Martedì 18 mattino

Esercizi ogni mercoledì dalle 14 in aula 2
Meccanica Applicata 1° piano C5

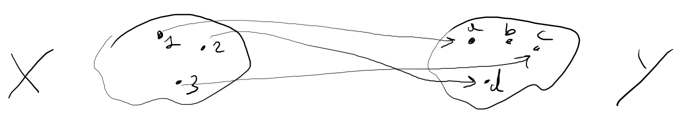
Grafico di una funzione

$$f : X \to Y$$

$$\Gamma_f = \{ (x,y) \in X \times Y \; ; \; y = f(x) \}$$

$f : X \to Y$ si dice iniettiva se

$$x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$$

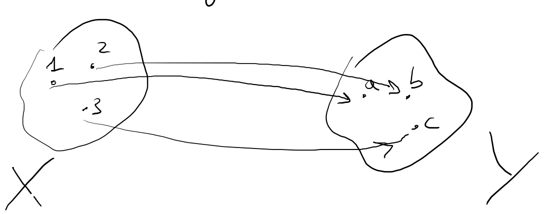


iniettiva

$f : X \to Y$ si dice suriettiva se $\forall y \in Y$

$\exists x \in X$ t.c. $f(x) = y$.

Le funzioni che sono sia iniettive che suriettive vengono dette biettive.



Data $f : X \to Y$ biettiva, si può definire la funzione inversa $f^{-1} : Y \to X$

dove $f^{-1}(y) = x \iff f(x) = y$

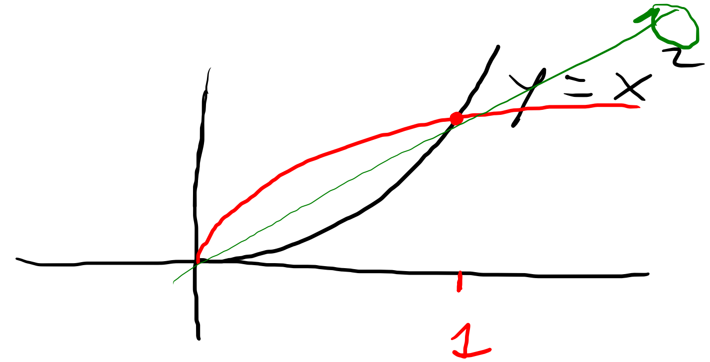
$$\Gamma_f = \{ (x, y) \in X \times Y : y = f(x) \}$$

$$\Gamma_{f^{-1}} = \{ (y,x) \in Y \times X : x = f^{-1}(y) \} = $$
$$= \{ (y,x) \in Y \times X : y = f(x) \}$$

$f(x) = x^2$

$x \geq 0$

$f: [0,+\infty) \to [0,+\infty)$

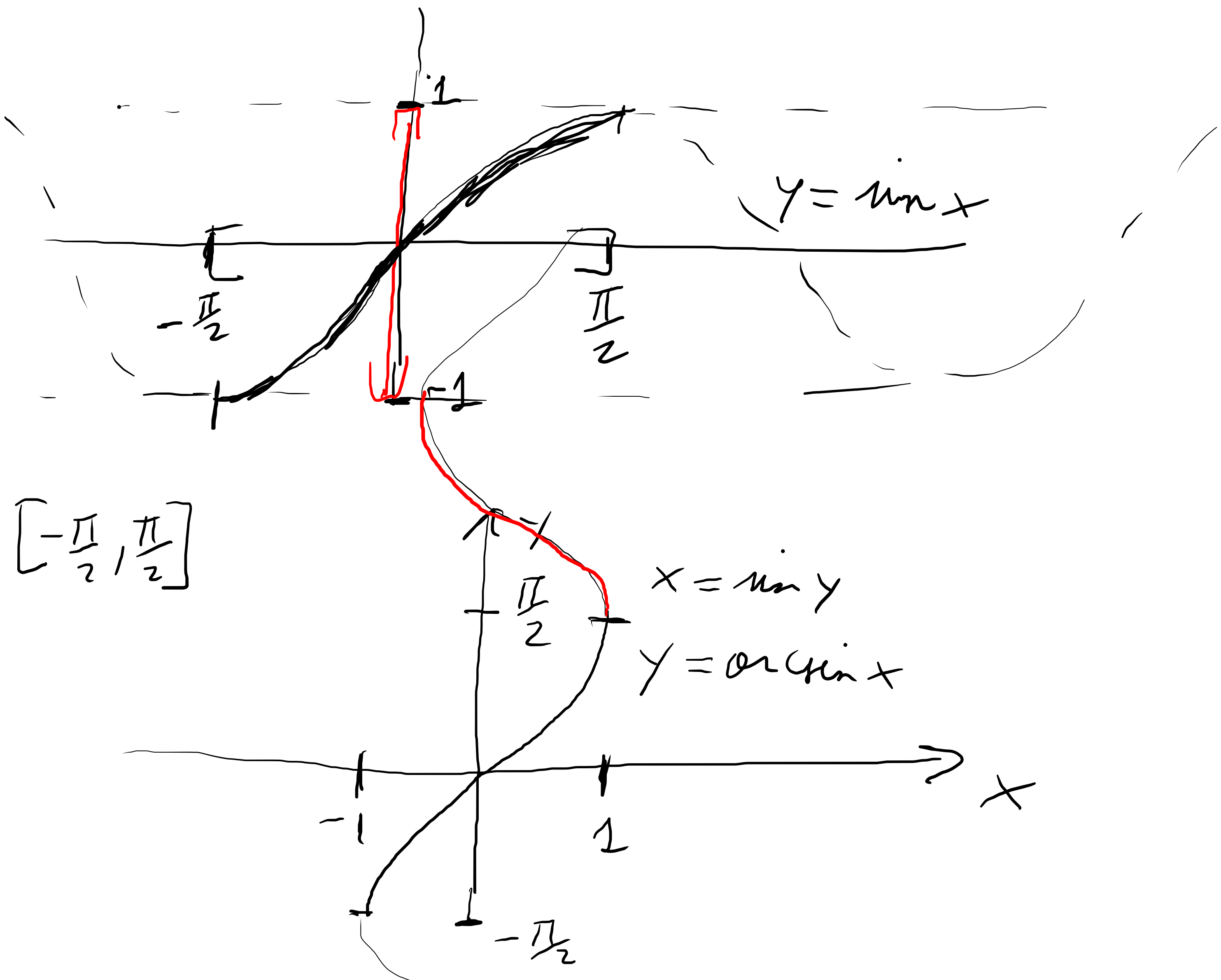


$g = f^{-1}$ $\qquad g(x) = \sqrt{x}$

sin x

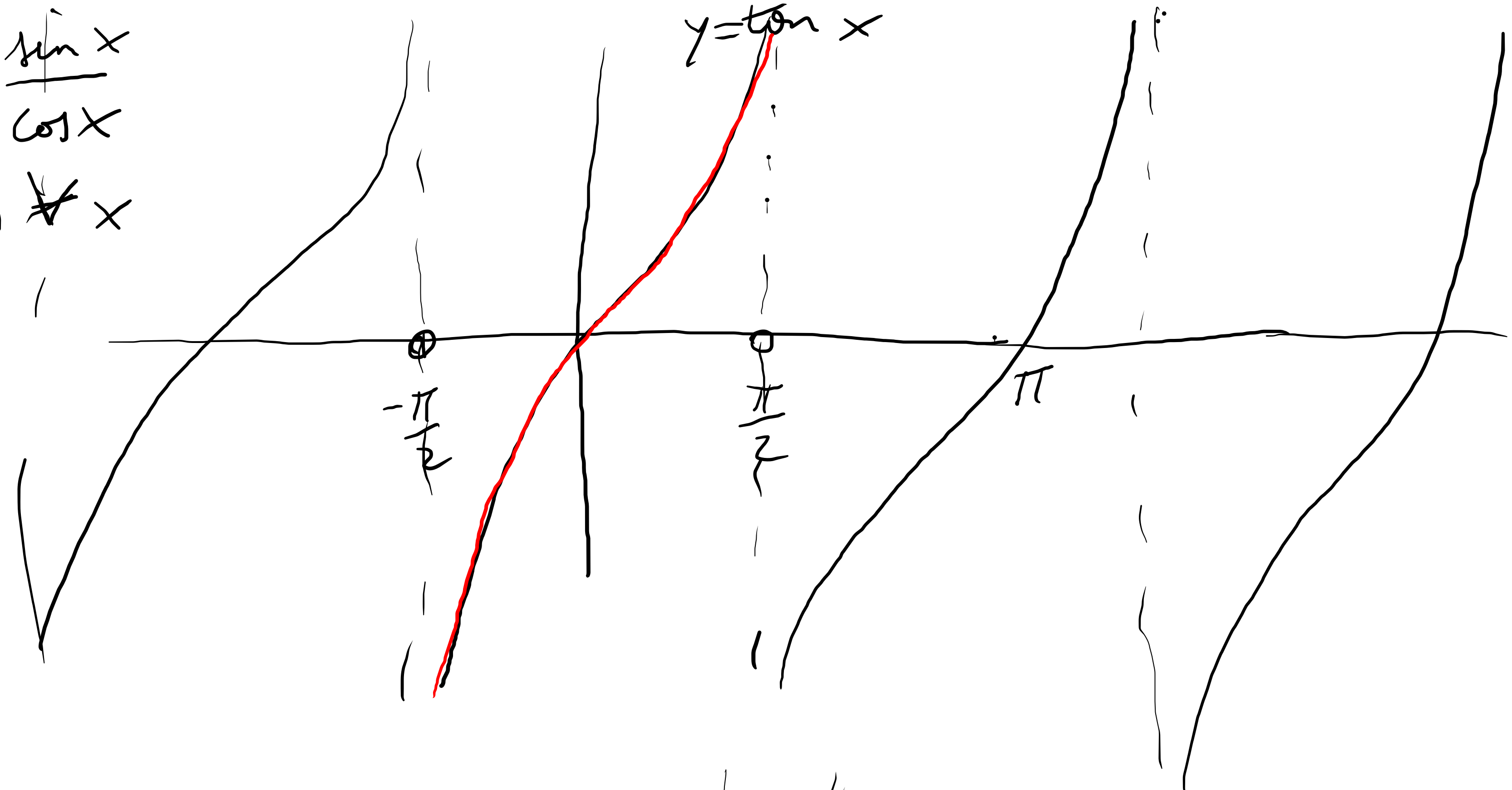
$\sin x : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$

$\arcsin : [-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$



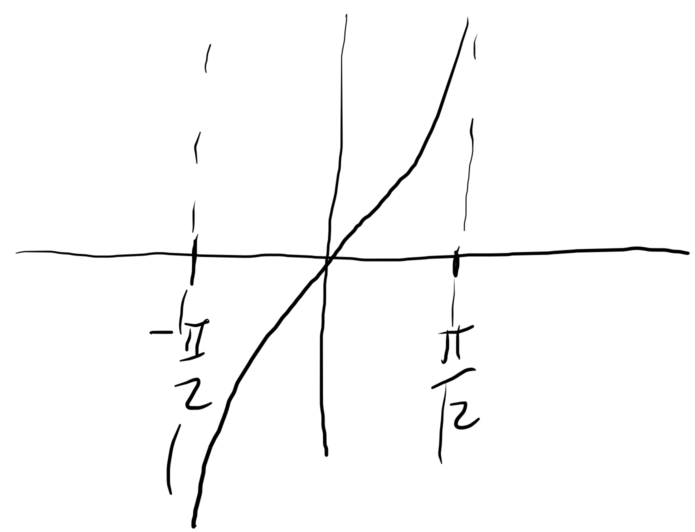
$\tan x = \dfrac{\sin x}{\cos x}$

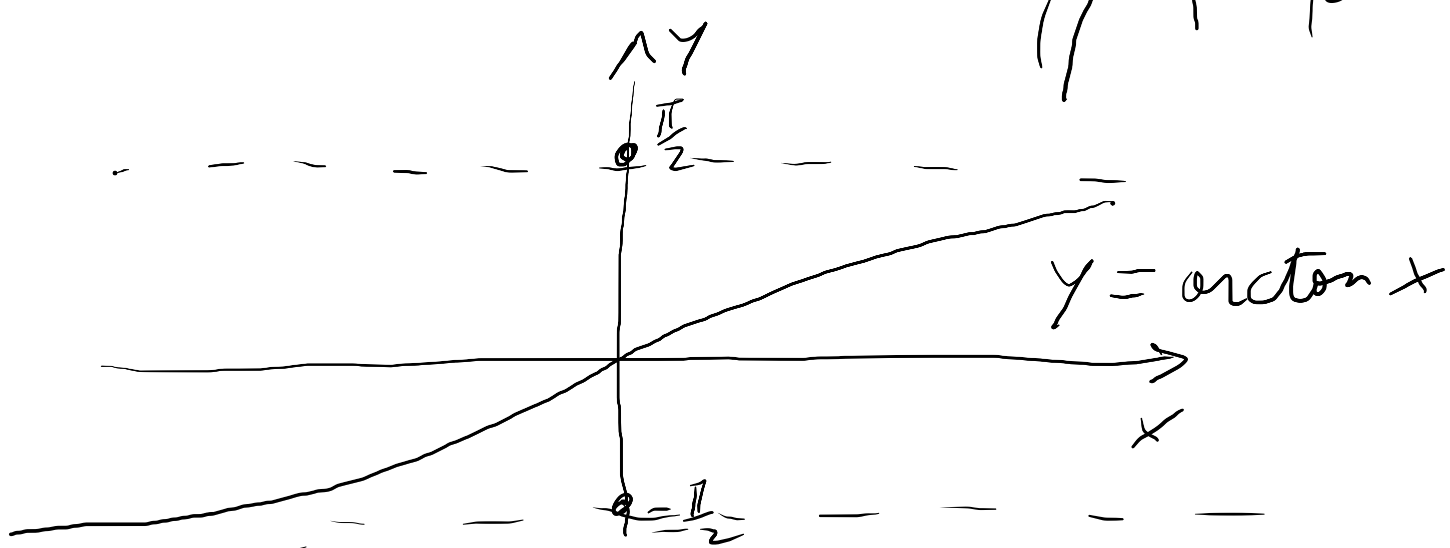
$\tan(x+\pi) = \tan(x) \ \forall x$



$\tan : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$

$\arctan : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



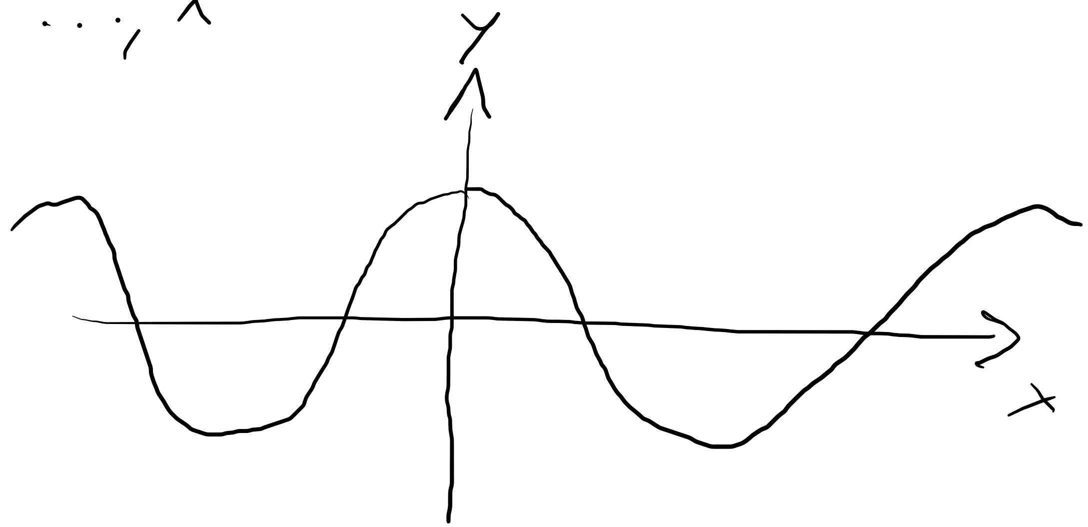


$f: \mathbb{R} \to \mathbb{R}$ è pari se

$f(x) = f(-x) \quad \forall\, x \in \mathrm{dom}\, f$

Es $f(x) = x^2, x^4, \dots, x^{2n}$

$f(x) = \cos x$



$$D(x) = \begin{cases} 1 & \text{se } x \in \mathbb{Q} \\ 0 & \text{se } x \notin \mathbb{Q} \end{cases}$$

è pari perché

$x \in \mathbb{Q} \iff -x \in \mathbb{Q}$ e quindi $D(x) = D(-x)$ $\forall x \in \mathbb{R}$

$[x]$ è pari?

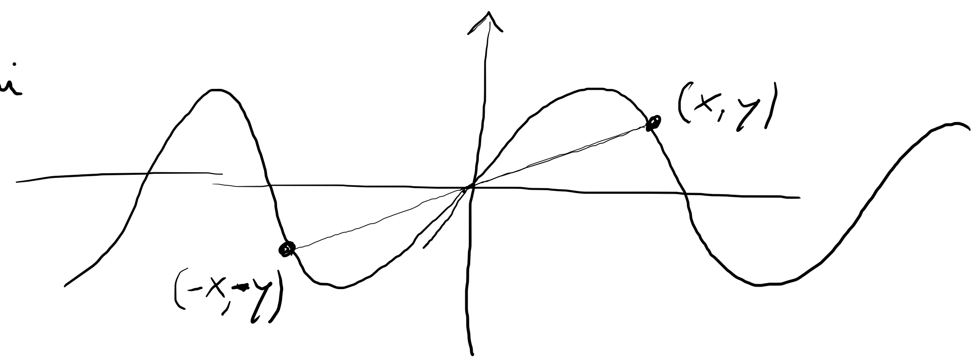
$[10] = 10 \qquad [-10] = -10 \implies [10] \neq [-10]$

$f: \mathbb{R} \to \mathbb{R}$ si dice dispari se $f(-x) = -f(x)$

$\forall x \in \text{dom} f$

Esempi $f(x) = x, \; x^3, \; x^5, \; x^{2n+1}$ $\forall n \in \mathbb{R}$

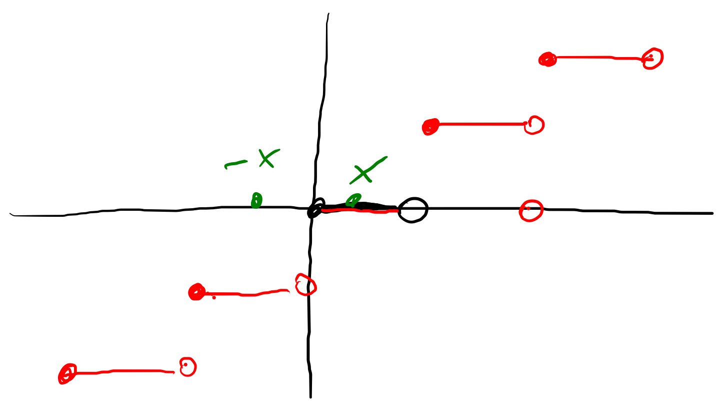
$\sin(x)$ è dispari



$[x]$ è dispari ?

$$[-x] \overset{?}{=} -[x]$$

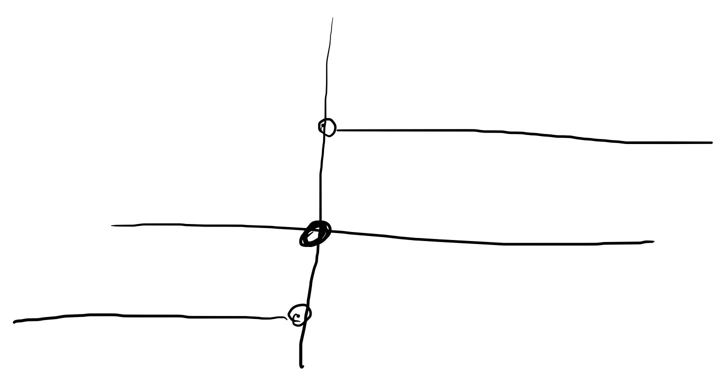
per $x \in (0, 1)$



$[x] = 0$

$[-x] = -1$

$$\text{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$



$f(-0) = -f(0)$

"$f(0) =$

$|z|^4 = r^4$

Esercizio $\frac{z^8}{\sqrt{2}} + z^4|z|^4 = 1$ $z^4 = r^4(\cos(4\vartheta)+i\,\mathrm{sen}(4\vartheta))$

$z = r(\cos\vartheta + i\sin\vartheta)$

$$\frac{1}{\sqrt{2}} r^8\left(\cos(8\vartheta)+i\sin(8\vartheta)\right) + r^8\left(\cos(4\vartheta)+i\sin(4\vartheta)\right) = 1$$

$$\begin{cases} \frac{1}{\sqrt{2}} r^8 \cos(8\vartheta) + r^8\cos(4\vartheta) = 1 \\ \frac{1}{\sqrt{2}} \cancel{r^8} \sin(8\vartheta) + \cancel{r^8}\sin(4\vartheta) = 0 \end{cases} \qquad r \neq 0$$

$$r^8\left(\frac{2\sin(4\vartheta)\cos(4\vartheta)}{\sqrt{2}} + \sin(4\vartheta)\right) = 0$$

$$\sin(4\vartheta)\left(\sqrt{2}\cos(4\vartheta)+1\right) = 0 \qquad \cos(8\vartheta) = 2\cos^2(4\vartheta) - 1$$

$= 1$

per $\sin(4\vartheta) = 0 \Rightarrow \cos(4\vartheta) = \pm 1 \Rightarrow \cos(8\vartheta) = 1$

$\frac{1}{\sqrt{2}} r^8 \pm r^8 = 1$ il caso $\cos(4\vartheta) = -1$

non è ammissibile $\left(\frac{1}{\sqrt{2}} - 1\right) r^8 = 1$ non ha soluzioni

perché $\left(\frac{1}{\sqrt{2}} - 1\right) r^8 \leq 0 < 1$

Se invece $\cos(4\vartheta) = 1$ $\left(\frac{1}{\sqrt{2}} + 1\right) r^8 = 1$

$$r^8 = \frac{1}{\frac{1}{\sqrt{2}}+1} \qquad r = \sqrt[8]{\frac{1}{\frac{1}{\sqrt{2}}+1}}$$

$$\begin{cases} \cos(4\vartheta) = 1 \\ \sin(4\vartheta) = 0 \end{cases} \qquad 4\vartheta = 2\pi k \qquad k \in \mathbb{Z}$$

$$\vartheta = \frac{2\pi k}{4} \qquad k = 0, 1, 2, 3$$

$$\cos(4\vartheta) = -\frac{1}{\sqrt{2}} \qquad \sin(4\vartheta) = \pm\frac{1}{\sqrt{2}}$$

$$\overbrace{\frac{r^8}{\sqrt{2}}\cos(8\vartheta)}^{0} + \cos(4\vartheta) r^8 = 1$$

$$\cos(8\vartheta) = 2\cos^2(4\vartheta) - 1 = 2\left(-\frac{1}{\sqrt{2}}\right)^2 - 1 = 2\,\frac{1}{2} - 1 = 0$$

$-\frac{1}{\sqrt{2}} r^8 = 1$ non ha soluzioni

$-\frac{1}{\sqrt{2}} r^8 \leq 0 < 1$ $\forall\ r \geq 0$

Se $f: X \longrightarrow Y$ e un $A \subseteq X$

resta definito $f_{|_A} : \cancel{A} \longrightarrow Y$

$$f_{|_A}(a) = f(a) \quad \forall a \in A \subseteq X.$$

Per $A \subseteq X$ con $f(A) = \{ y \in Y : y = f(a) \text{ per } a \in A \}$

Se $B \subseteq Y$ $\quad f^{-1}(B) = \{ x \in X : f(x) \in B \}$

$$
\begin{tikzcd}
X \arrow[r, "f"] \arrow[rr, bend right] & Y \arrow[r, "g"] & Z
\end{tikzcd}
$$

$$
\begin{tikzcd}
x \arrow[r] \arrow[rr, bend right, "g \circ f"'] & f(x) \arrow[r] & g(f(x))
\end{tikzcd}
$$

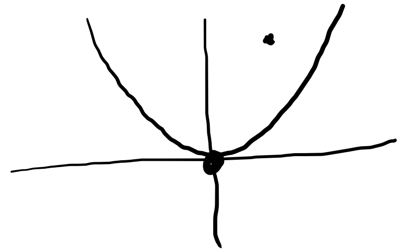
$$g \circ f(x) = g(f(x))$$

$$f(x) = \sin(x) \qquad g(x) = x + 2$$

$$g(f(x)) = g(\sin(x)) = \sin(x) + 2$$

$$f(g(x)) = \sin(g(x)) = \sin(x+2) \neq \sin(x) + 2$$

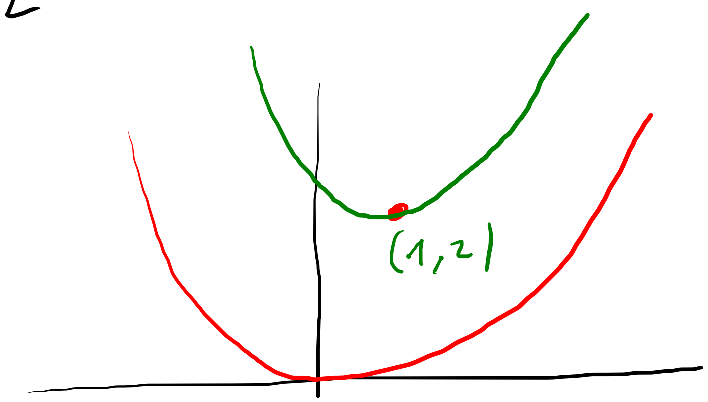
$y = x^2$



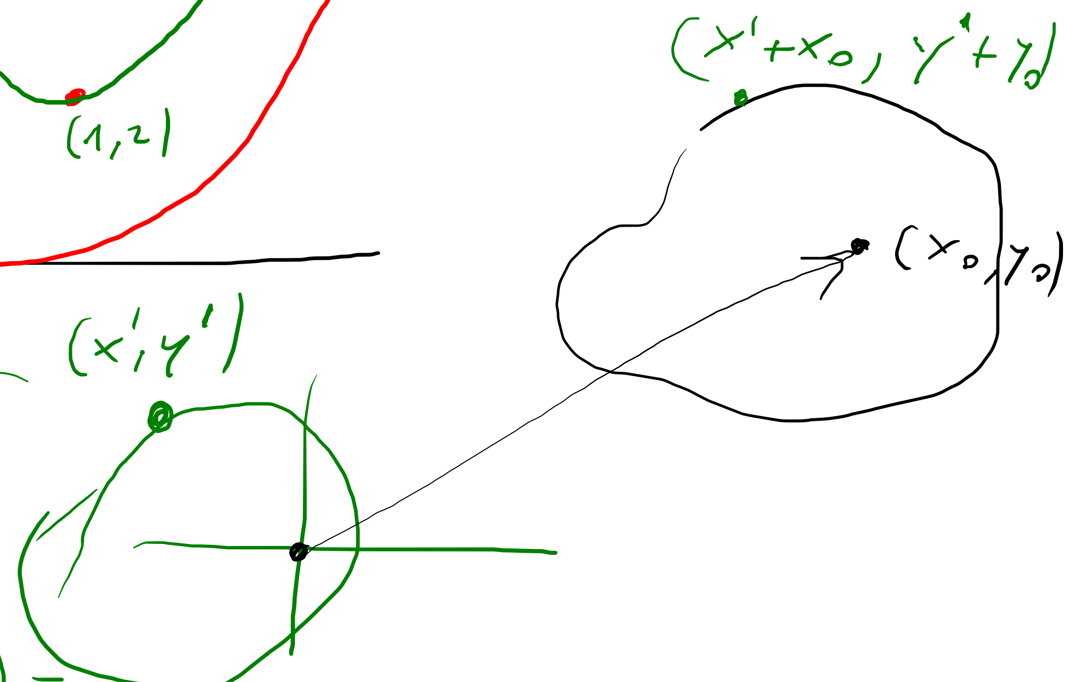
$y - y_0 = (x - x_0)^2$

$(x_0, y_0) = (1, 2)$

$y - 2 = (x - 1)^2$



$f(x, y) = 0$

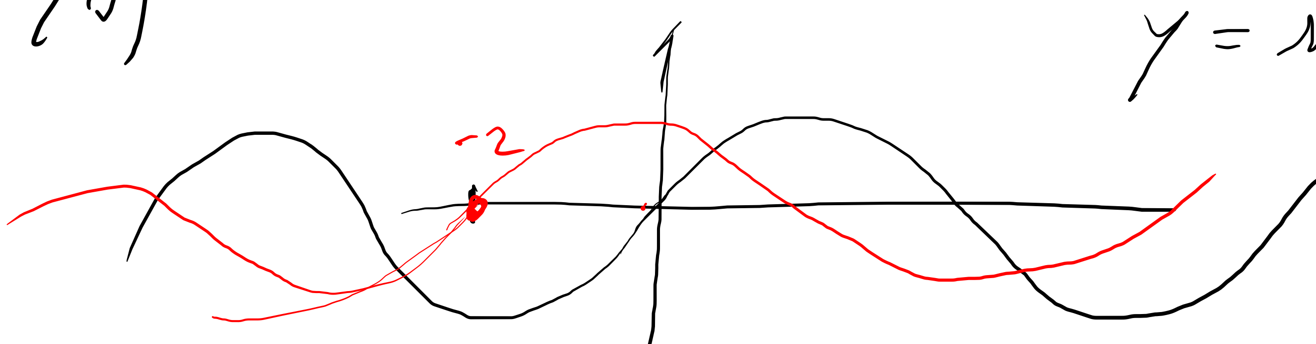


se $f(x', y') = 0$

segue che $f((x' + \cancel{x_0}) - \cancel{x_0}, (y' + \cancel{y_0}) - \cancel{y_0}) =$

$f(x - x_0, y - y_0) = 0$ $\quad = f(x', y') = 0$

$y = \sin(x + 2)$



$y = \sin(x)$

$f : \mathbb{R} \to \mathbb{R}$ è crescente se

$x_1 \leq x_2 \Rightarrow f(x_1) \leq f(x_2)$

$\left(f(x_1) < f(x_2) \right.$ strettamente crescente)

decrescente

$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$

$\left(f(x_1) > f(x_2) \right.$ strettamente decrescente

$[x]$ è crescente ma non è strettamente crescente $0 < \frac{1}{2}$ $[0] = [\frac{1}{2}] = 0$

$D(x)$ non è crescente perché

$D(0) = 1$ $D(\pi) = 0$ ma non è

nemmeno decrescente perché

$D(-\pi) = 0$ $D(0) = 1$.