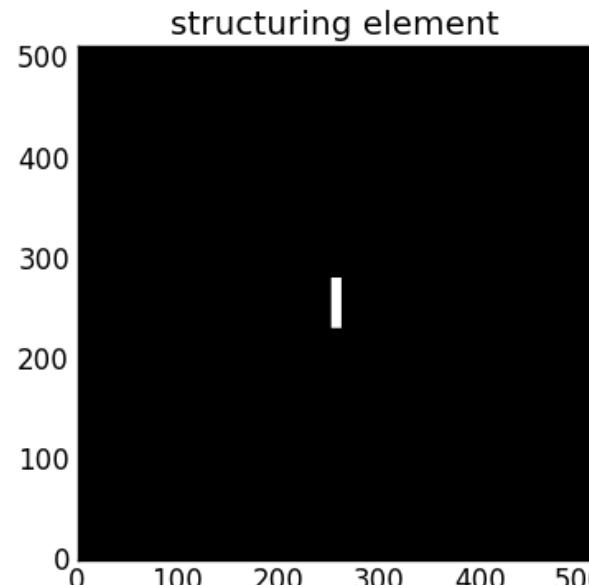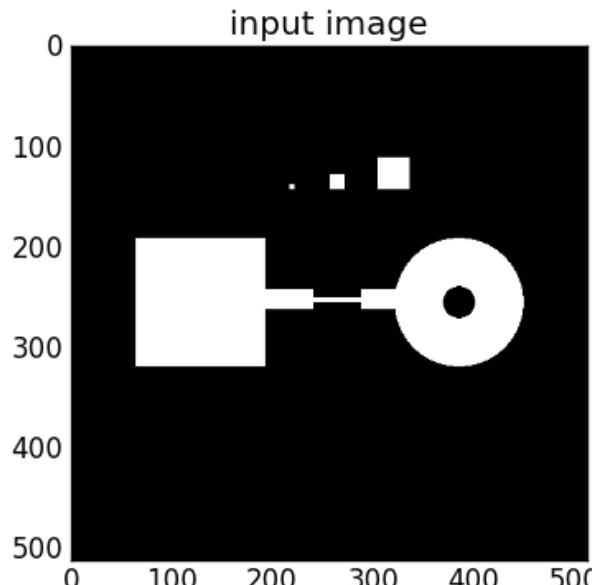
- dilation

# Morphological operations
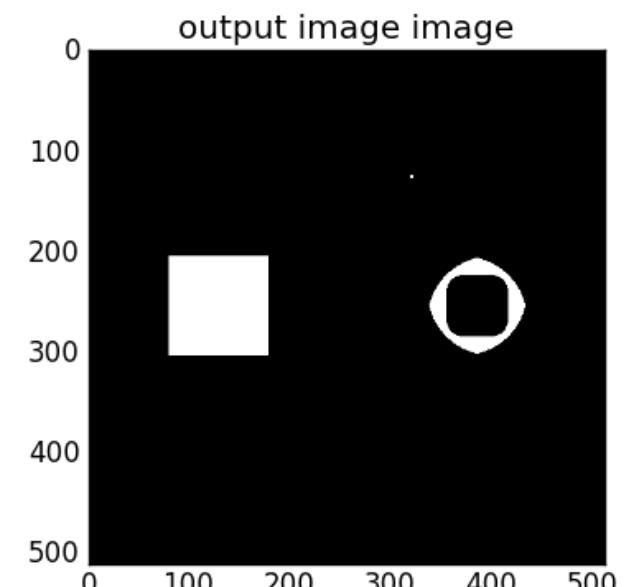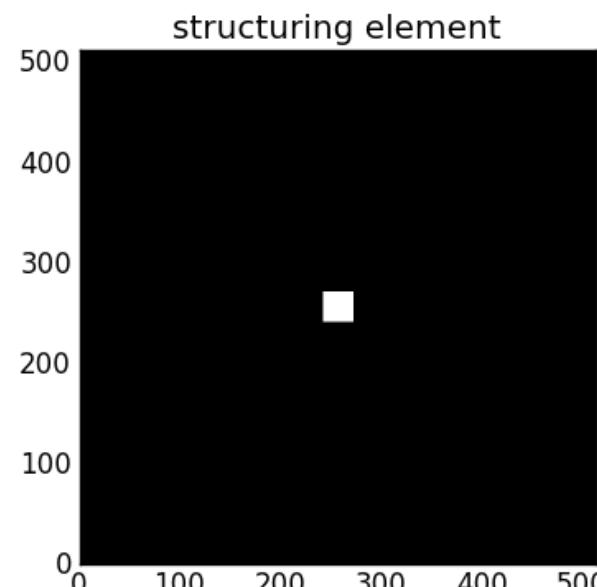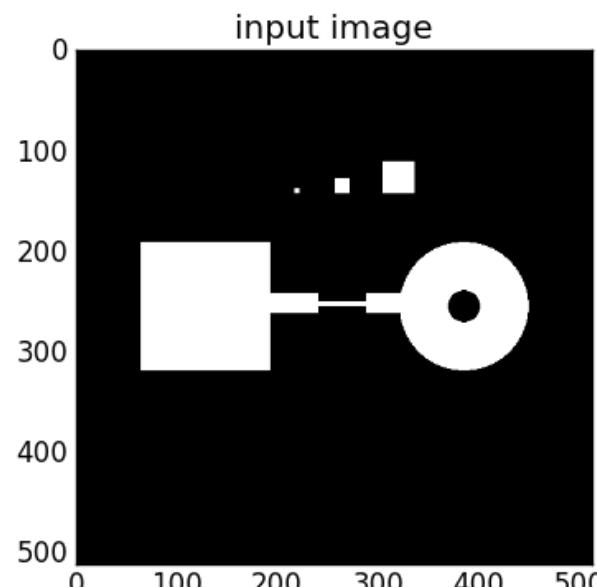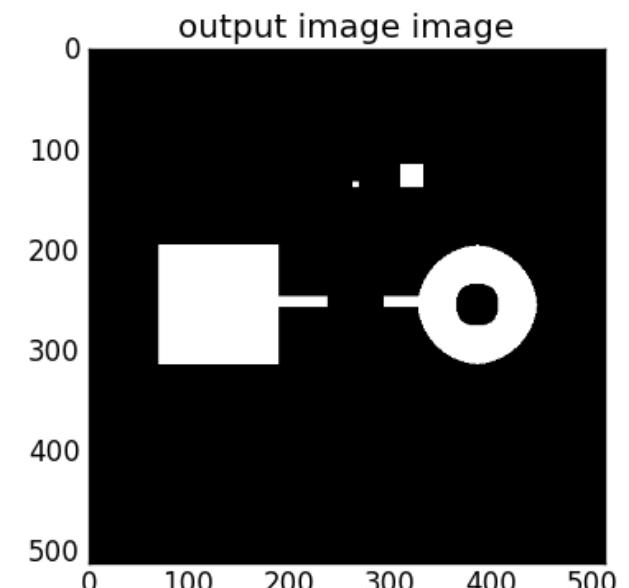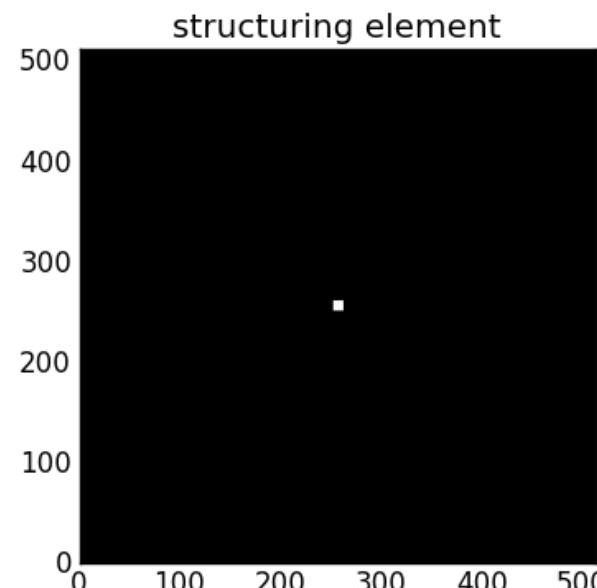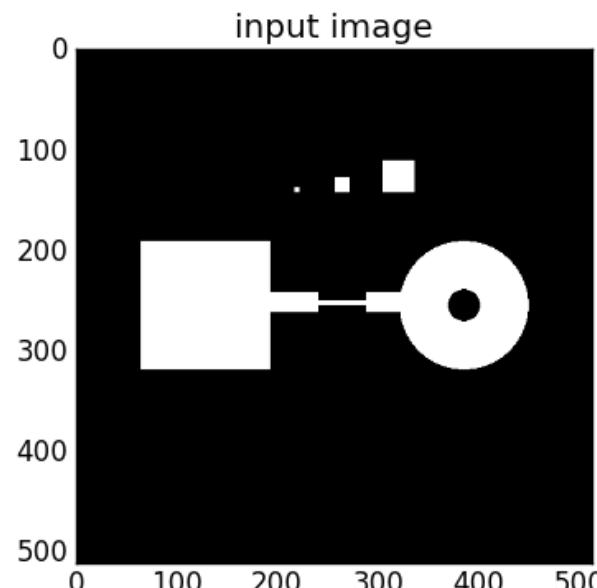
- erosion

Morphological operations
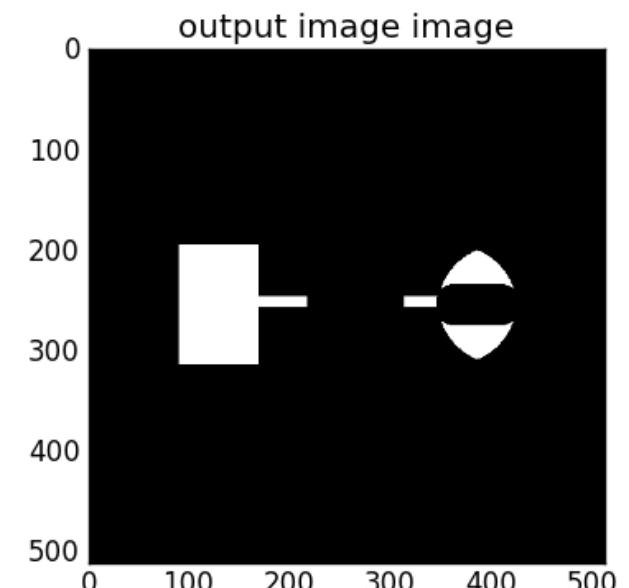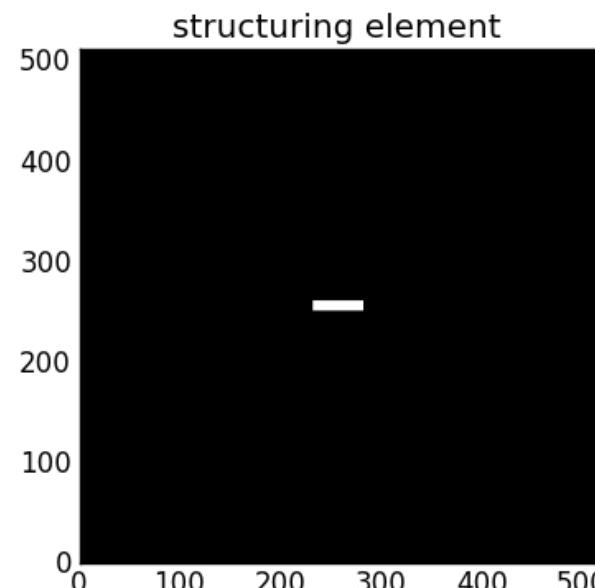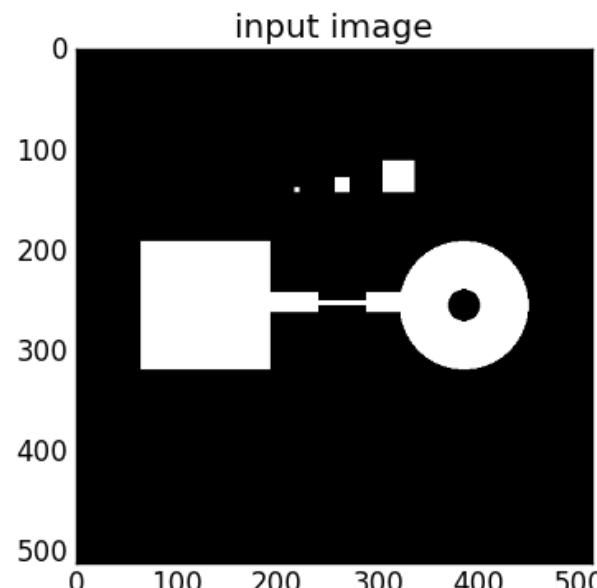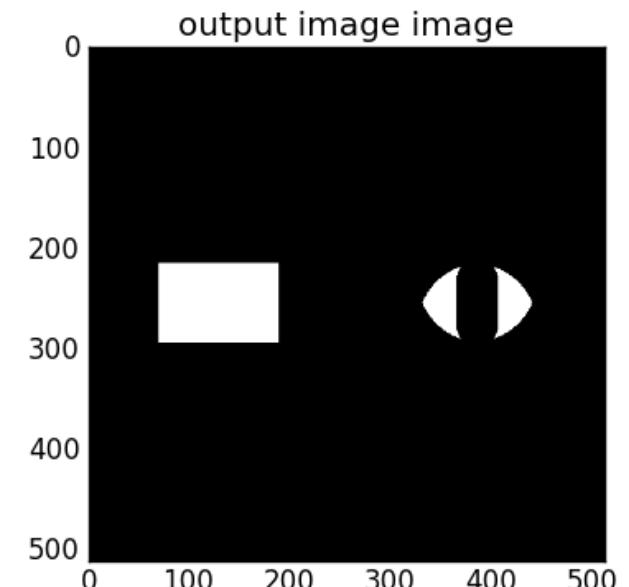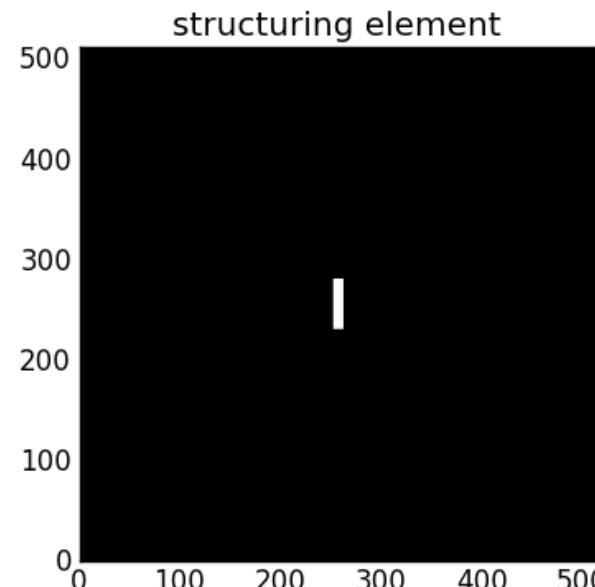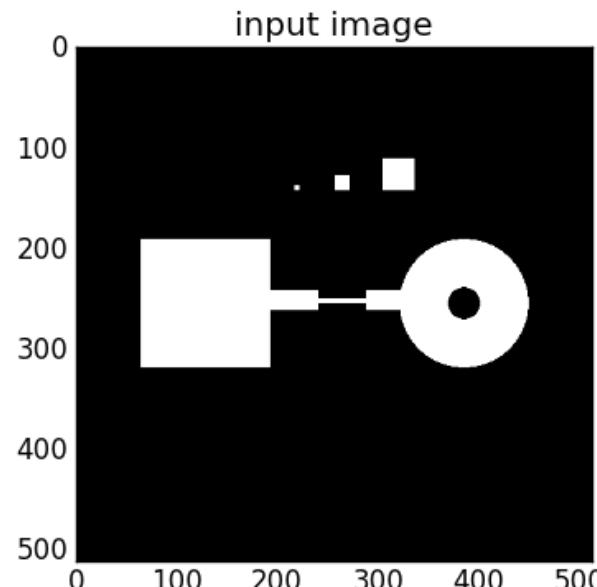
- erosion

Morphological operations

- opening: first erosion, then dilation
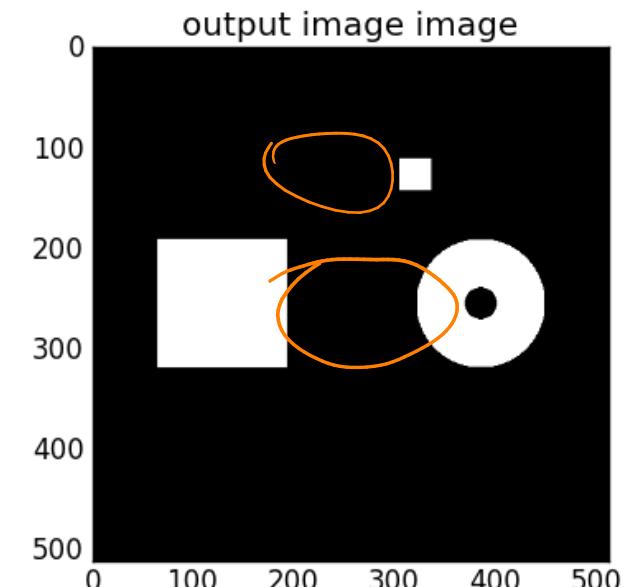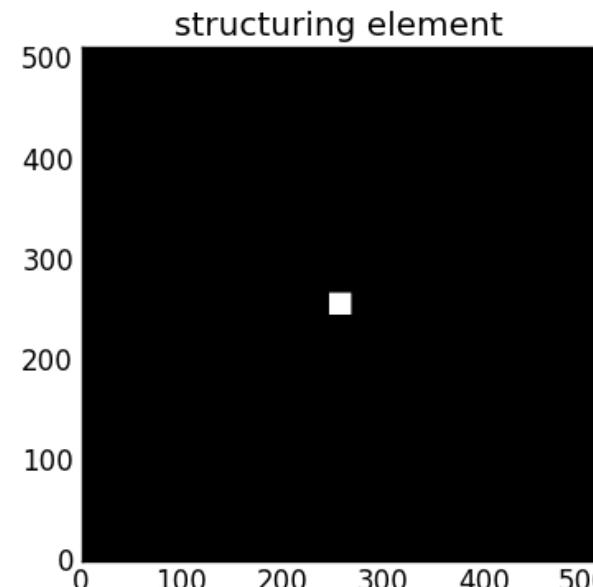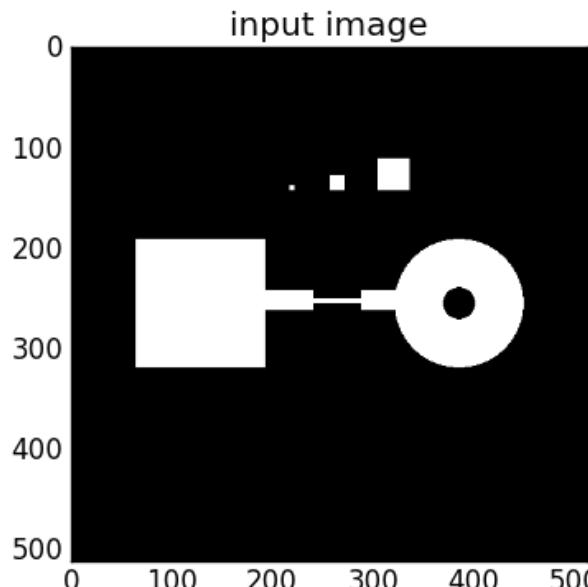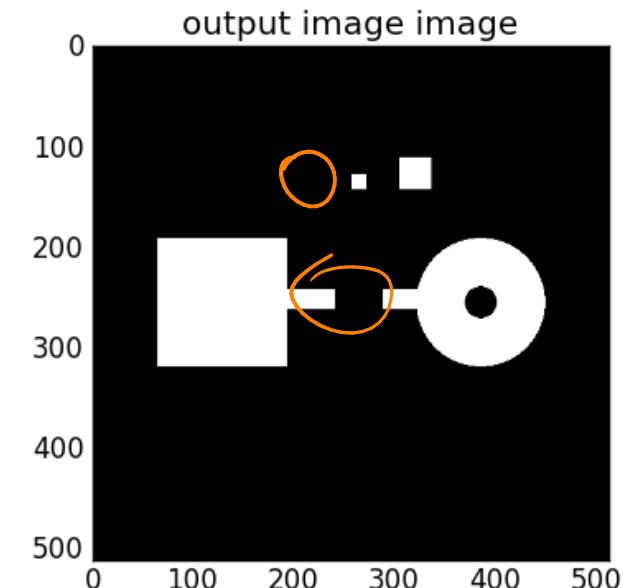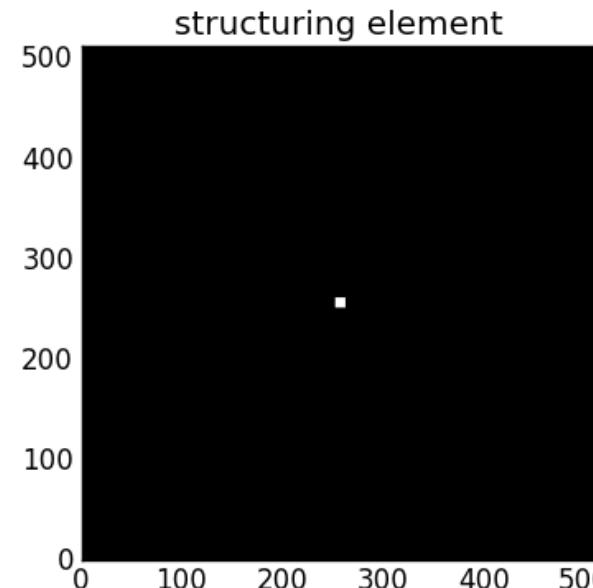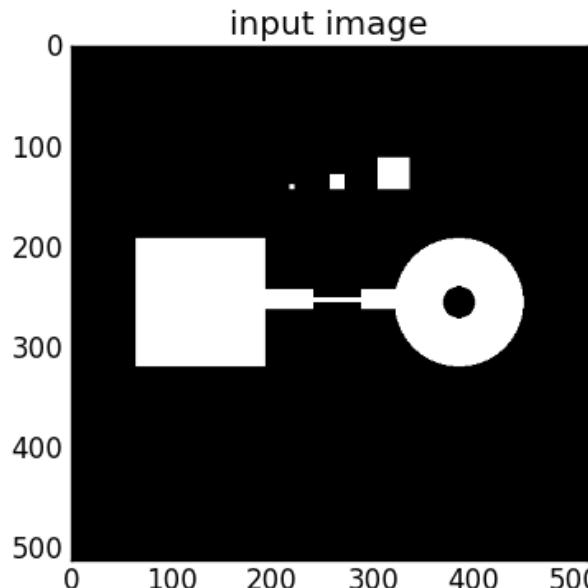
Morphological operations

- closing: first dilation, then erosion   *fill holes*
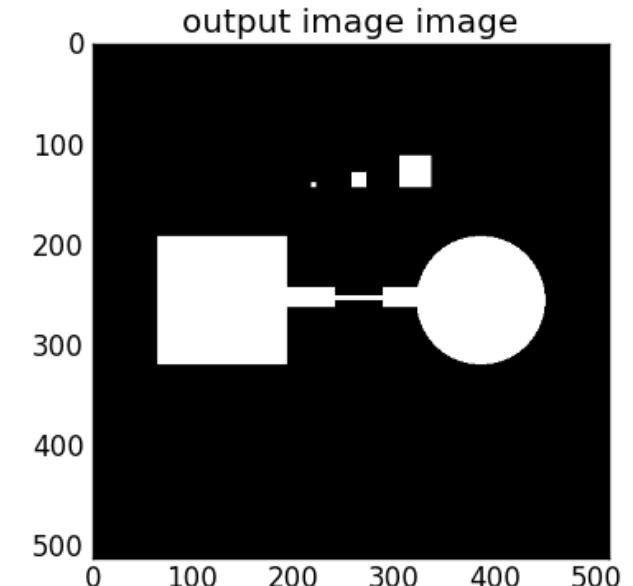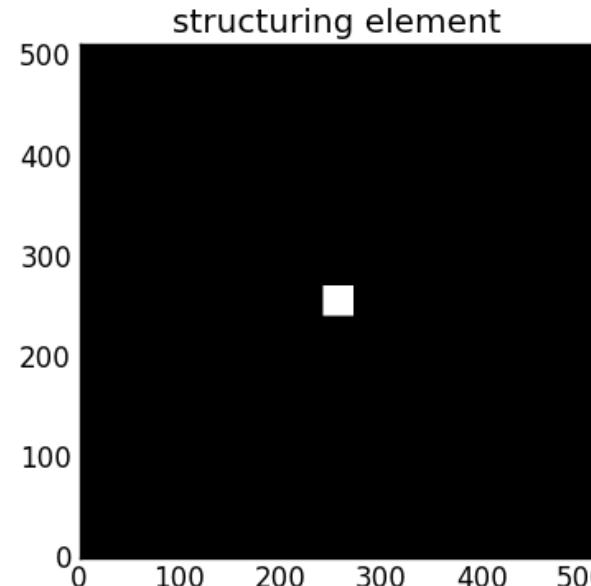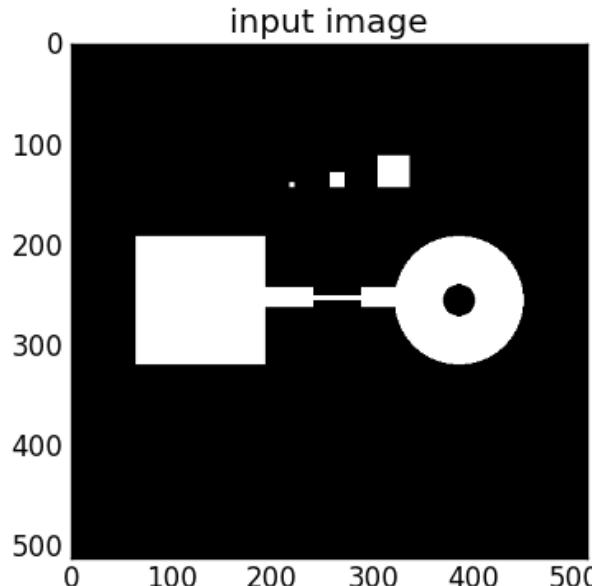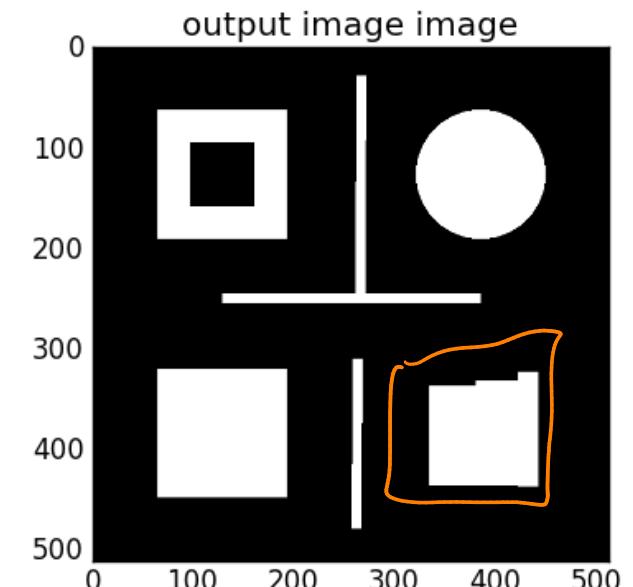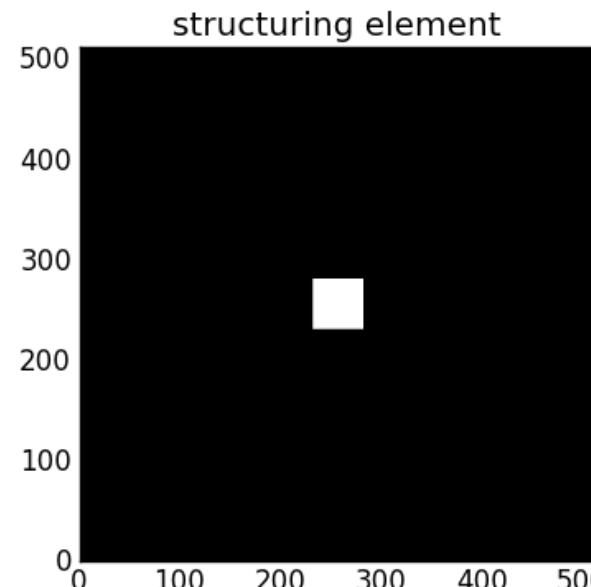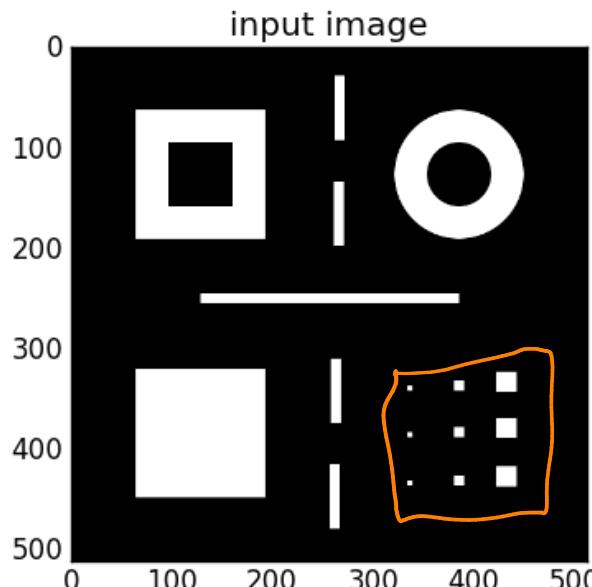
Morphological operations

- boundary: original – erosion
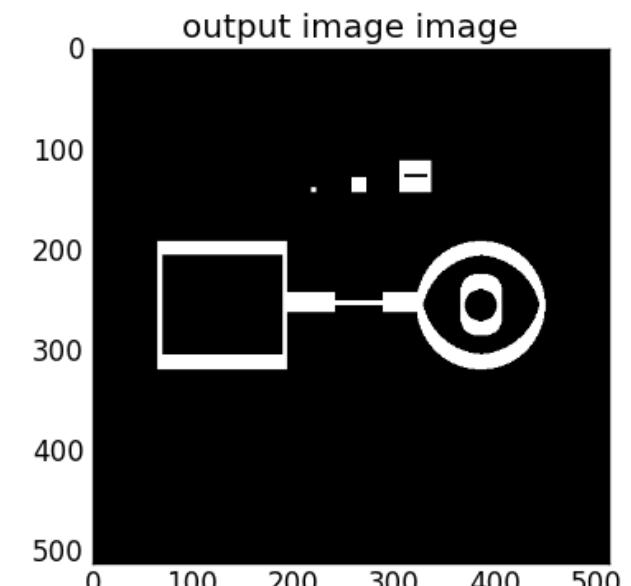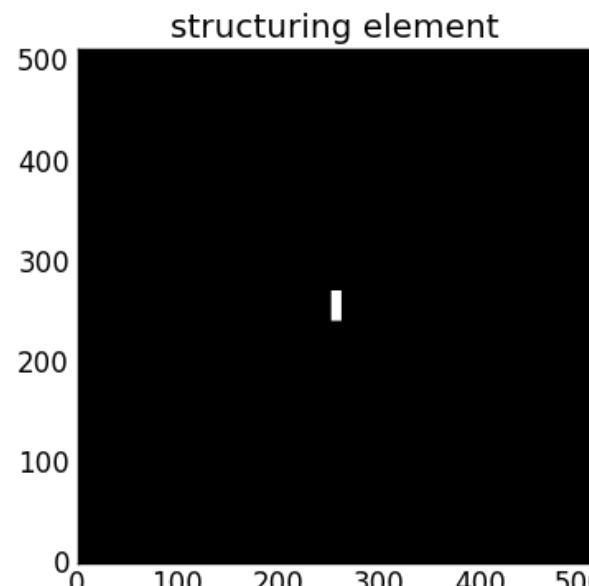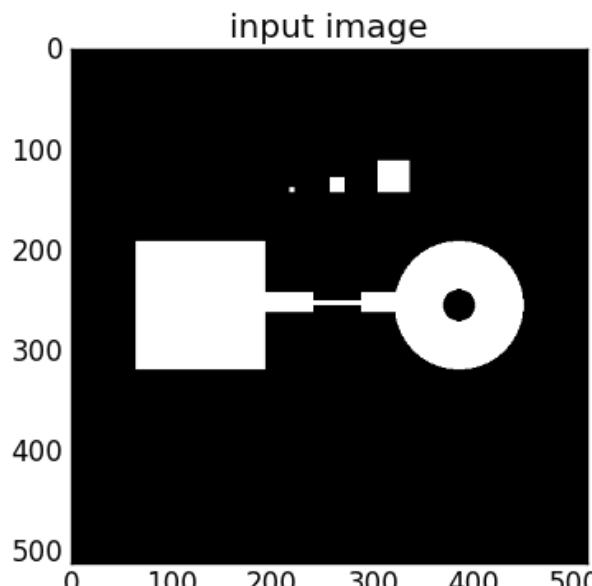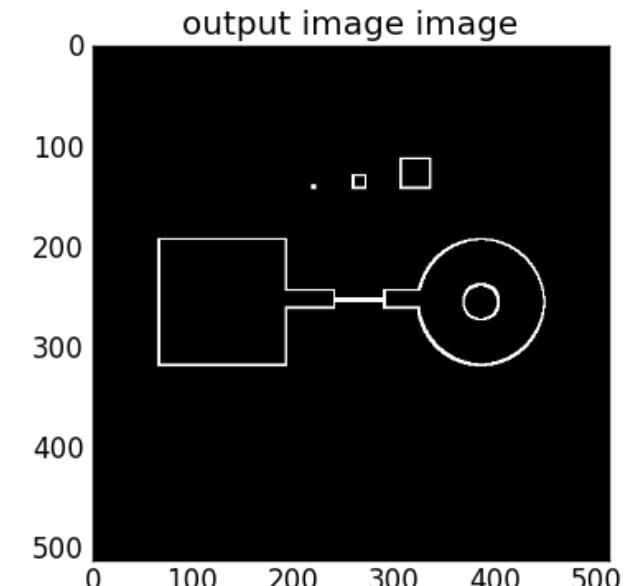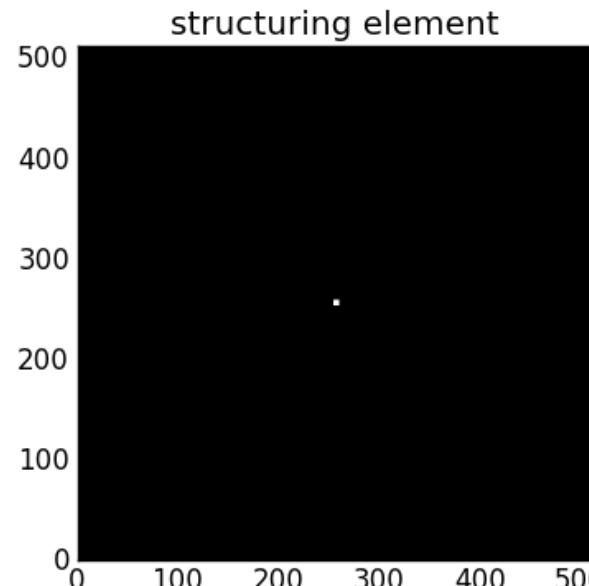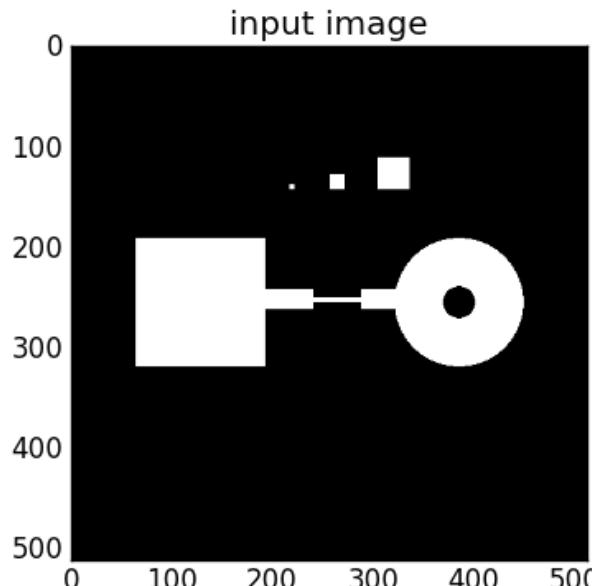
Segmentation: Motivation

- Partitioning of image by regions-of-interest *(ROI)*

- various methods available *or masks*

  - by morphology

  - by intensity

  - by region

  - by boundary

  - ...

Segmentation by morphology

original 512x512 — opening — opening — opening — opening — resulting segmentation

Image manipulations in the spatial domain

Segmentation by morphology

original 512x512   threshold   opening

closing+dilation   boundary   resulting segmentation

Image manipulations in the spatial domain
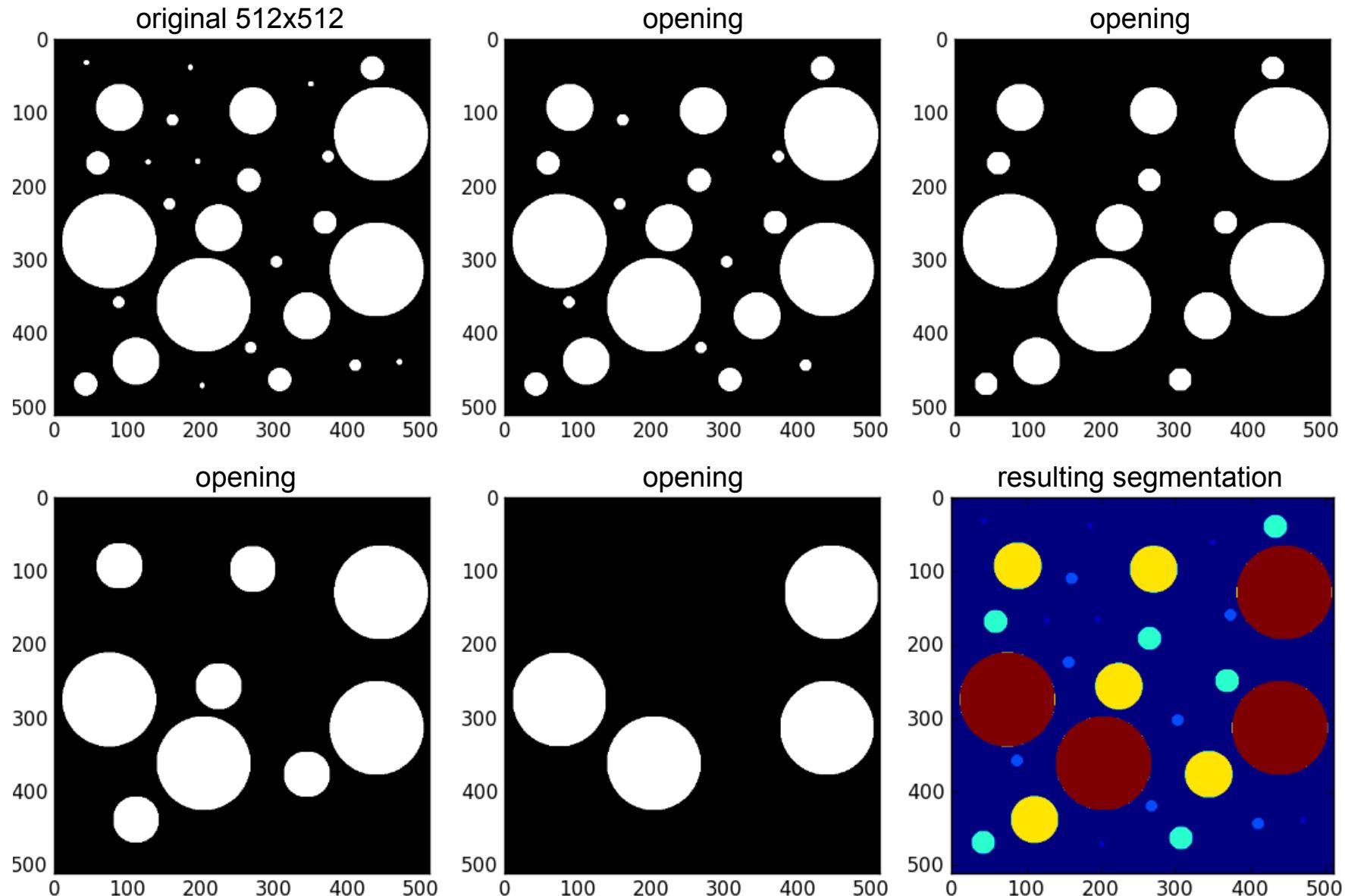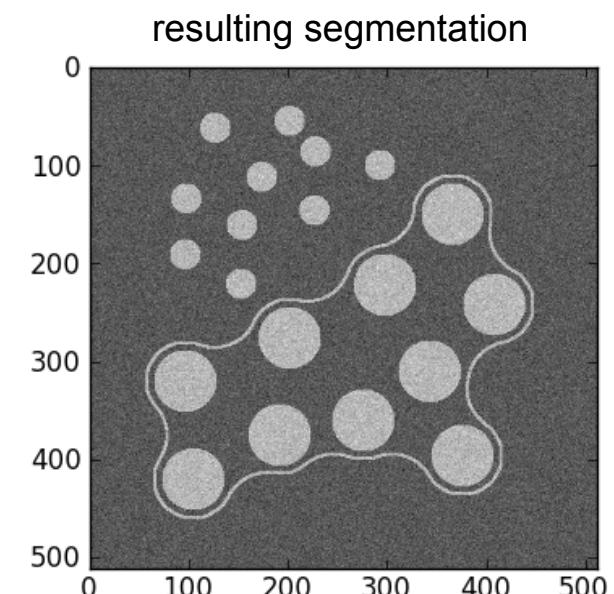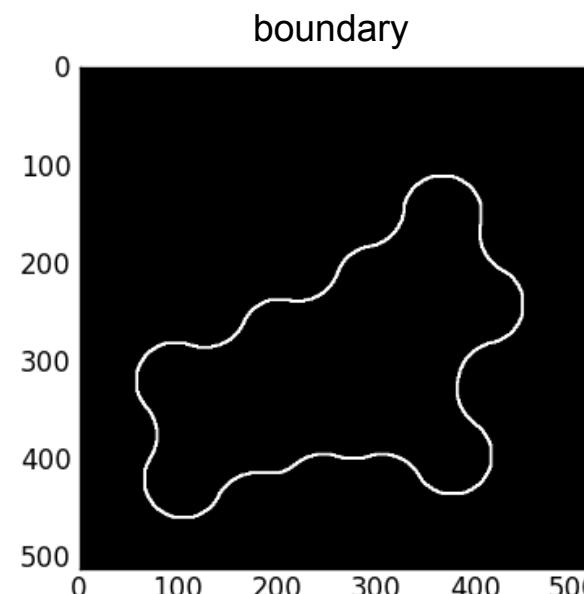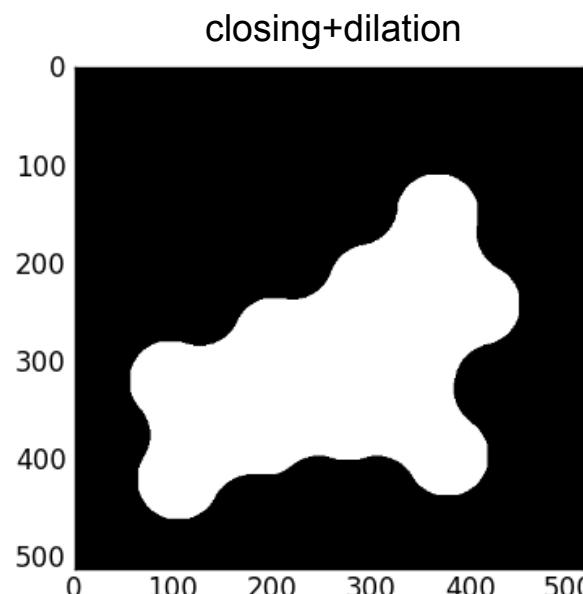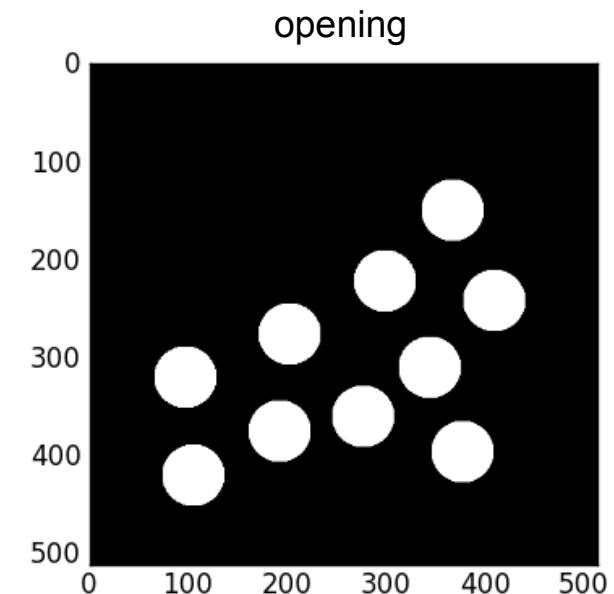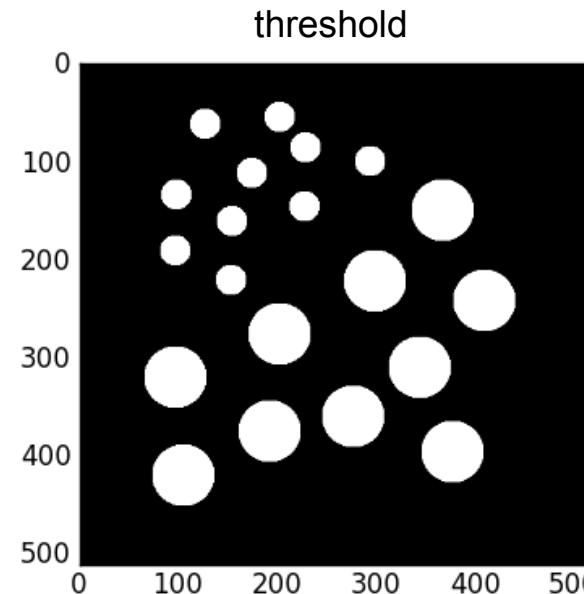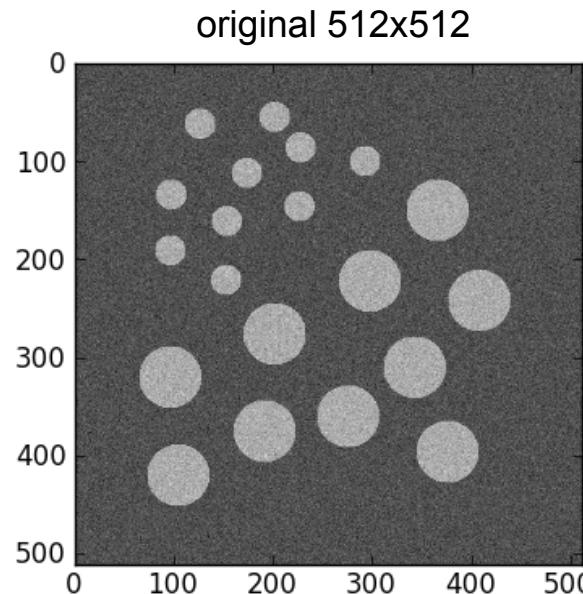
Segmentation by intensity

- easy

- widely used

original

high window

mid window

low window
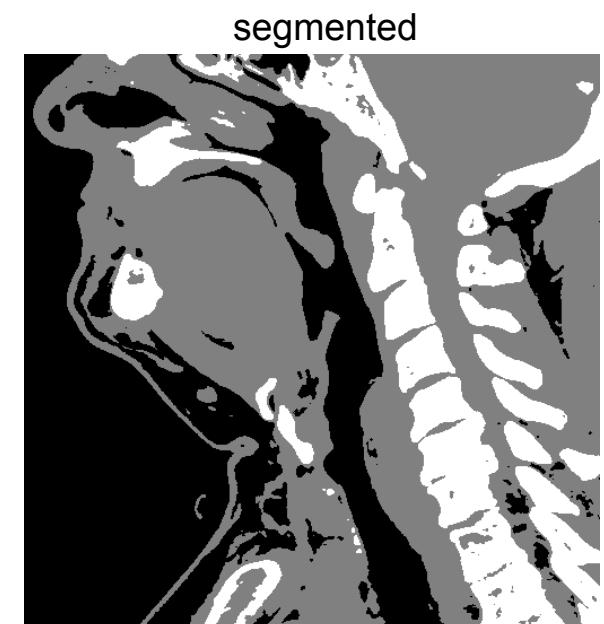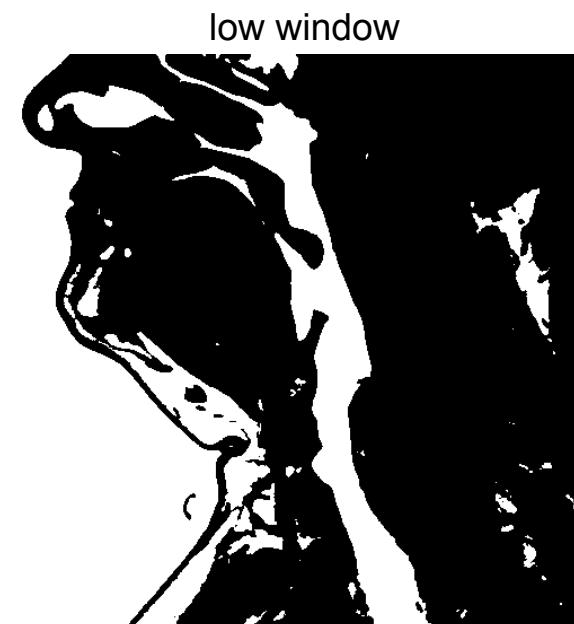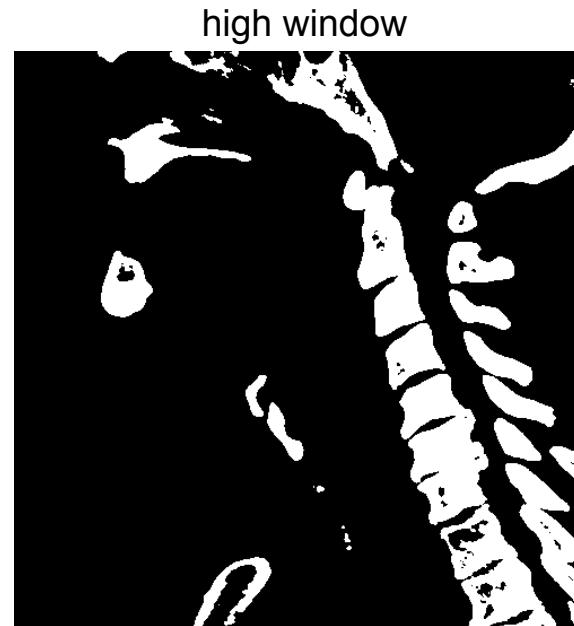
segmented

- noise prone

- no connectivity

Image manipulations in the spatial domain

Segmentation by region growth

- start with seed

- check intensity in neighborhood

- if intensity within window, set to 1

- iterate until no change

original

seed

Image manipulations in the spatial domain

Segmentation by boundaries

– look for sharp changes in intensity

– more next week...

original

laplace

Image manipulations in the spatial domain