EXAMPLE CALCULATION OF STATIC FORCES

Example 1. Compute force and torque, per unit width, on the cylindrical body hinged in A. Consider H=2m; h=5m; R=1m; $\gamma=10^4 N/m^3$.

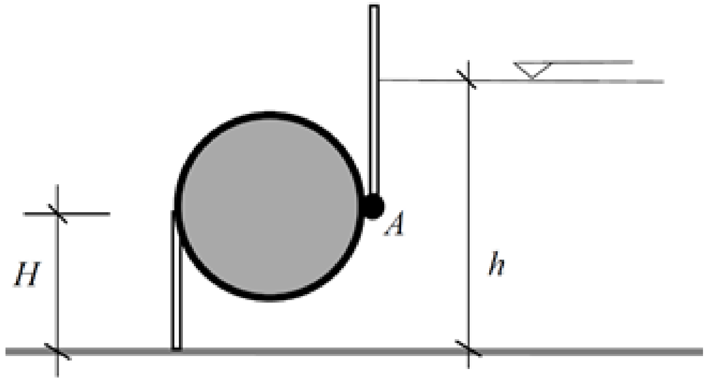

Figure.1. Sketch of the static system in example 1.

Solution:

$$F_H = 0\ ;$$
$$F_V = \gamma V + \gamma(h-H)2R = \gamma V + \gamma(h-H)2R = \gamma\left(\frac{\pi}{2}R^2 + 2R(h-H)\right) = 7.57\ 10^4\ \frac{\mathrm{N}}{\mathrm{m}}\ ;$$
$$T_A = F_V \cdot R\ .$$

Example 2. Compute force and torque, per unit width, for the entire surface, made of half a circle plus a straight part, hinged in A. Consider: h=6m; R=2m; γ=10^4 N/m^3.

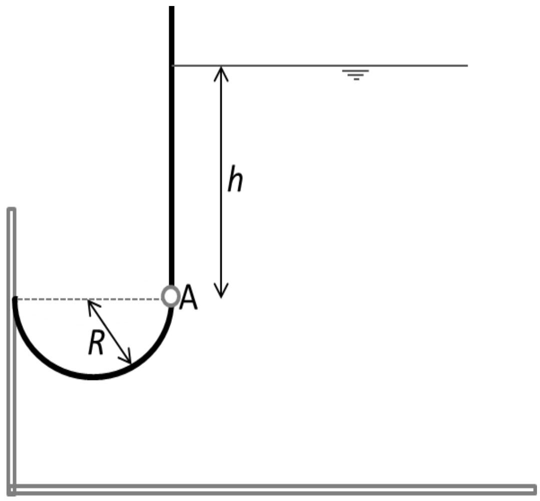

Figure 2. Sketch of the static system in example 2.

Solution:

$$F_H = \gamma\frac{h^2}{2} = 18\ 10^4\mathrm{N}\ , \quad r_H = \frac{h}{3} = 2\mathrm{m}\ ;$$
$$F_V = \gamma\left(2hR + \frac{\pi}{2}R^2\right) = 30.3\ 10^4\ \mathrm{N}, \quad r_V = R = 2\mathrm{m}\ ;$$
$$T_A = -F_H r_H + F_V r_V = 24.6\ 10^4\ \mathrm{J}\ .$$

Example 3. Compute force and torque, per unit width, for the surface made of two semicircles, hinged in upper edge *A*.

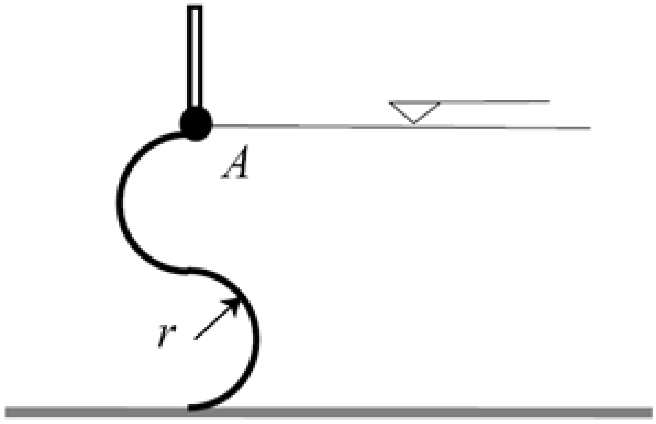

Figure **Errore. Nel documento non esiste testo dello stile specificato.**3. Sketch of the static system in example 3.

Solution:

$$F_H = \gamma 8r^2, \quad r_H = \frac{2}{3}4r, \quad F_V = \gamma\frac{\pi}{2}r^2 - \gamma\frac{\pi}{2}r^2 = 0\,;$$

notice that F_V is a force couple with arm $r_V = \frac{4}{3\pi}r$.

$$T_A = \gamma 8r^2 \cdot \frac{2}{3}4r - \gamma\pi r^2 \cdot \frac{4}{3\pi}r = 20\,\gamma r^3\,.$$

Example 4. Define the volume V of the sphere, of negligible weight, such that its buoyancy is as strong as the downward force on the lower surface of area *A*.

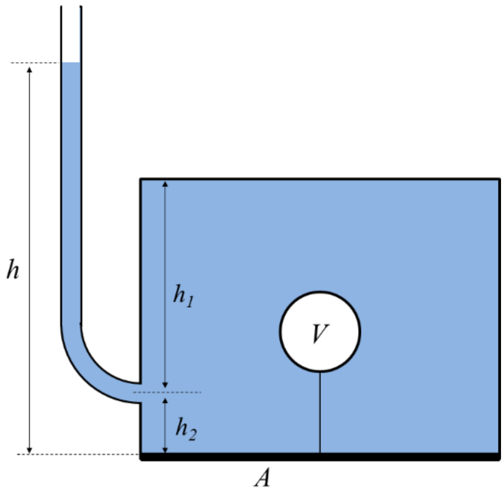

Figure 4. Sketch of the static system in example 4.

Solution:

$$\gamma hA = \gamma V, \quad V = hA\,.$$

Example 5. A plane surface, made of homogeneous material of negligible thickness, and hinged in point C, separates two reservoirs. Compute the weight W of the surface (per unit of width) to ensure that it remains in equilibrium. Consider HA=1.0 m, HB=2.0 m.

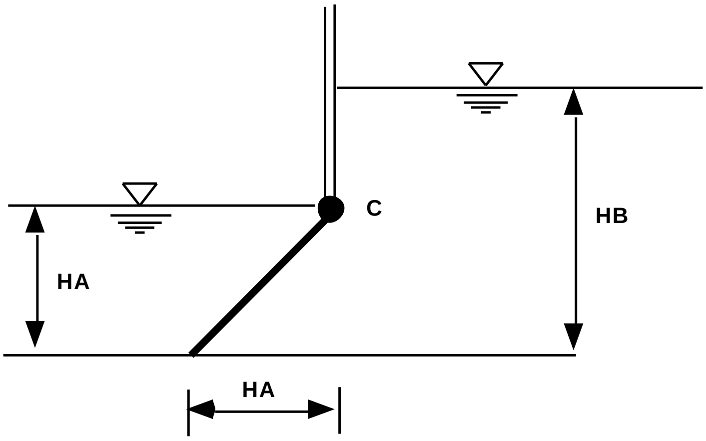

Figure 5. Sketch of the static system in example 5.

Solution:

Fluid torque is

$$F = \gamma(\mathrm{HB} - \mathrm{HA})\mathrm{HA}\sqrt{2}, \quad r = \mathrm{HA}\frac{\sqrt{2}}{2}, \quad T_f = F \cdot r = \gamma(\mathrm{HB} - \mathrm{HA})\mathrm{HA}^2\,;$$

the same result could be found considering separately the left and right sides of the surface

$$F_L = \gamma \mathrm{HA}^2\frac{\sqrt{2}}{2}, \quad r = \frac{2\sqrt{2}}{3}\mathrm{HA}, \quad M_L = \gamma\frac{2}{3}\mathrm{HA}^2;$$
$$F_{R1} = \gamma(\mathrm{HB} - \mathrm{HA})\mathrm{HA}\sqrt{2}, \quad r = \frac{\sqrt{2}}{2}\mathrm{HA}, \quad M_{R1} = \gamma(\mathrm{HB} - \mathrm{HA})\mathrm{HA}^2;$$
$$F_{R2} = \gamma \mathrm{HA}^2\frac{\sqrt{2}}{2}, \quad r = \frac{2\sqrt{2}}{3}\mathrm{HA}, \quad M_{R2} = \gamma\frac{2}{3}\mathrm{HA}^2;$$

noticing that F_L and F_{R2} cancel each other.

Solid weight torque is

$$T_s = W\frac{\mathrm{HA}}{2}\,.$$

Equilibrium

$$W = 2\gamma(\mathrm{HB} - \mathrm{HA})\mathrm{HA} = 20\ \mathrm{KN}\,.$$

Example 6. Evaluate the weight of the cover such that it equilibrates the force exerted by the fluid from below. Consider γ_{water}=10^4 N/m^3 e γ_{oil}=6800 N/m^3; let heights be h_1=2m e h_3=36cm, and diameters D=5m, d=20cm.

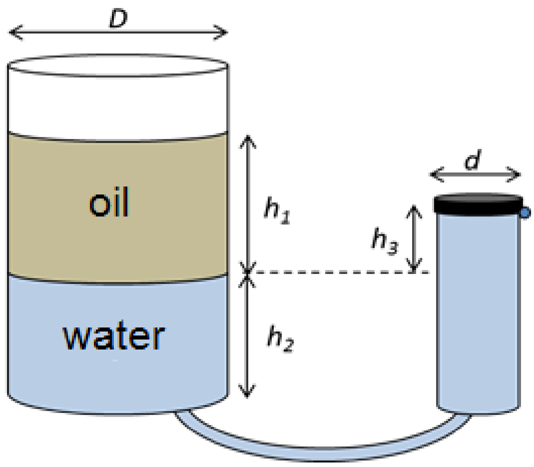

Figure 6. Sketch of the static system in example 6.

Solution:

$$F = (\gamma_{\text{oil}} h_1 - \gamma_{\text{water}} h_3)\pi \frac{d^2}{4} = 314\text{N}\,.$$

Example 7. Compute force and torque relative to the hinge in *A*, per unit width, on the surface of the object, made of half a cylinder or radius *R*=2m with a cylindrical cavity of radius $r = R/4$. Consider the fluid of specific gravity $\gamma=10^4\ N/m^3$.

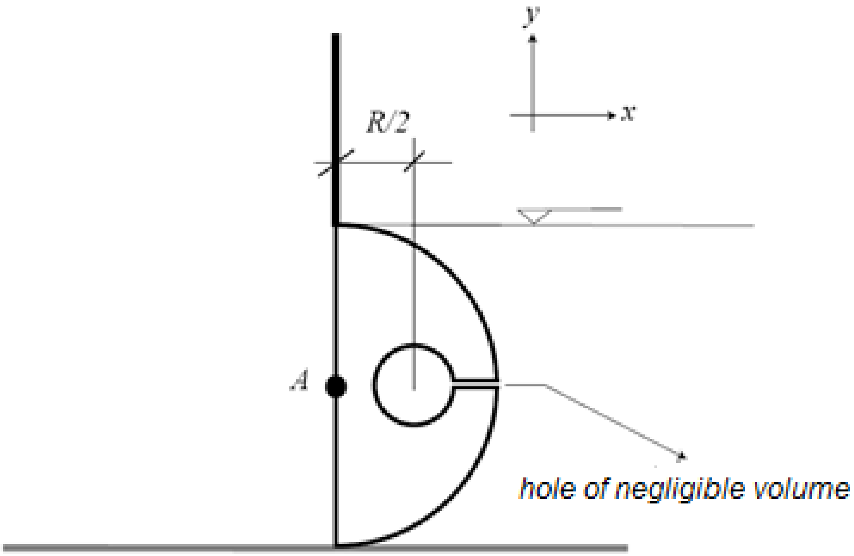

Figure 7. Sketch of the static system in example 7.

Solution:

$$F_x = -\gamma 2R^2, \quad F_y = \gamma\left(\pi\frac{R^2}{2} - \pi r^2\right).$$

Pressure on the outer cylindrical surface acts radially and gives no momentum. Momentum about A is only due to the pressure on the inner cylinder surface:

$$T_A = \gamma \pi r^2 \frac{R}{2}.$$

Example 8. Compute the torque acting on the rectangular cover of length d=50cm, and unitary width. Consider the specific weigh of fluid γ=10^4 N/m^3 and that of mercury γ_m=130 N/dm^3, in the differential manometer whose reading is Δ=40 mm. Consider the following dimensions h_1=1.12m, h_2=62cm, a=120cm, b=50dm.

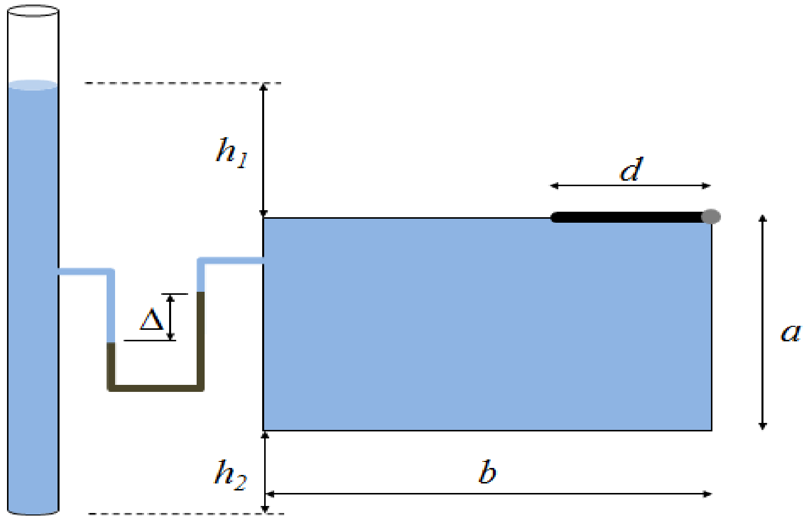

Figure 8. Sketch of the static system in example 8.

Solution:

$$h = h_1 - \left(\frac{\gamma_m - \gamma}{\gamma}\right)\Delta = 0.64\text{m}, \quad F = \gamma h d = 3.2\text{KN}, \quad T = F\frac{d}{2} = 800\text{J}.$$

Example 9. Compute the tilting moment on the structure. Consider γ=10^4 N/m^3 and measures h=6m, b=50 cm, B=2m.

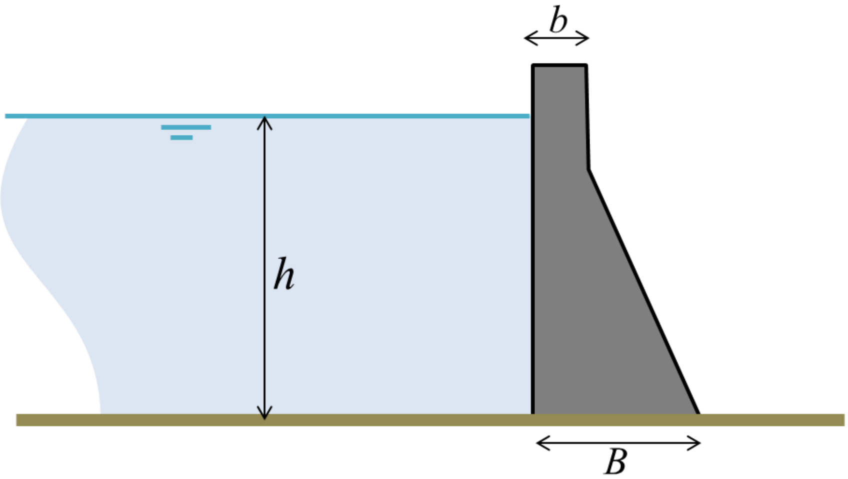

Figure 9. Sketch of the static system in example 9.

Solution:

$$M = \gamma \frac{h^2}{2} \frac{h}{3} = \gamma \frac{h^3}{6} = 3.6\ 10^5 \text{J} \cdot$$

Example 10. Compute the torque, relative to the basis, made by the fluid on the oblique septum. Geometric measures are a=2m, b=4m, e Δ=20cm. The fluid specific gravity is γ=9810N/m^3, pressure in the gas chamber is P_1=10^4Pa at height h_1=1m.

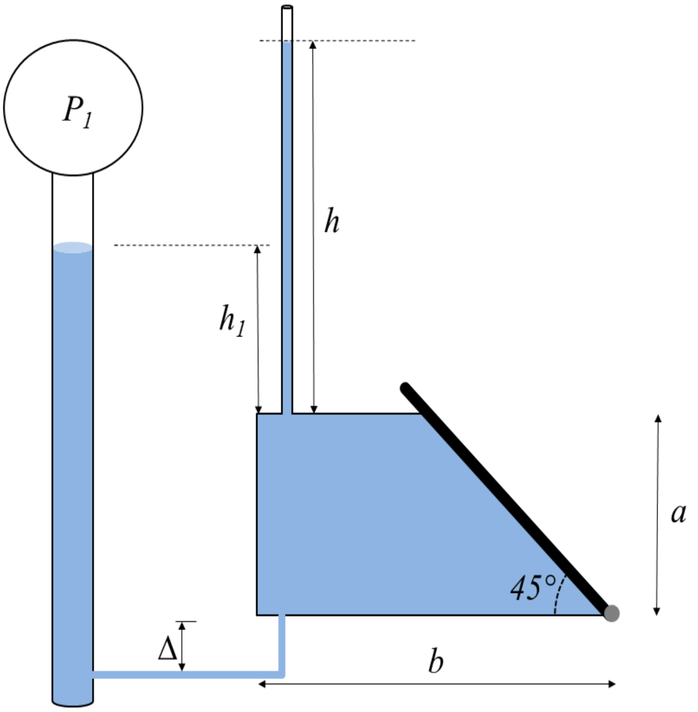

Figure 10. Sketch of the static system in example 10.

Solution:

$$h = h_1 + \frac{P_1}{\gamma} = 2.02\text{m}, \qquad F = \gamma \left(h + \frac{a}{2}\right) a\sqrt{2} = 83.78\text{KN},$$

$$\zeta_C = \frac{2}{3}\frac{(h+a)^3 - h^3}{(h+a)^2 - h^2} = 3.13\text{m}, \quad r = (h + a - \zeta_C)\sqrt{2} = 1.258\text{m}, \quad T = F \cdot r = 105\text{KJ}\,.$$

Torque could be evaluated by considering separately the contribution of the square and triangular distribution of pressure

$$F_{sq} = \gamma h a\sqrt{2} = 56\text{KN}, \quad T_{sq} = F_{sq}\frac{a}{2}\sqrt{2} = 79\text{KJ},$$

$$F_{tr} = \gamma \frac{a}{2} a\sqrt{2} = 28\text{KN}, \quad T_{tr} = F_{tr}\frac{a}{3}\sqrt{2} = 26\text{KJ}, \quad T = T_{sq} + T_{tr} = 105\text{KJ}\,.$$

Example 11. Define the width X of the base such that the surface is in equilibrium to rotation with respect to the rightmost edge.

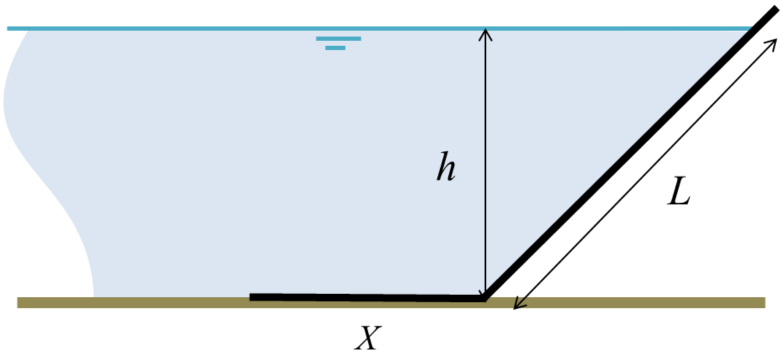

Figure 11. Sketch of the static system in example 11.

Solution:

$$\gamma \frac{hL}{2}\frac{L}{3} = \gamma hX\frac{X}{2}, \quad X = \frac{L}{\sqrt{3}} .$$

Example 12. Compute the force on the hemisphere at the base of bowl. Assume γ=9810 N/m^3 and pressure on the upper gas P_0=9810Pa; fluid height is h=2.2m and radius at the base R=1m.

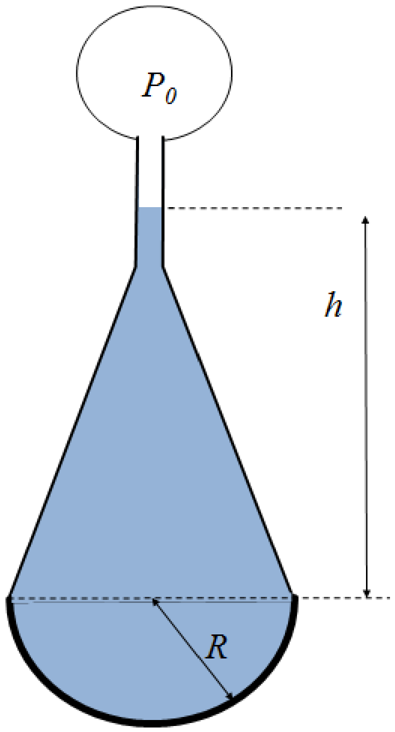

Figure 12. Sketch of the static system in example 12.

Solution:

$$H = h + \frac{P_0}{\gamma} = 3.2\text{m}, \quad F = \gamma H\pi R^2 + \gamma\frac{2}{3}\pi R^3 = 120\text{KN} .$$

Example 13. Consider the rigid surface, made of a horizontal wall of width L and a vertical wall of height h. Compute the value of the former such that it is in equilibrium to ration around the hinge A.

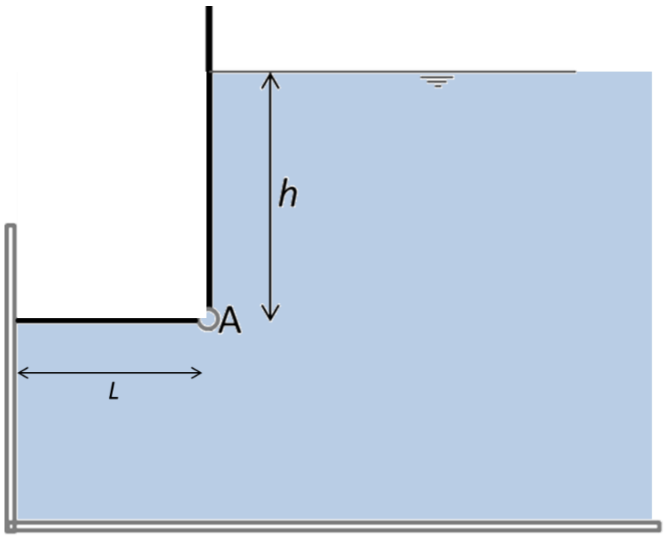

Figure 13. Sketch of the static system in example 13.

Solution:

$$\gamma \frac{h^3}{6} = \gamma h \frac{L^2}{2}, \quad L = \frac{h}{\sqrt{3}}.$$

Example 14. Compute the torque about the hinge in A for the surface made of two rectangular walls and a central semicircle. Consider $D = 6m$, and $\gamma = 10^4 \, N/m^3$.

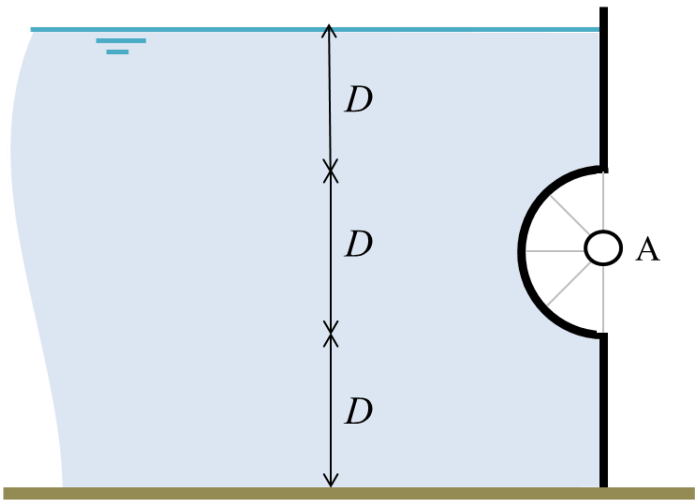

Figure 14. Sketch of the static system in example 14.

Solution:

The circular surface does not generate torque about A. The upper and lower straight walls give, respectively

$$T_u = \gamma\frac{D^2}{2}\left(\frac{D}{3}+\frac{D}{2}\right) = 9\ 10^5\text{J}\ ;$$
$$T_l = \gamma\frac{D^2}{2}\left(\frac{2}{3}D+\frac{D}{2}\right) + \gamma 2D^2\left(\frac{D}{2}+\frac{D}{2}\right) = 55.8\ 10^5\text{J}\ ;$$
$$T = T_l - T_u = 46.8\ 10^5\text{J}.$$

Example 15. Compute the ratio between vertical and horizontal components of the force made by fluid on the wall made of a rectangular wall above a semi cylindrical surface. Consider $H = D/2$.

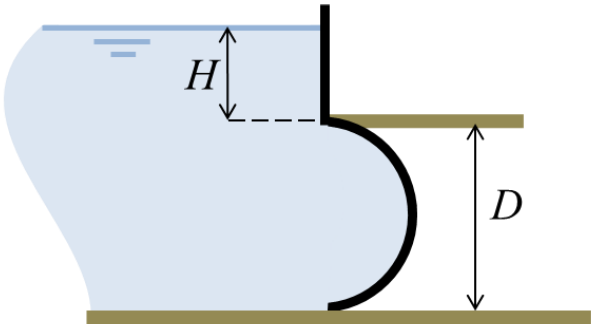

Figure 15. Sketch of the static system in example 15.

Solution:

$$F_H = \gamma\left(H+\frac{D}{2}\right)D + \gamma\frac{H^2}{2} = \frac{9}{8}\gamma D^2,\quad F_V = \gamma\pi\frac{D^2}{8},\quad \frac{F_V}{F_H} = \frac{\pi}{9}.$$

Example 16. Compute the function $M(h)$, of the tilting moment about the hinge A, per unit width, of the oblique wall as a function of the height h. Consider the quote $a = 2h/3$ and gas pressure in the chamber $P_0 = \gamma h$.

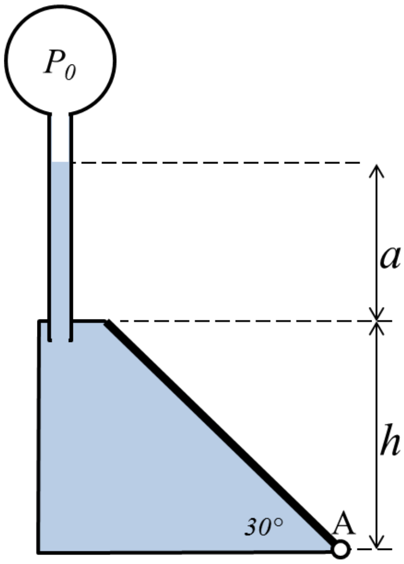

Figure 16. Sketch of the static system in example 16.

Solution:

The length of the wall is $L = \frac{h}{\sin(30°)} = 2h$, and pressure at the top of the wall is equal to that given by a depth H below a free surface, where

$$H = a + \frac{P_0}{\gamma} = \frac{5}{3}h\,.$$

Considering the pressure distribution as the sum of square plus triangular profiles, respectively

$$F_{sq} = \gamma HL, \quad M_{sq} = \gamma HL \cdot \frac{L}{2} = \frac{10}{3}\gamma h^3,$$
$$F_{tr} = \gamma h\frac{L}{2}, \quad M_{tr} = \gamma h\frac{L}{2}\cdot\frac{L}{3} = \frac{2}{3}\gamma h^3;$$

from which the result follows

$$M(h) = \frac{10}{3}\gamma h^3 + \frac{2}{3}\gamma h^3 = 4\gamma h^3.$$

The same result could be found considering the momentum of the entire force acting con the center C of pressure distribution

$$F = \gamma\left(H + \frac{h}{2}\right)L = \frac{13}{3}\gamma h^2, \quad \zeta_C = \frac{2}{3}\frac{(h+H)^3 - H^3}{(h+H)^2 - H^2} = \frac{86}{39}h, \quad r = 2(H + h - \zeta_C) = \frac{12}{13}h;$$

from which the same result follows

$$M(h) = F\cdot r = \frac{13}{3}\gamma h^2 \cdot \frac{12}{13}h = 4\gamma h^3.$$

Example 17. Compute force and torque, about the hinge in A, on the vertical surface (per unit width). Consider P_0=150*mmHg*, h=5m, a=2m; and the specific gravity γ=10KN/m^3.

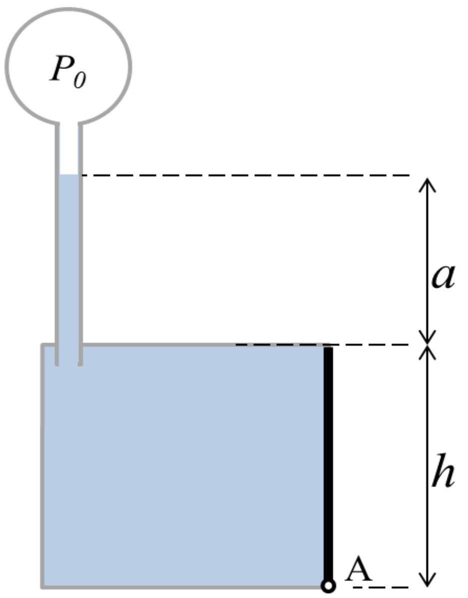

Figure 17. Sketch of the static system in example 17.

Solution:

$$P_0 = 150\text{mmHg} = 20\text{KPa} = \gamma \cdot 2\text{m}\,.$$

Considering the pressure distribution as the sum of square plus triangular pressure profiles, respectively

$$F = F_{sq} + F_{tr} = (P_0 + \gamma a)h + \gamma \frac{h^2}{2} = 200\text{KN} + 125\text{KN} = 325\text{KN}\,,$$
$$T = F_{sq} \cdot \frac{h}{2} + F_{tr} \cdot \frac{h}{3} = 500\text{KJ} + 208\text{KJ} = 708\text{KJ}\,.$$

In alternative, considering the entire force at once

$$h_1 = \frac{P_0}{\gamma} + a = 4\text{m}, \quad h_2 = h_1 + h = 9\text{m}, \quad h_C = \frac{2}{3}\frac{h_2^3 - h_1^3}{h_2^2 - h_1^2} = 6.82\text{m}, = 2.18\text{m}\,;$$
$$F = \left(P_0 + \gamma\left(a + \frac{h}{2}\right)\right)h = 325\text{KN}, \quad T = F \cdot (h_2 - h_C) = 708\text{KJ}\,.$$

Example 18. Compute the static force acting on the semispherical surface. Consider H=3.6m, R=1.6m, γ=9810 N/m^3.

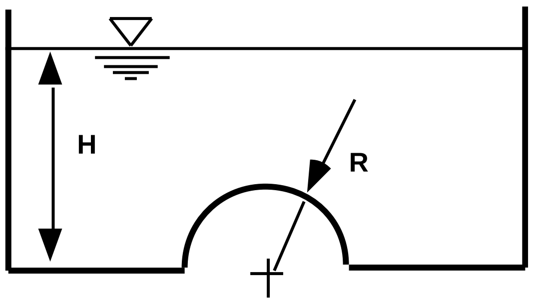

Figure 18. Sketch of the static system in example 18.

Solution:

$$F = \gamma V = \gamma\left(\pi \text{R}^2\text{H} - \frac{2}{3}\pi \text{R}^3\right) = 200\text{KN}\,.$$

Example 19. Compute the horizontal and vertical components of the force made by the fluid on the semispherical surface. Assume *H=50cm*, *D=1m* and the fluid specific gravity γ=9810 N/m^3.

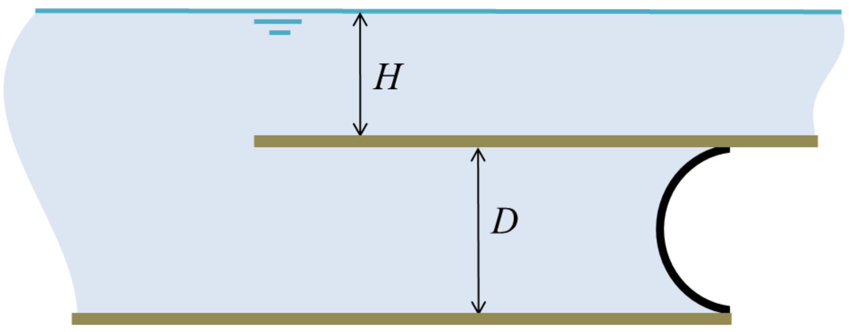

Figure 19. Sketch of the static system in example 19.

Solution:

$$F_H = \gamma\left(H + \frac{D}{2}\right)\pi\frac{D^2}{4} = 7705\text{N}, \quad F_V = \gamma\frac{2}{3}\pi\frac{D^3}{8} = 2568\text{N}\,.$$

Example 20. Compute the force acting on the semispherical surface at the bottom of the bowl. Consider the following relationships $\gamma_{oil} = 2/3\gamma$, $h_1 = 1.8R$, $h_2 = 0.2R$, $h_3 = 1.5R$, where R=3m and γ=9810N/m^3.

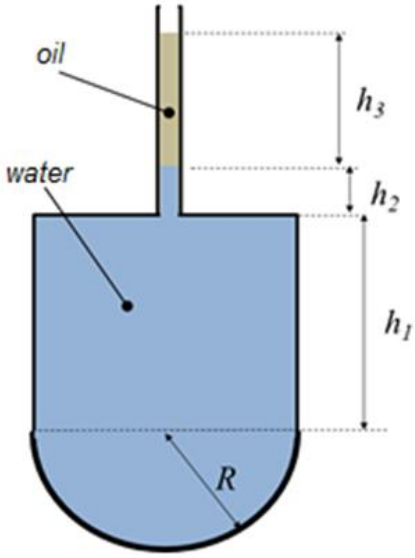

Figure .20. Sketch of the static system in example 20.

Solution:

$$h = h_1 + h_2 + \frac{\gamma_{oil}}{\gamma}h_3 = 3R, \quad F_H = 0, \quad F_V = \gamma h\pi R^2 + \gamma\frac{2}{3}\pi R^3 = \gamma\frac{11}{3}\pi R^3 = 3.06\text{MN}\,.$$