Recap for SEISMIC RISK: SEISMIC WAVES

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UNIVERSITÀ DEGLI STUDI DI TRIESTE

Some basic definitions - 1

Principles of mechanics applied to bulk matter:

Mechanics of fluids   Mechanics of solids

Continuum Mechanics

A material can be called solid (rather than -perfect- fluid) if it can support a shearing force over the time scale of some natural process.

Shearing forces are directed parallel, rather than perpendicular, to the material surface on which they act.

Some basic definitions - 2

When a material is loaded at sufficiently low temperature, and/or short time scale, and with sufficiently limited stress magnitude, its deformation is fully recovered upon uploading:

the material is **elastic**

If there is a permanent (plastic) deformation due to exposition to large stresses:

the material is **elastic-plastic**

If there is a permanent deformation (viscous or creep) due to time exposure to a stress, and that increases with time:

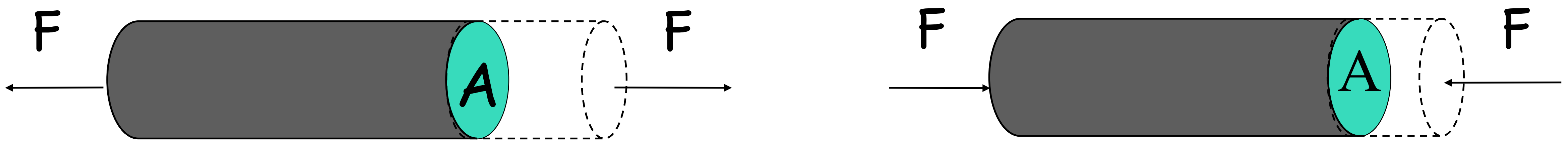
the material is **viscoelastic** (with elastic response), or

the material is **visco-plastic** (with partial elastic response)

Stress as a measure of Force

Normal stress acts perpendicular to the surface

(F=normal force)

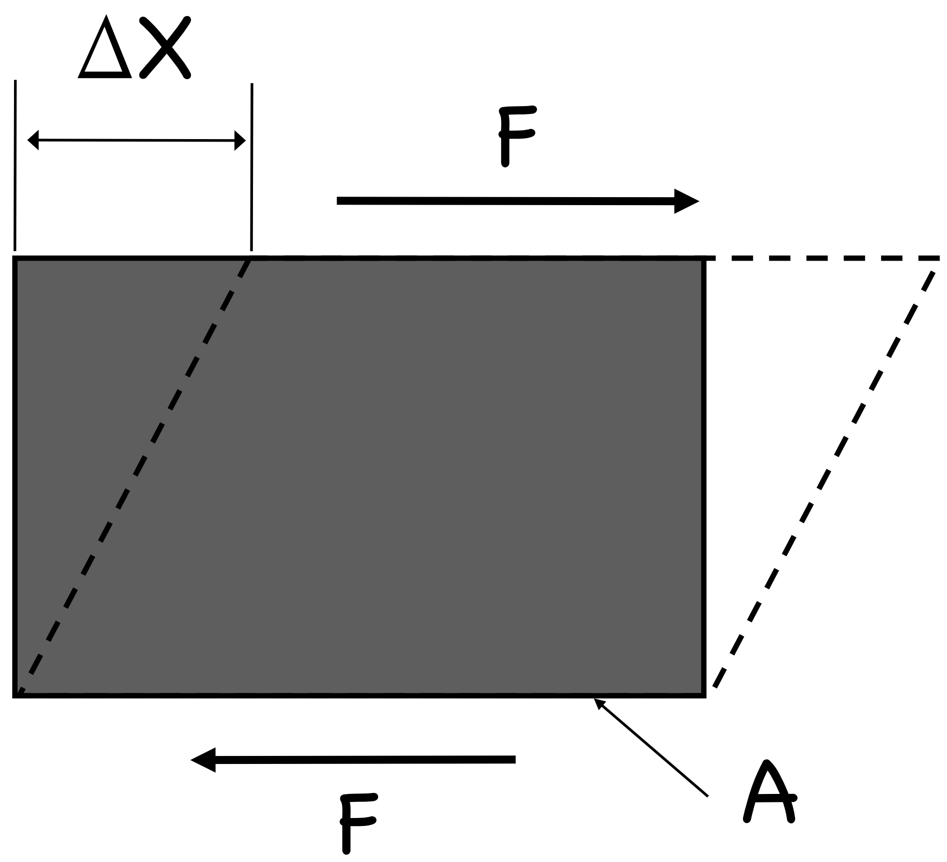


Tensile causes elongation

Compressive causes shrinkage

$$\sigma = \frac{\text{stretching force}}{\text{cross sectional area}}$$

Shear Stress as a measure of Force



$$\tau = \frac{\text{shear force}}{\text{tangential area}}$$

Linear Elastic Properties

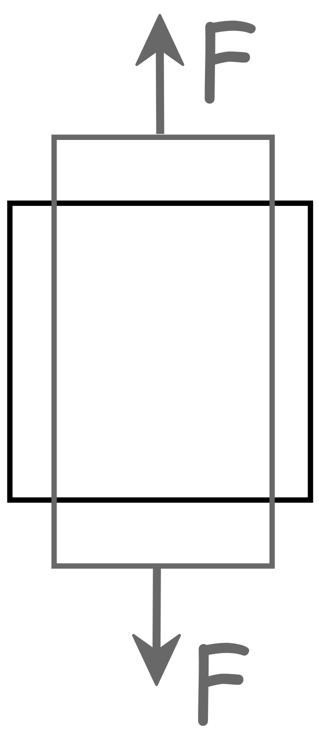
Modulus of Elasticity, E:

(also known as Young's modulus)

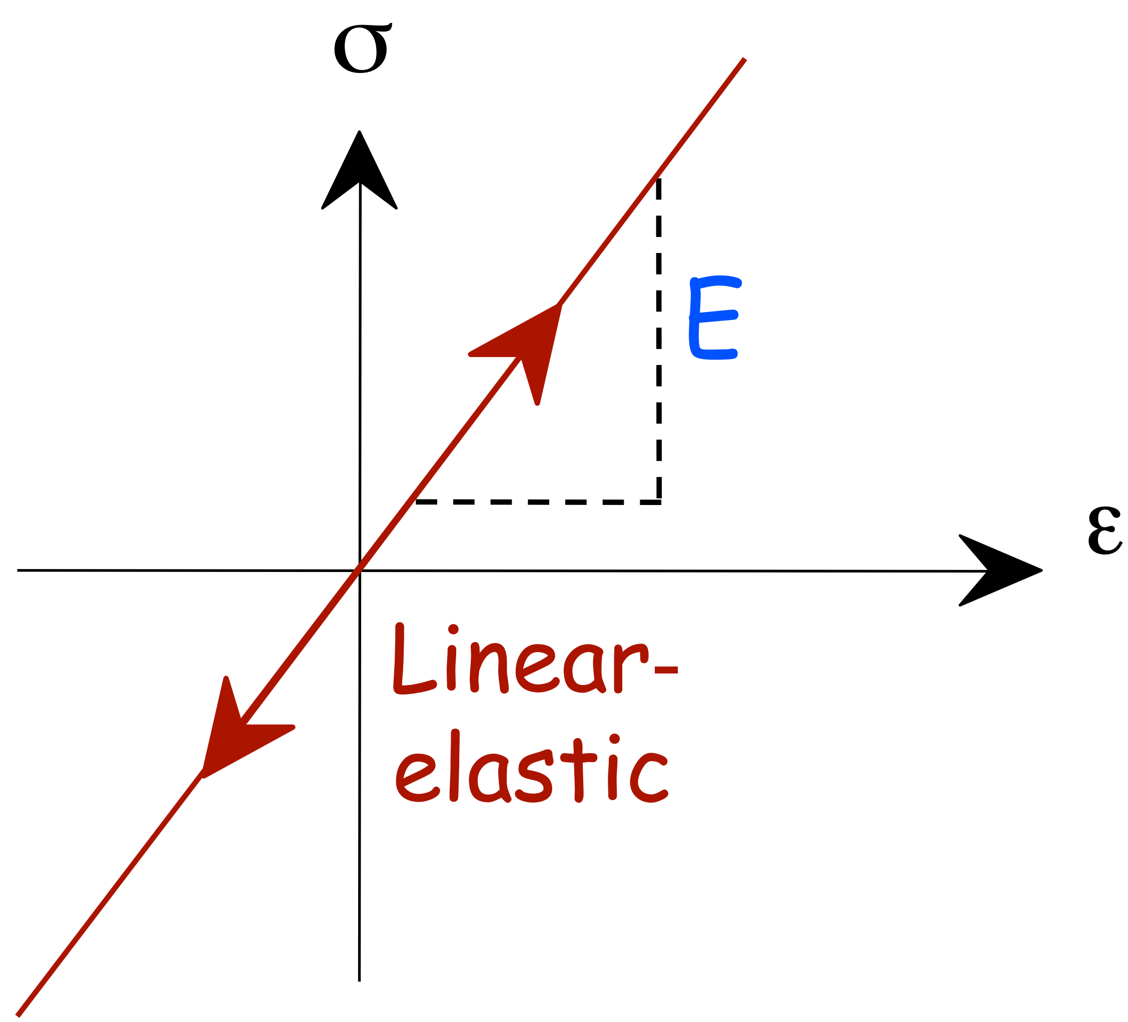
- **Hooke's Law:**

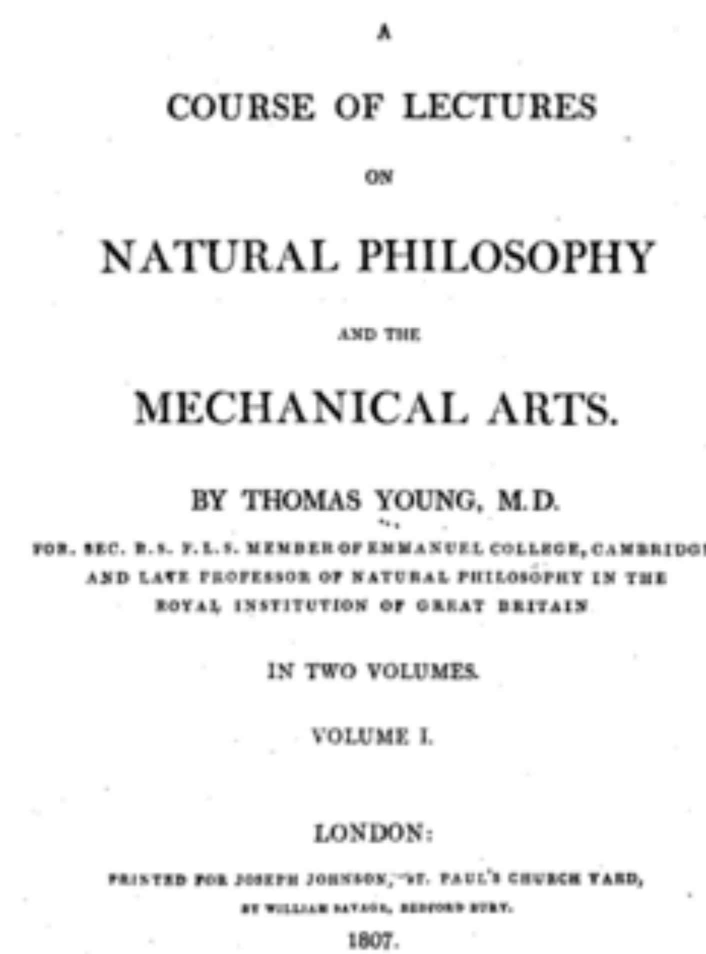
$$\sigma = E\,\varepsilon$$

E: **stiffness** (material's resistance to elastic deformation)



simple tension test




A

COURSE OF LECTURES

ON

NATURAL PHILOSOPHY

AND THE

MECHANICAL ARTS.

BY THOMAS YOUNG, M.D.

FOR. SEC. R.S. F.L.S. MEMBER OF EMMANUEL COLLEGE, CAMBRIDGE, AND LATE PROFESSOR OF NATURAL PHILOSOPHY IN THE ROYAL INSTITUTION OF GREAT BRITAIN

IN TWO VOLUMES.

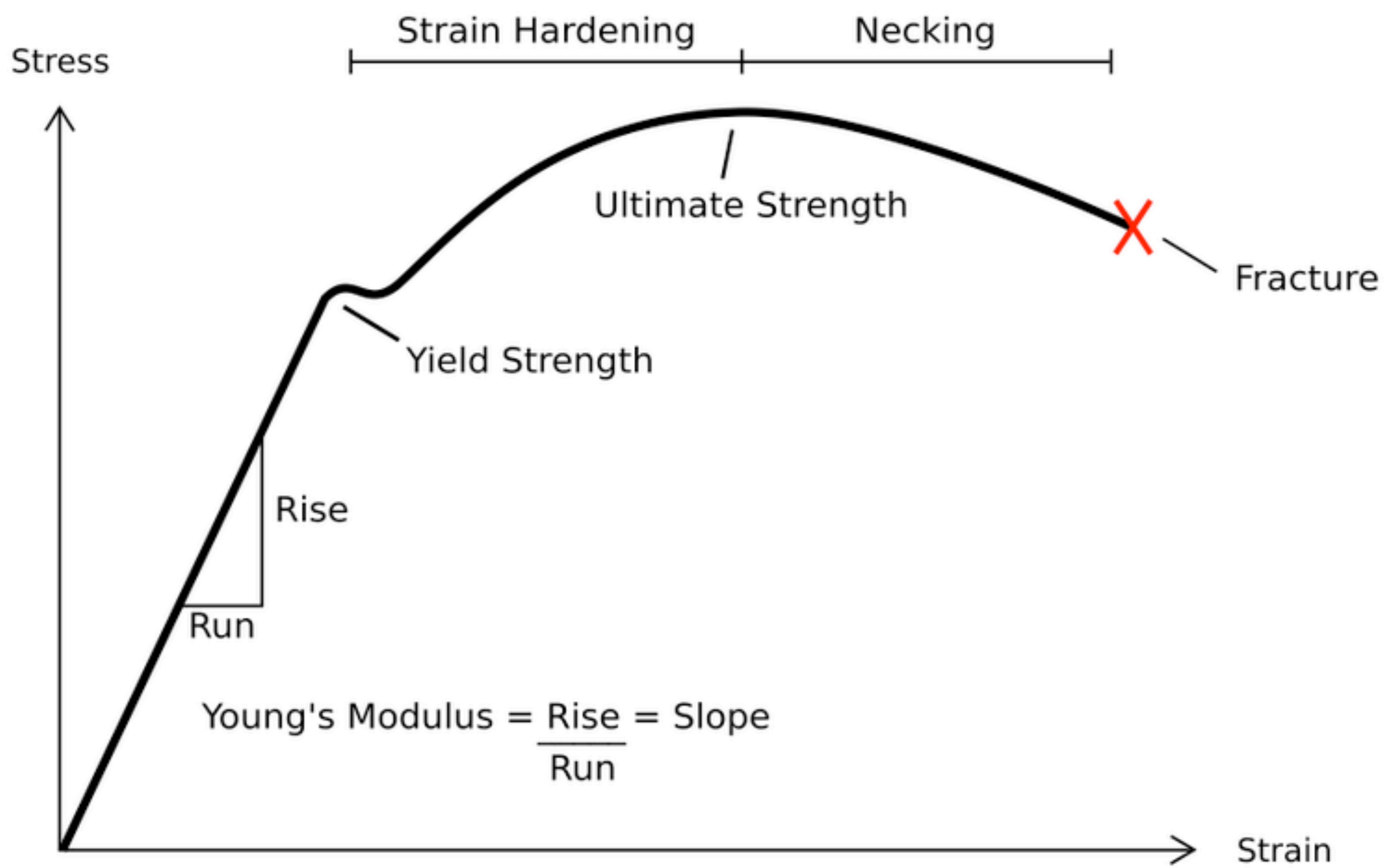
VOLUME I.

LONDON:

PRINTED FOR JOSEPH JOHNSON, ST. PAUL'S CHURCH YARD, BY WILLIAM SAVAGE, BEDFORD BURY.

1807.

Young's modulus



Elasticity Theory

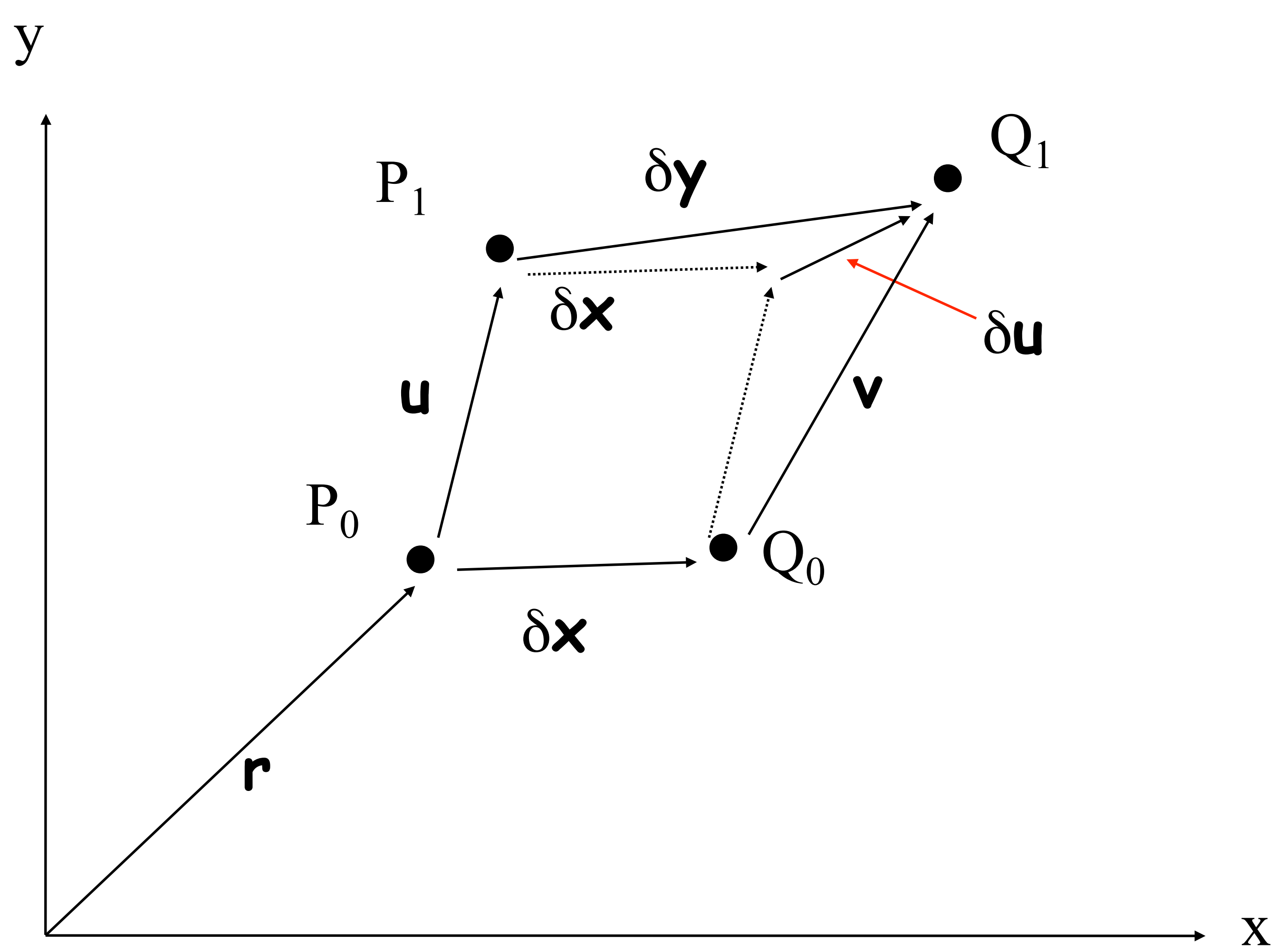
A time-dependent perturbation of an elastic medium (e.g. a rupture, an earthquake, a meteorite impact, a nuclear explosion etc.) generates elastic waves emanating from the source region. These disturbances produce local changes in stress and strain.

To understand the propagation of elastic waves we need to describe kinematically the deformation of our medium and the resulting forces (stress). The relation between deformation and stress is governed by elastic constants.

The time-dependence of these disturbances will lead us to the elastic wave equation as a consequence of conservation of energy and momentum.

Deformation

Let us consider a point P_0 at position r relative to some fixed origin and a second point Q_0 displaced from P_0 by dx



Unstrained state:

Relative position of point P_0 w.r.t. Q_0 is **δx.**

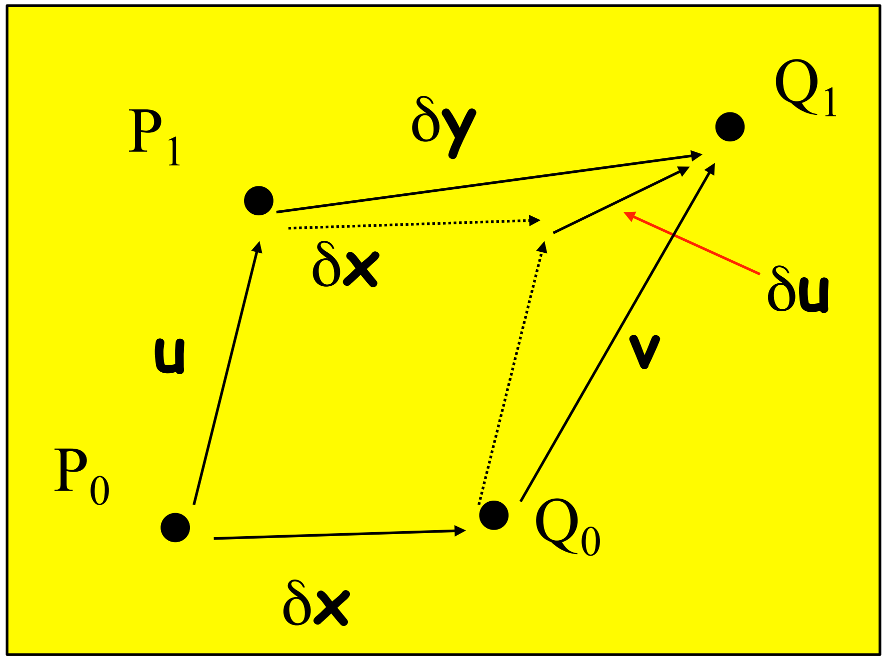
Strained state:

Relative position of point P_0 has been displaced a distance **u** to P_1 and point Q_0 a distance **v** to Q_1.

Relative position of point P_1 w.r.t. Q_1 is **δy= δx+ δu.** The change in relative position between Q and P is just **δu.**

Linear Elasticity

The relative displacement in the unstrained state is **u**(**r**). The relative displacement in the strained state is **v**=**u**(**r**+ δ**x**).



So finally we arrive at expressing the **relative displacement** due to strain:

δ**u**=**u**(**r**+ δ**x**)-**u**(**r**)

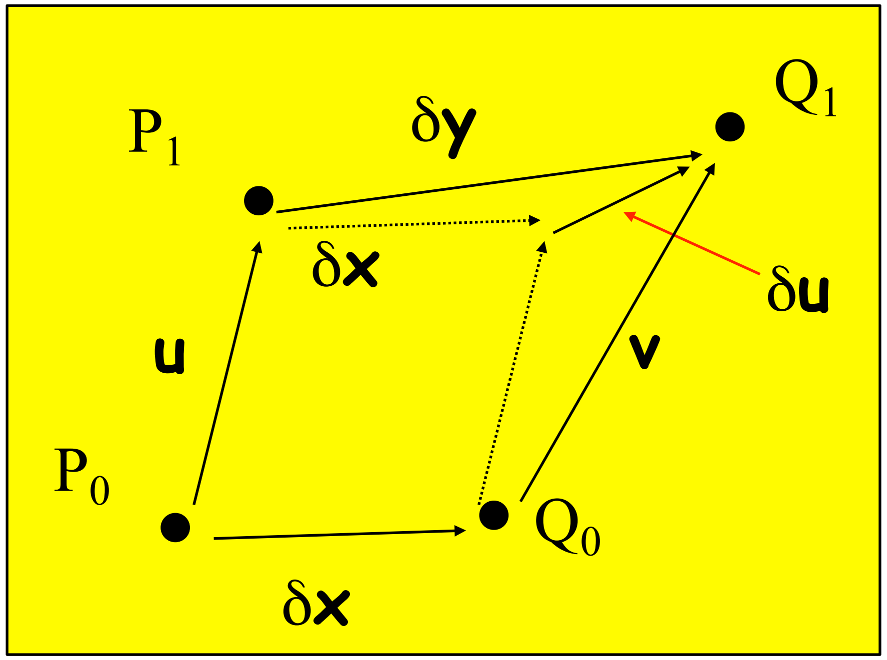
We now apply Taylor's theorem in 3-D to arrive at:

$$\delta u_i = \sum_{k=1,3} \frac{\partial u_i}{\partial x_k} \delta x_k \equiv \frac{\partial u_i}{\partial x_k} \delta x_k$$

What does this equation mean?

The partial derivatives of the vector components

$$\frac{\partial u_i}{\partial x_k}$$



represent a **second-rank tensor** which can be resolved into a **symmetric** and anti-symmetric part:

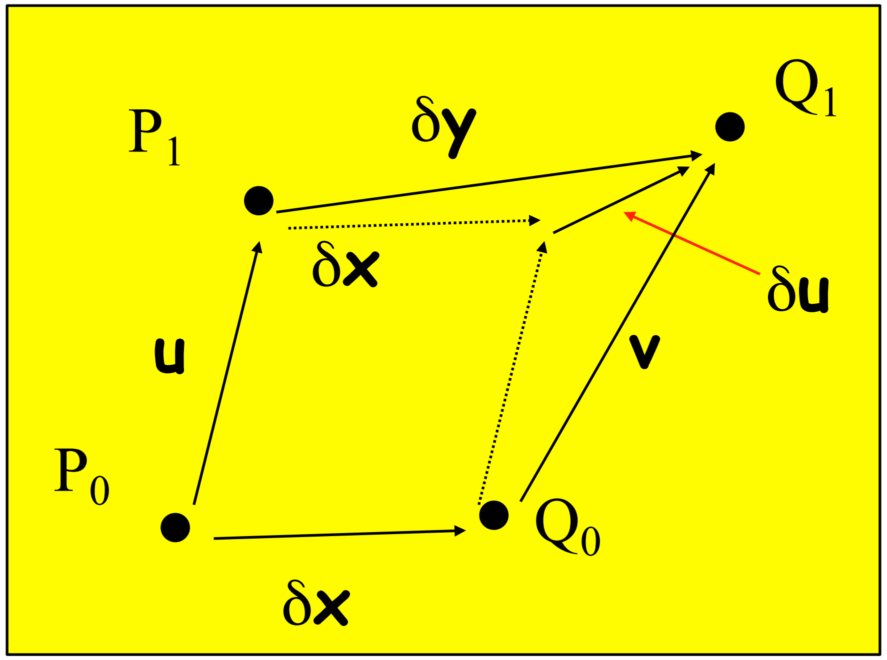
$$\delta u_i = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\delta x_k - \frac{1}{2}\left(\frac{\partial u_k}{\partial x_i} - \frac{\partial u_i}{\partial x_k}\right)\delta x_k$$

- **symmetric**
- **strain**

- antisymmetric
- pure rotation

The symmetric part is called the **strain tensor**

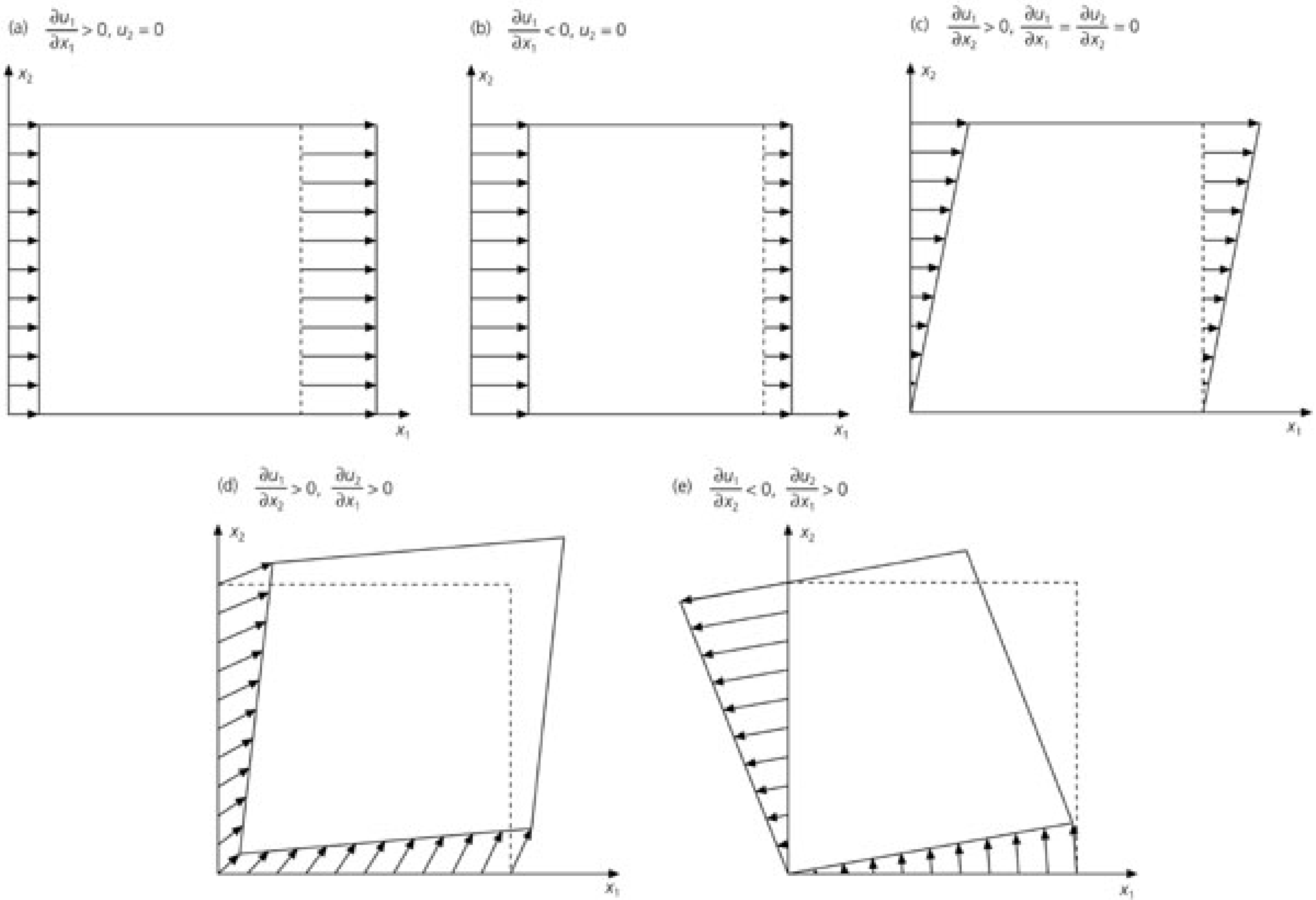
$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$



and describes the relation between deformation and displacement in linear elasticity. In 2-D this tensor looks like

$$\varepsilon_{ij}= \begin{bmatrix} \frac{\partial u_1}{\partial x} & \frac{1}{2}\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right) & \frac{\partial u_2}{\partial y} \end{bmatrix}$$

Figure 2.3-12: Some possible strains for a two-dimensional element.



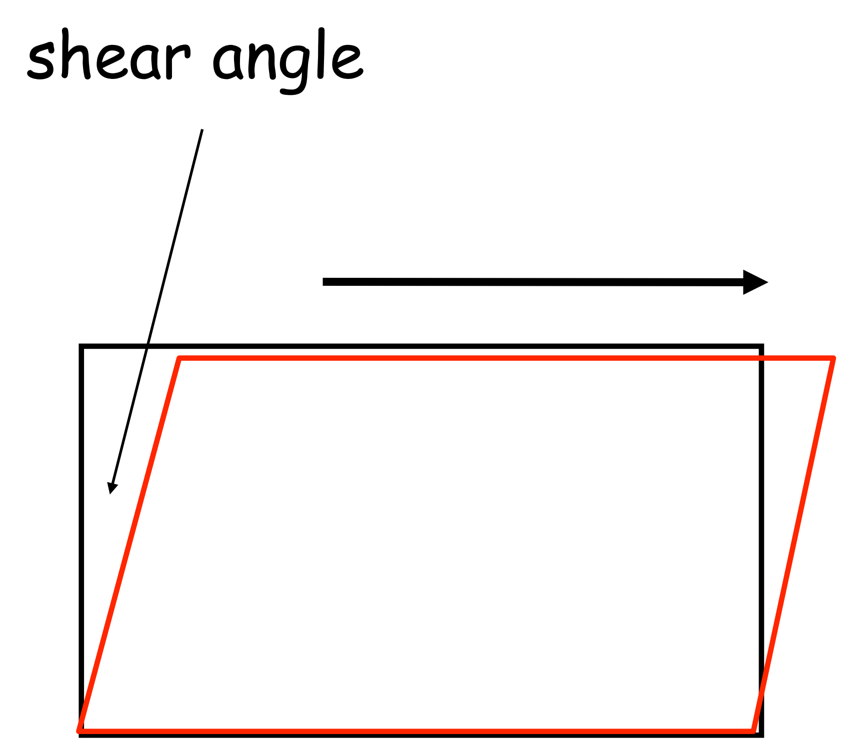
Thus

$$u_1 = \lambda_1 y_1 \qquad u_2 = \lambda_2 y_2 \qquad u_3 = \lambda_3 y_3$$

... in other words ...

the eigenvalues are the relative change of length along the three coordinate axes

$$\lambda_1 = \frac{u_1}{y_1}$$



In arbitrary coordinates the diagonal elements are the relative change of length along the coordinate axes and the off-diagonal elements are the infinitesimal shear angles.

Deformation tensor - trace

The trace of a tensor is defined as the sum over the diagonal elements. Thus:

$$\varepsilon_{ii} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

This trace is linked to the volumetric change after deformation. Before deformation the volume was V_0. Because the diagonal elements are the relative change of lengths along each direction, the new volume after deformation is

$$V = L_1(1+\varepsilon_{11})L_2(1+\varepsilon_{22})L_3(1+\varepsilon_{33})$$

... and neglecting higher-order terms ...

$$V = L_1 L_2 L_3 \left(1+\varepsilon_{ii}\right) \text{ or } V_0\left(1+\varepsilon_{ii}\right)$$

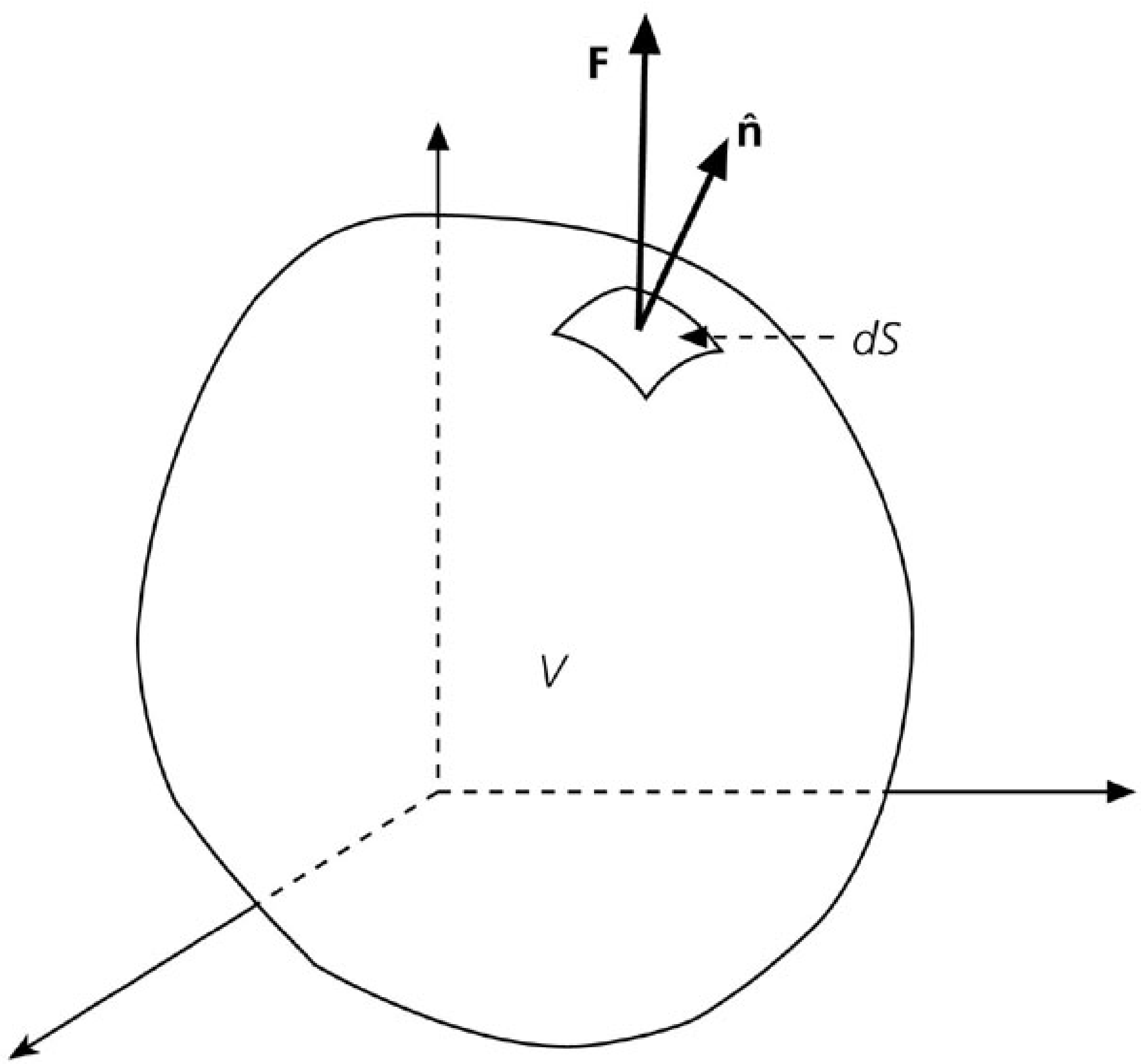
$$\theta = \frac{\Delta V}{V_0} = \varepsilon_{ii} = \frac{\partial u_i}{\partial x_i} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \mathrm{div}\,\mathbf{u} = \nabla \bullet \mathbf{u}$$

Stress - Traction (vector)

In an elastic body there are restoring forces if deformation takes place. These forces can be seen as acting on planes inside the body. **Forces divided by an areas are called stresses.**

In order for the deformed body to remain deformed these forces have to compensate each other.

Figure 2.3-1: Surface force on a volume element.



Traction vector cannot be completely described without the specification of the force ($\Delta \mathbf{F}$) and the surface ($\Delta \mathbf{S}$) on which it acts:

$$\mathbf{T}(\mathbf{n}) = \lim_{\Delta S \to 0} \frac{\Delta F}{\Delta S} = \frac{dF}{dS}$$

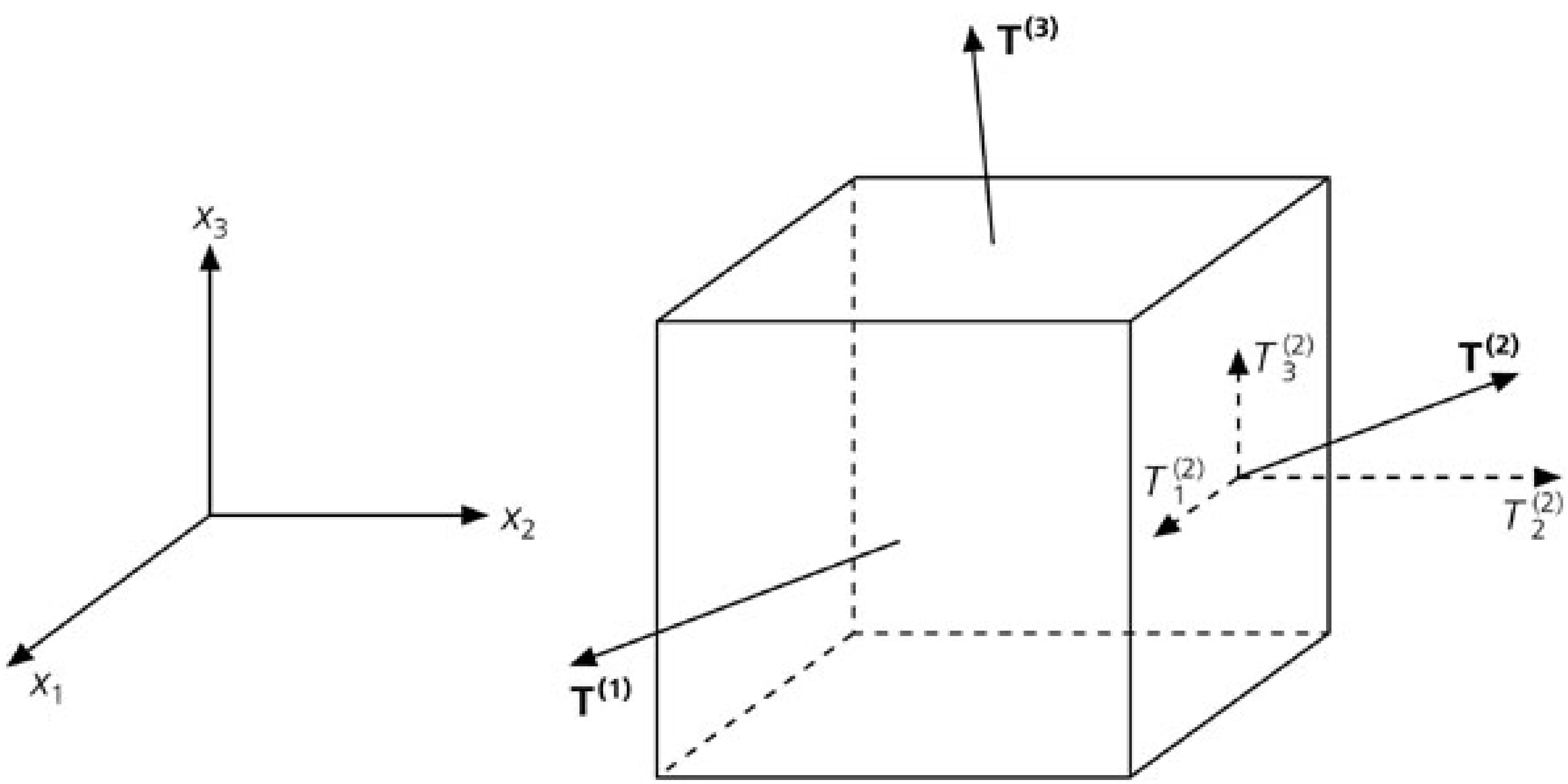
And from the linear momentum conservation, we can show that:

$$\mathbf{T}(-\mathbf{n}) = -\mathbf{T}(\mathbf{n})$$

Stress-Traction (cont.)

Stress acting on a given internal plane can be decomposed in 3 mutually orthogonal components: one normal (direct stress), tending to change the volume of the material, and two tangential (shear stress), tending to deform, to the surface. If we consider an infinitely small cube, aligned with a Cartesian reference system:

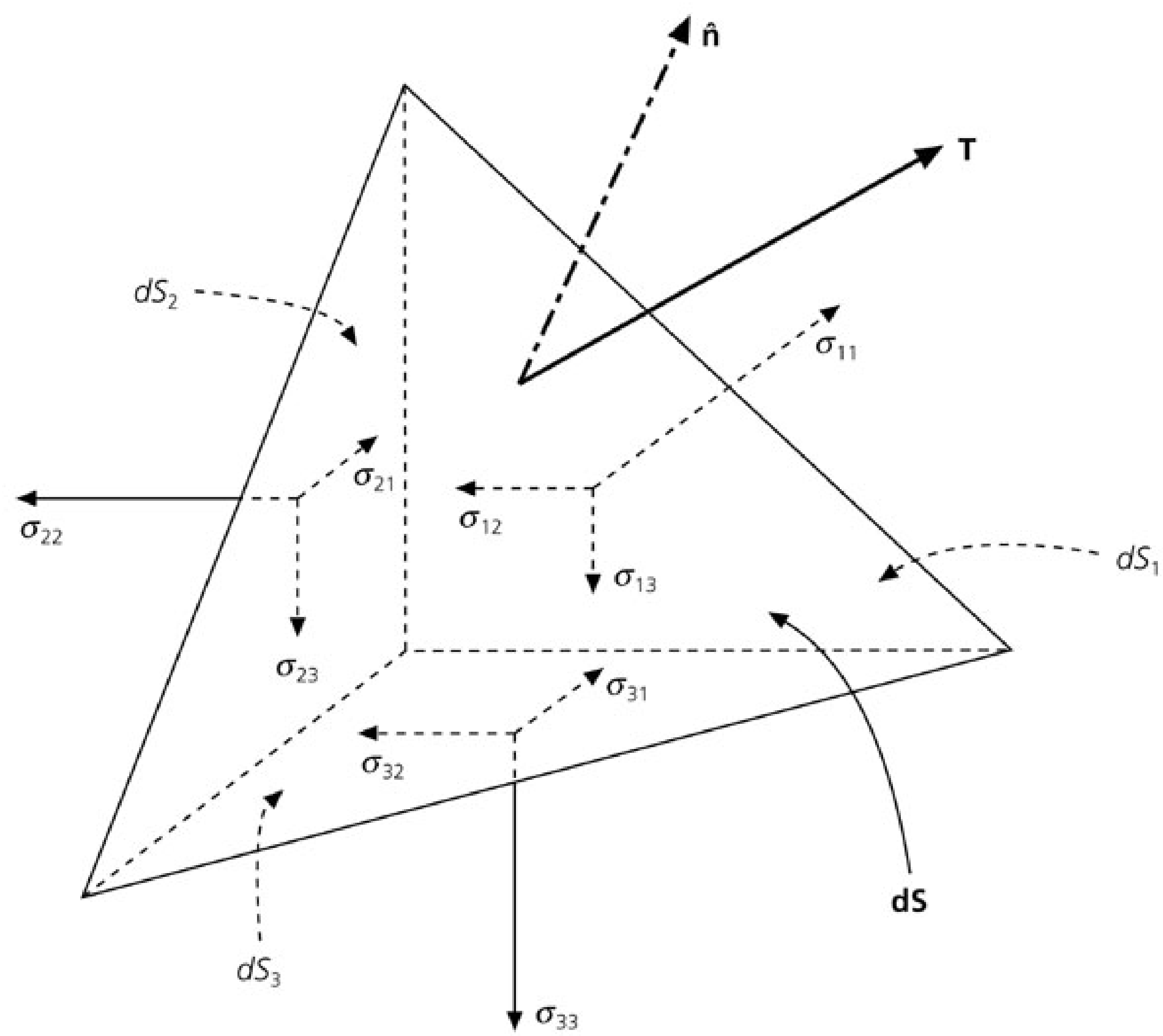
Figure 2.3-2: Traction vectors on the faces of a volume element.



$$\mathbf{T}^{(n)} = n_i \mathbf{T}^{(i)} = n_i T_j^{(i)} \mathbf{e}_j = n_i \sigma_{ij} \mathbf{e}_j$$

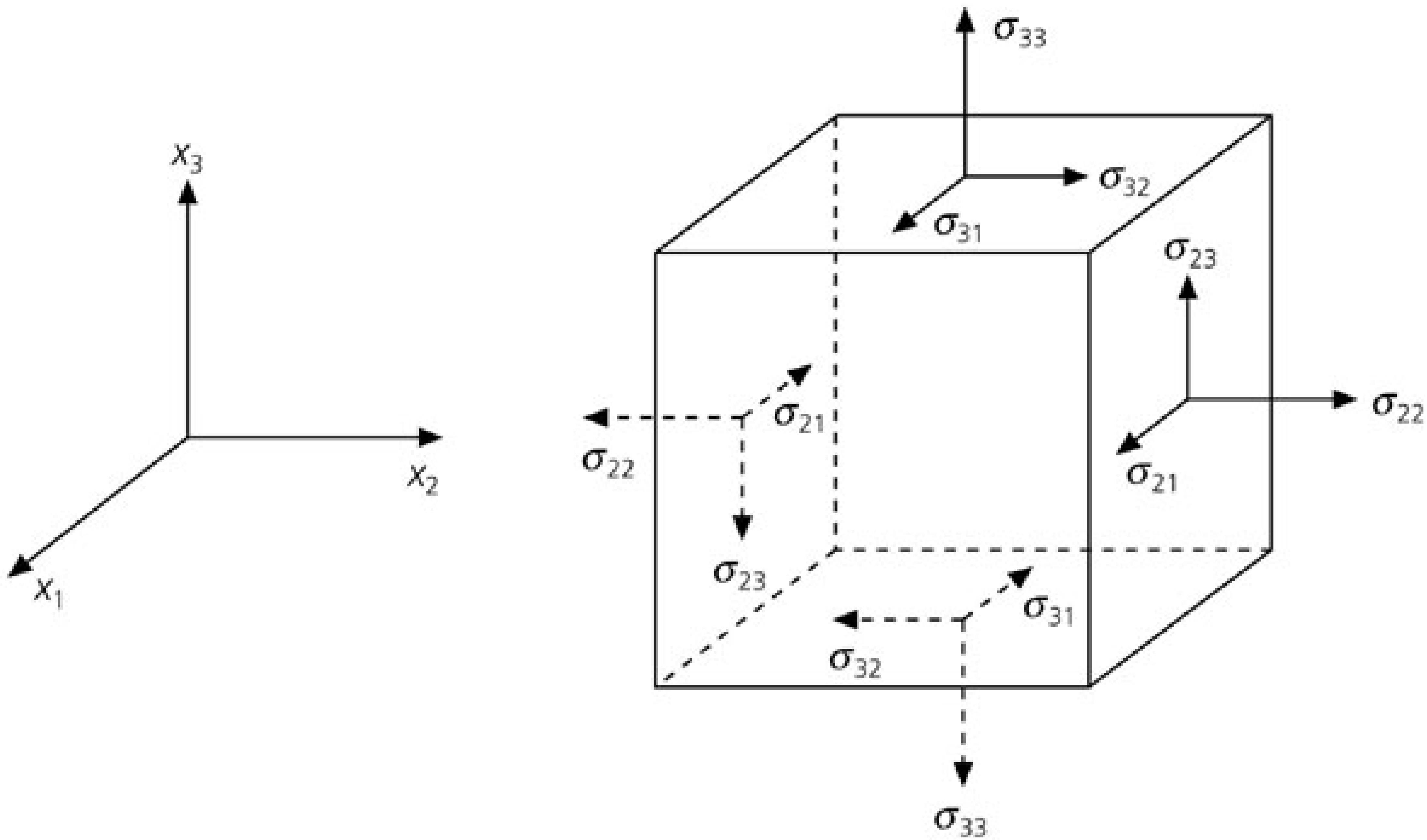
Consider an infinitively small tethraedrum, whose 3 faces are oriented normally to the reference axes. The components of traction **T**, acting on the face whose normal is **n** can be written using the directional cosines referred to versor system **ê**

$$\mathbf{T}^{(n)} = n_i \mathbf{T}^{(i)} = n_i T_j^{(i)} \mathbf{e}_j = n_i \sigma_{ij} \mathbf{e}_j$$



...and the stress state in a point of the material can be expressed with:

Figure 2.3-4: Stress components on the faces of a volume element.



Stress-strain relation

The relation between stress and strain in general is described by the tensor of elastic constants c_{ijkl}

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$

Generalised Hooke's Law

From the symmetry of the stress and strain tensor and a thermodynamic condition if follows that the maximum number if independent constants of c_{ijkl} is 21. In an isotropic body, where the properties do not depend on direction, the relation reduces to

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}$$

Hooke's Law

where λ and μ are the Lame parameters, θ is the dilatation and δ_{ij} is the Kronecker delta.

$$\theta\delta_{ij} = \varepsilon_{kk}\delta_{ij} = \left(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}\right)\delta_{ij}$$

Elastic parameters

Rigidity is the ratio of pure shear strain and the applied shear stress component

$$\mu = \frac{\sigma_{ij}}{2\varepsilon_{ij}}$$

Bulk modulus of incompressibility is defined the ratio of pressure to volume change. Ideal fluid means no rigidity (μ = 0), thus λ is the incompressibility of a fluid.

$$K = -\frac{P}{\theta} = \lambda + \frac{2}{3}\mu$$

Consider a stretching experiment where tension is applied to an isotropic medium along a principal axis (say x).

Poisson's ratio $\equiv \nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = \frac{\lambda}{2(\lambda + 2\mu)}$

Young's modulus $\equiv E = -\frac{\sigma_{11}}{\varepsilon_{11}} = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$

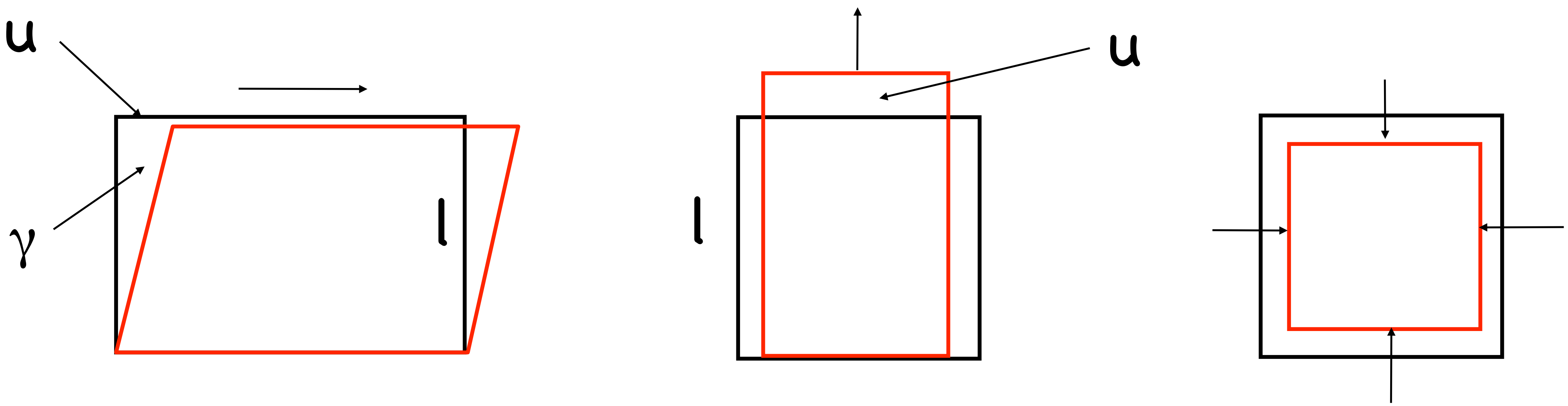
$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

For Poisson's ratio we have $0<\nu<0.5$.

A useful approximation is $\lambda=\mu$ (Poisson's solid), then $\nu=0.25$ and for fluids $\nu=0.5$

Stress-strain - significance

As in the case of deformation the stress-strain relation can be interpreted in simple geometric terms:



$$\sigma_{12} = \mu\gamma = \mu\varepsilon_{12} \qquad \sigma_{22} = E\frac{u}{l} = K\varepsilon_{ii} \qquad P = K\frac{\Delta V}{V} = K\varepsilon_{ii}$$

Remember that these relations are a generalization of Hooke's Law:

$F = Kx$

Elastic constants

Let us look at some examples for elastic constants:

Rock	K 10^{12} dynes/cm^2	E 10^{12} dynes/cm^2	μ 10^{12} dynes/cm^2	ν
Limestone		0.621	0.248	0.251
Granite	0.132	0.416	0.197	0.055
Gabbro	0.659	1.08	0.438	0.219
Dunite		1.52	0.6	0.27

Equations of elastic motion

We now have a complete description of the forces acting within an elastic body. Adding the inertia forces with opposite sign leads us from

$$f_i + \frac{\partial \sigma_{ij}}{\partial x_j} = 0$$

to

$$\rho \frac{\partial^2 u_i}{\partial t^2} = f_i + \frac{\partial \sigma_{ij}}{\partial x_j}$$

the equations of motion for dynamic elasticity

Equations of motion - P waves

$$\rho \partial_t^2 \mathbf{u} = \mathbf{f} + (\lambda + 2\mu)\nabla\nabla \bullet \mathbf{u} - \mu\nabla \times \nabla \times \mathbf{u}$$

Let us apply the **div** operator to this equation, we obtain

$$\rho \partial_t^2 \theta = (\lambda + 2\mu)\Delta\theta$$

where

$$\theta = \nabla \bullet \mathbf{u}$$

Acoustic wave equation

$$\frac{1}{\alpha^2}\partial_t^2\theta = \Delta\theta$$

or

P-wave velocity

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

Equations of motion - S waves

$$\rho \partial_t^2 \mathbf{u} = \mathbf{f} + (\lambda + 2\mu)\nabla\nabla \bullet \mathbf{u} - \mu\nabla \times \nabla \times \mathbf{u}$$

Let us apply the **curl** operator to this equation, we obtain

$$\rho \partial_t^2 \nabla \times \mathbf{u} = (\lambda + \mu)\nabla \times \nabla\theta + \mu\Delta(\nabla \times \mathbf{u})$$

we now make use of $\nabla \times \nabla\theta = 0$ and define

$$\varphi = \nabla \times \mathbf{u}$$

to obtain

Shear wave equation

$$\frac{1}{\beta^2}\partial_t^2\varphi = \Delta\varphi$$

S-wave velocity

$$\beta = \sqrt{\frac{\mu}{\rho}}$$

... what can we say about the direction of displacement, the **polarization** of seismic waves?

$$\mathbf{u} = \nabla\Phi + \nabla \times \Psi \qquad \Rightarrow \mathbf{u} = \mathbf{P} + \mathbf{S}$$

$$\mathbf{P} = \nabla\Phi \qquad \mathbf{S} = \nabla \times \Psi$$

... we now assume that the potentials have the well known form of plane harmonic waves

$$\Phi = A\exp[i(\mathbf{k} \bullet \mathbf{x} - \omega t)] \qquad \Psi = \mathbf{B}\exp[i(\mathbf{k} \bullet \mathbf{x} - \omega t)]$$

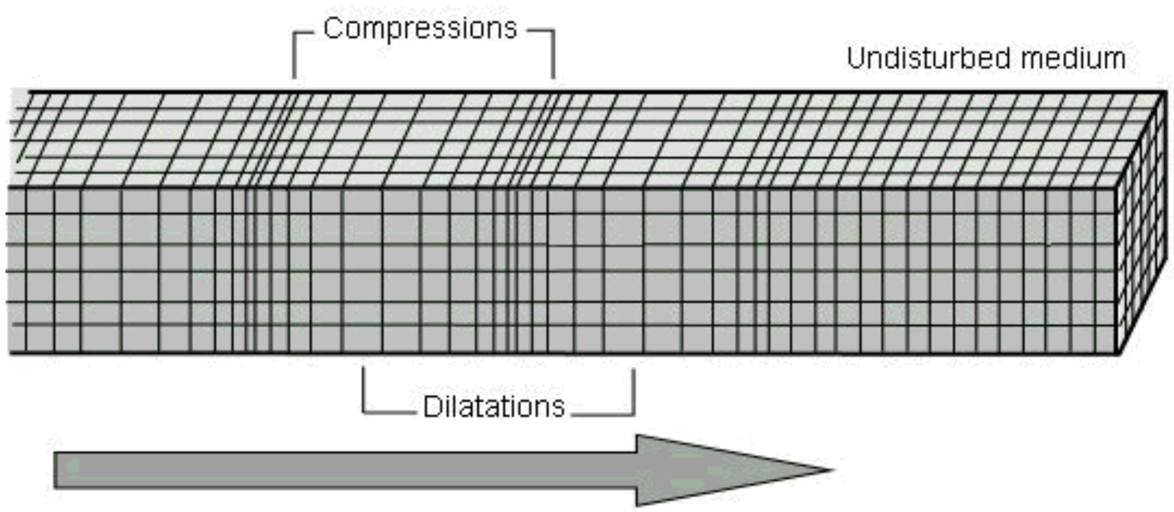
$$\mathbf{P} = A\mathbf{k}\exp[i(\mathbf{k} \bullet \mathbf{x} - \omega t)] \qquad \mathbf{S} = \mathbf{k} \times \mathbf{B}\exp[i(\mathbf{k} \bullet \mathbf{x} - \omega t)]$$

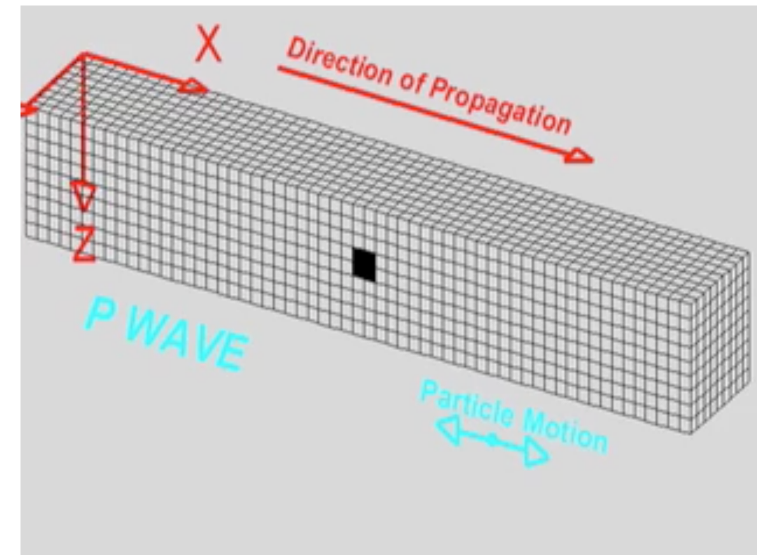
P waves are **longitudinal** as P is parallel to k

S waves are **transverse** because S is normal to the wave vector k

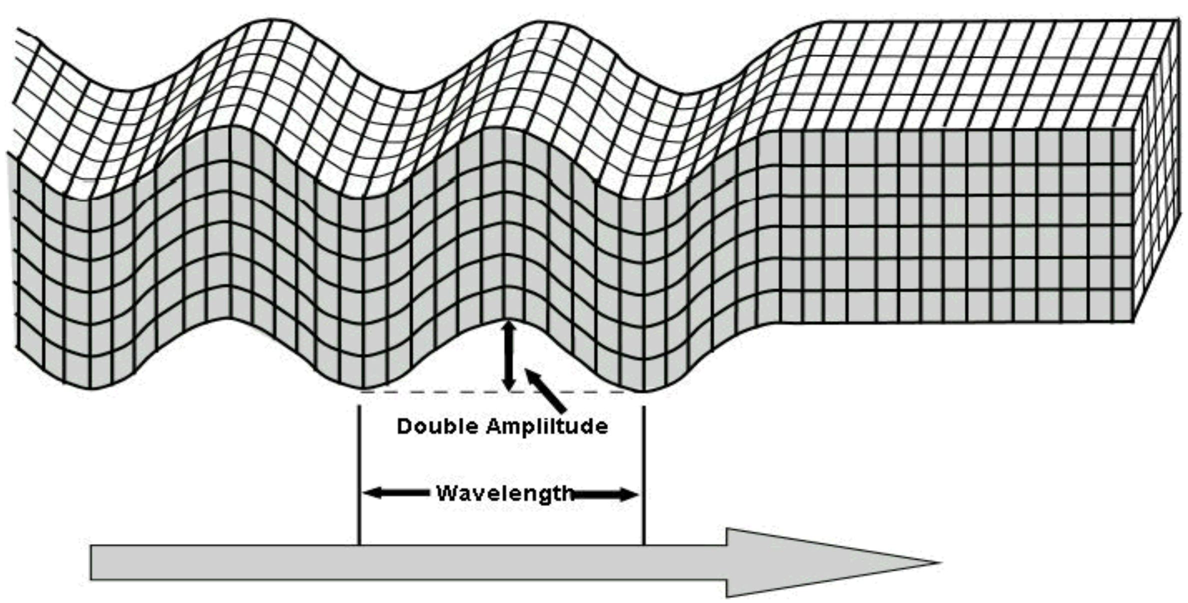
Wavefields visualization - body waves

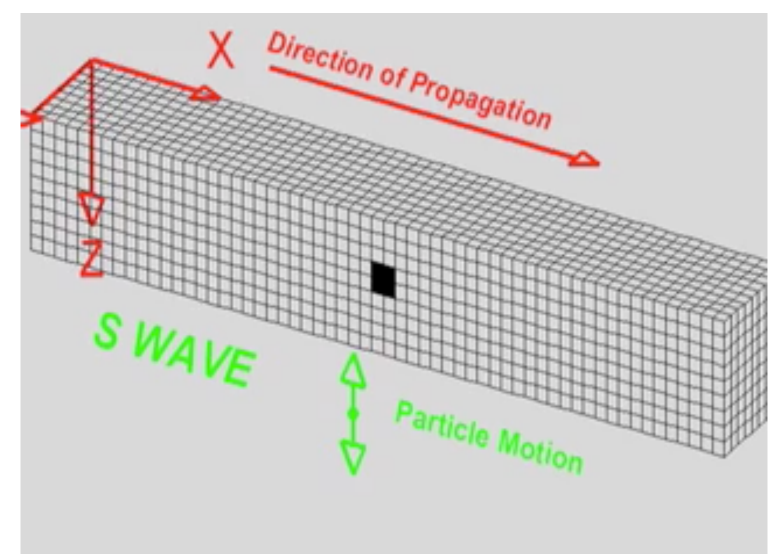
P Wave





S Wave





They are **spherical** waves
and decay as $(r)^{-1}$

https://www.iris.edu/hq/inclass/animation/seismic_wave_motions4_waves_animated

Seismic Velocities

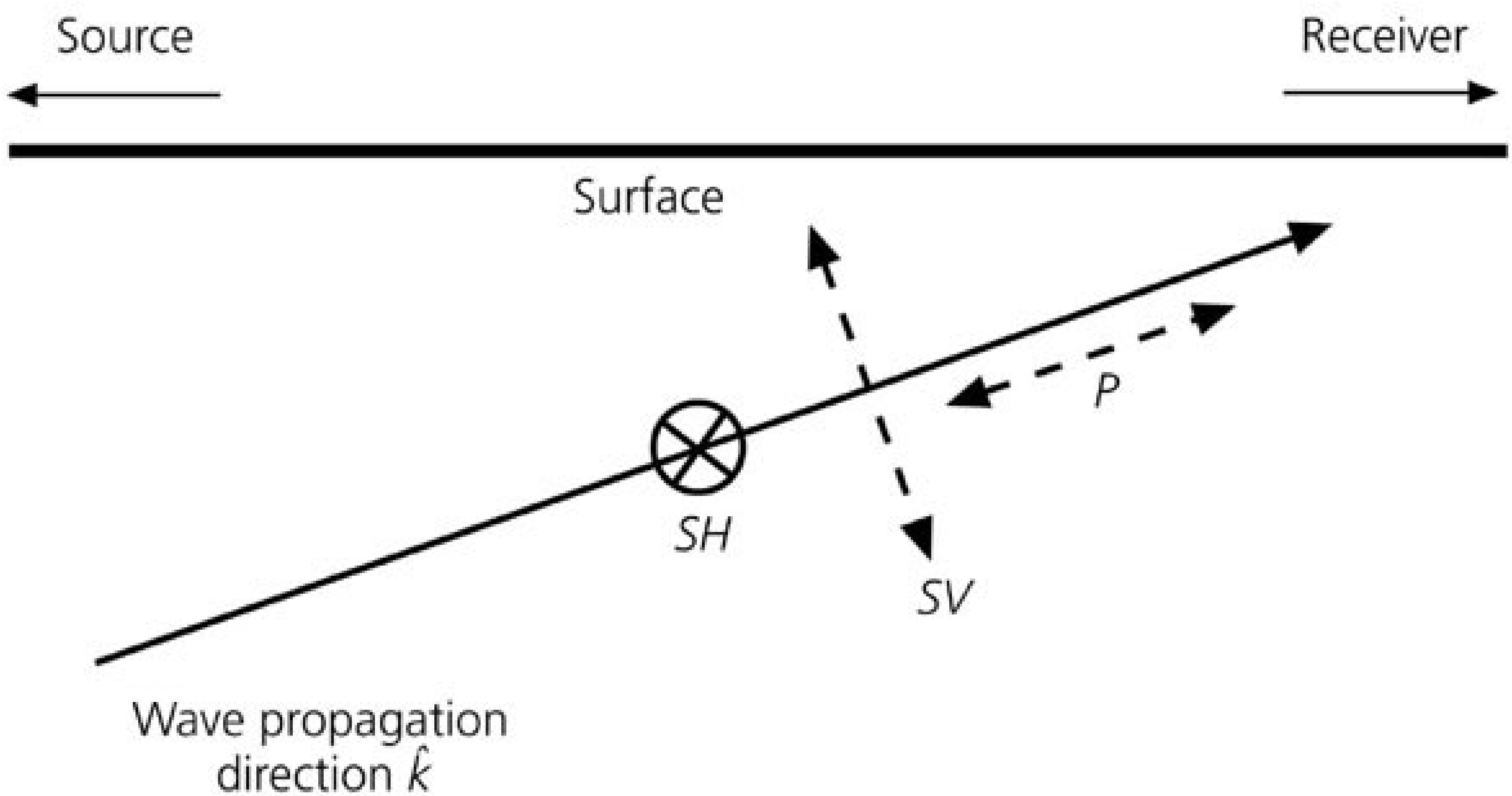
Material	P-wave velocity (m/s)	shear wave velocity (m/s)
Water	1500	0
Loose sand	1800	500
Clay	1100-2500	
Sandstone	1400-4300	
Anhydrite, Gulf Coast	4100	
Conglomerate	2400	
Limestone	6030	3030
Granite	5640	2870
Granodiorite	4780	3100
Diorite	5780	3060
Basalt	6400	3200
Dunite	8000	4370
Gabbro	6450	3420

Seismic Velocities

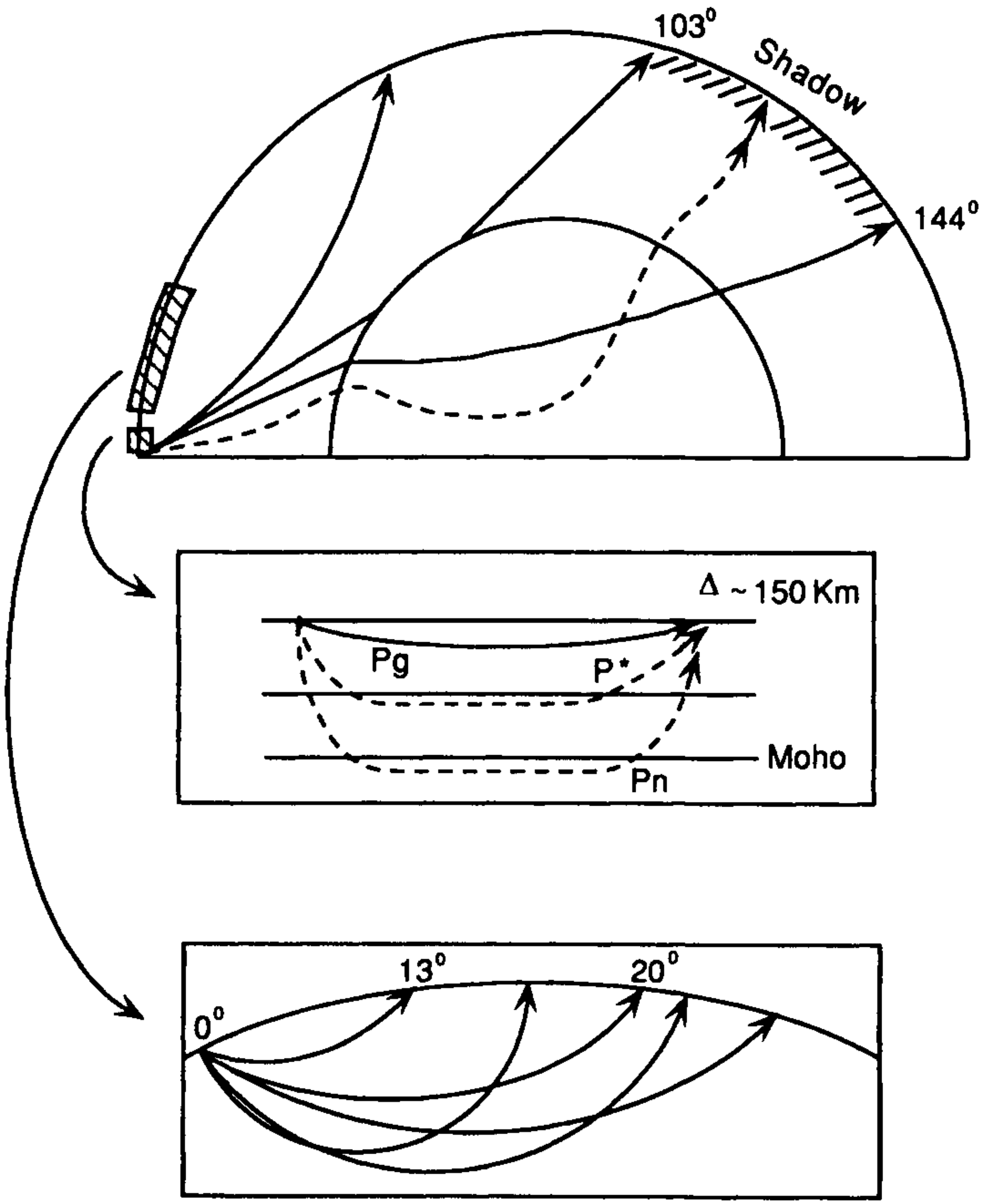
Material	V_p (km/s)
Unconsolidated material	
Sand (dry)	0.2-1.0
Sand (wet)	1.5-2.0
Sediments	
Sandstones	2.0-6.0
Limestones	2.0-6.0
Igneous rocks	
Granite	5.5-6.0
Gabbro	6.5-8.5
Pore fluids	
Air	0.3
Water	1.4-1.5
Oil	1.3-1.4
Other material	
Steel	6.1
Concrete	3.6

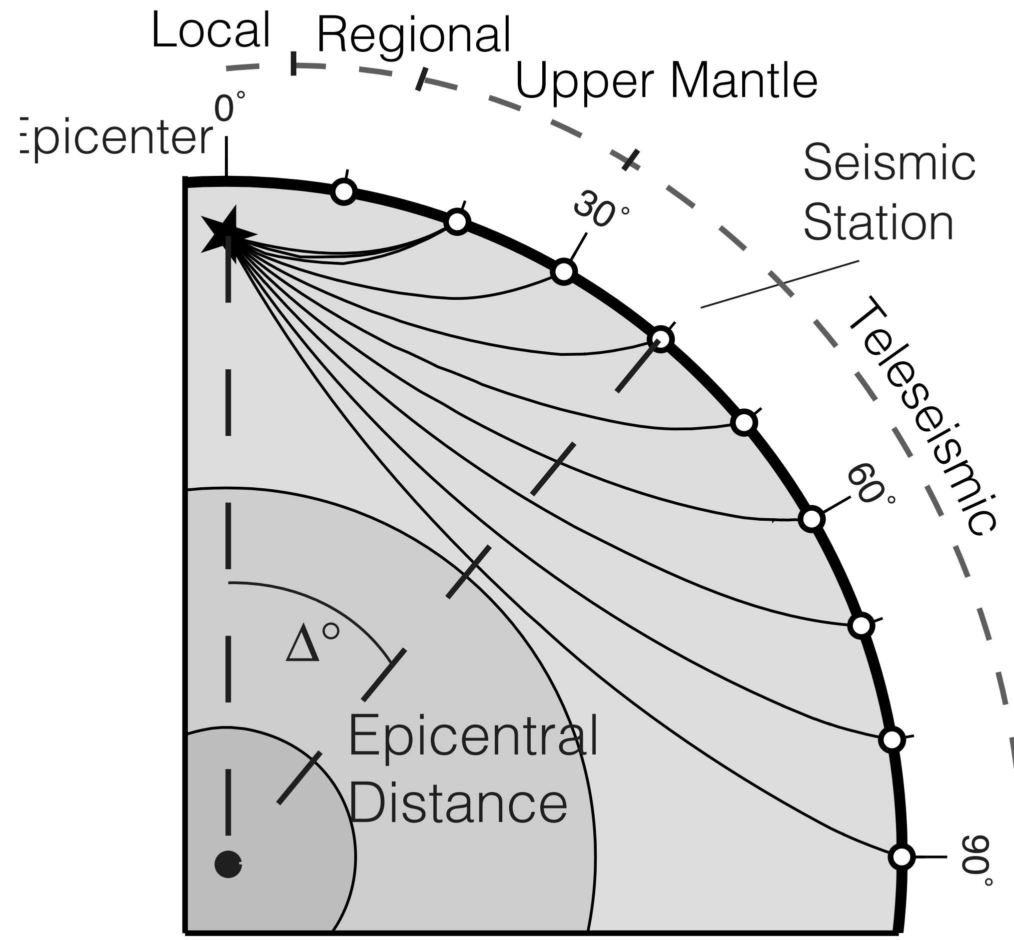
Free surface: P-SV-SH

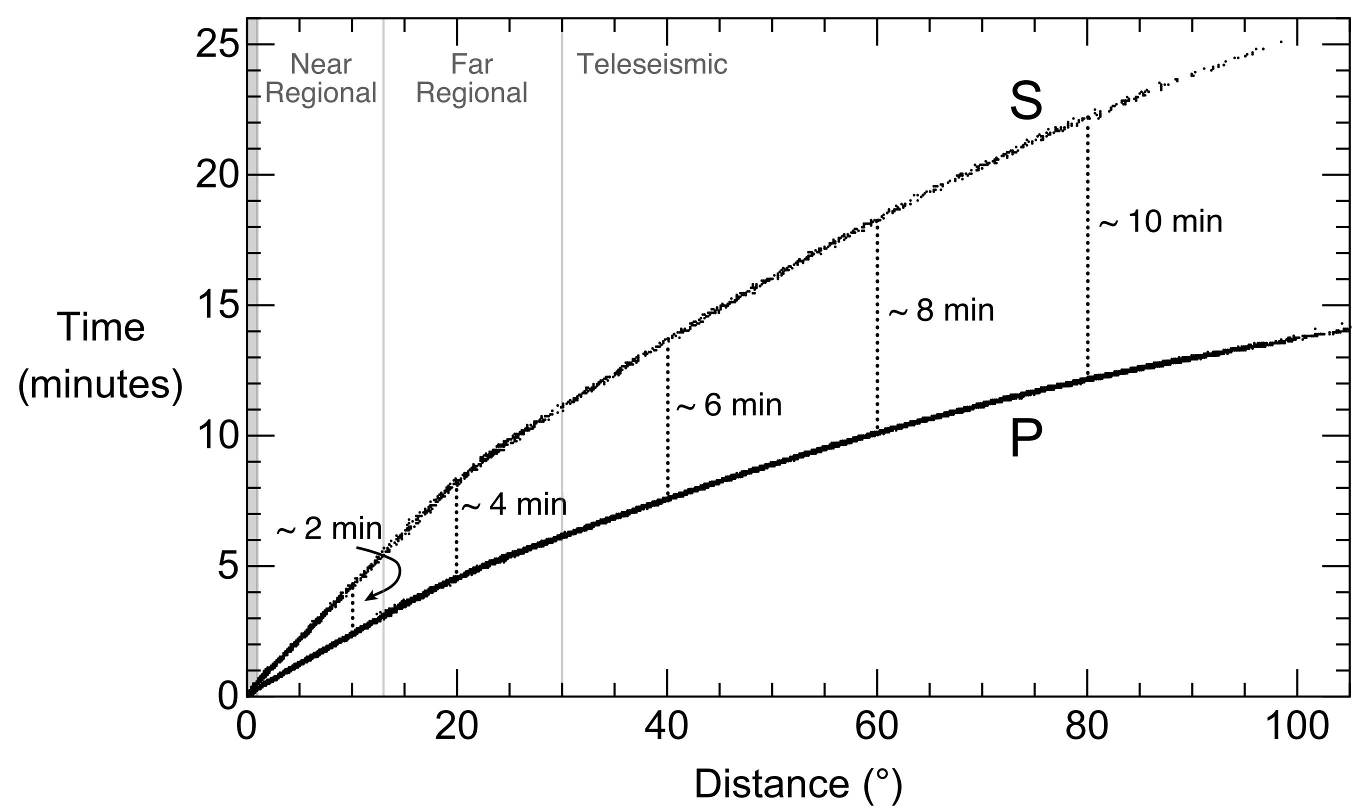
Figure 2.4-4: Displacements for *P*, *SV*, and *SH*.



Seismological (body waves) distance ranges

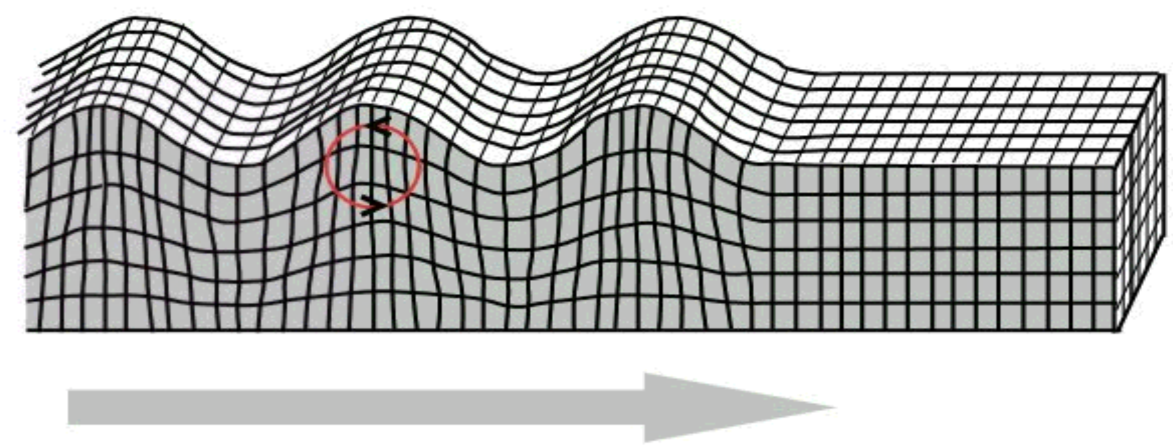




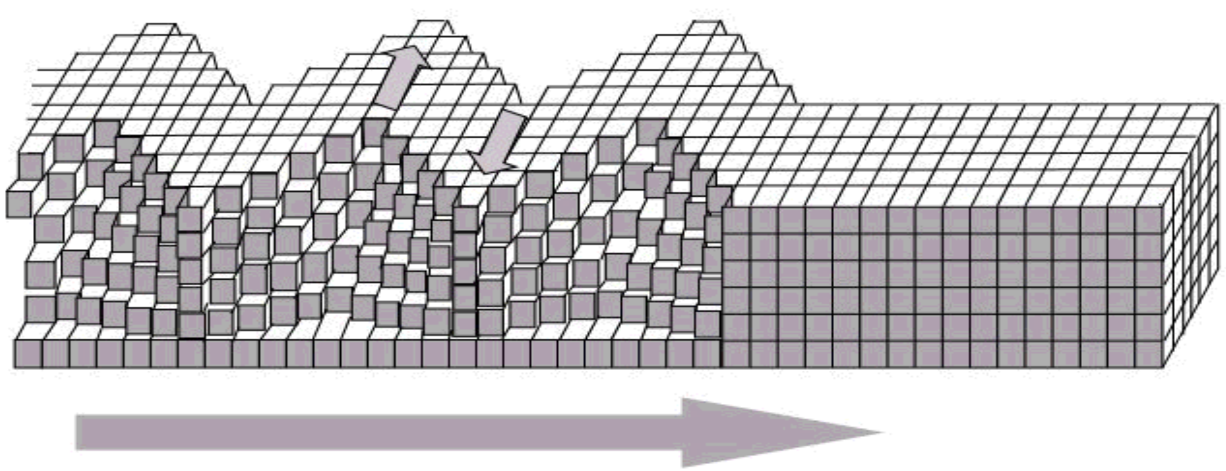


Wavefields visualization - surface waves

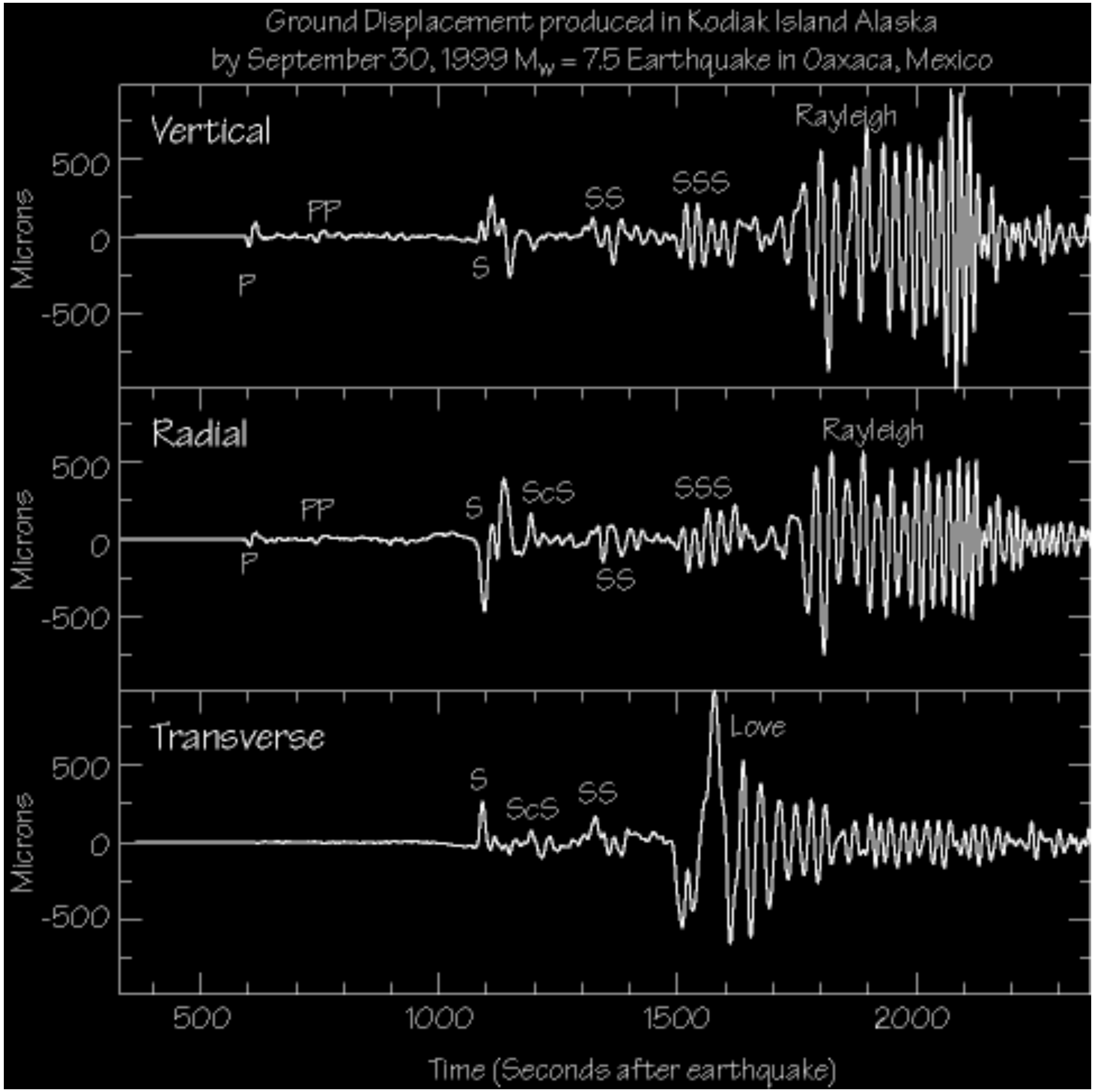
Interference of **P-SV** waves
at surfaces (e.g. free surface)
and velocity is roughly 92% of β



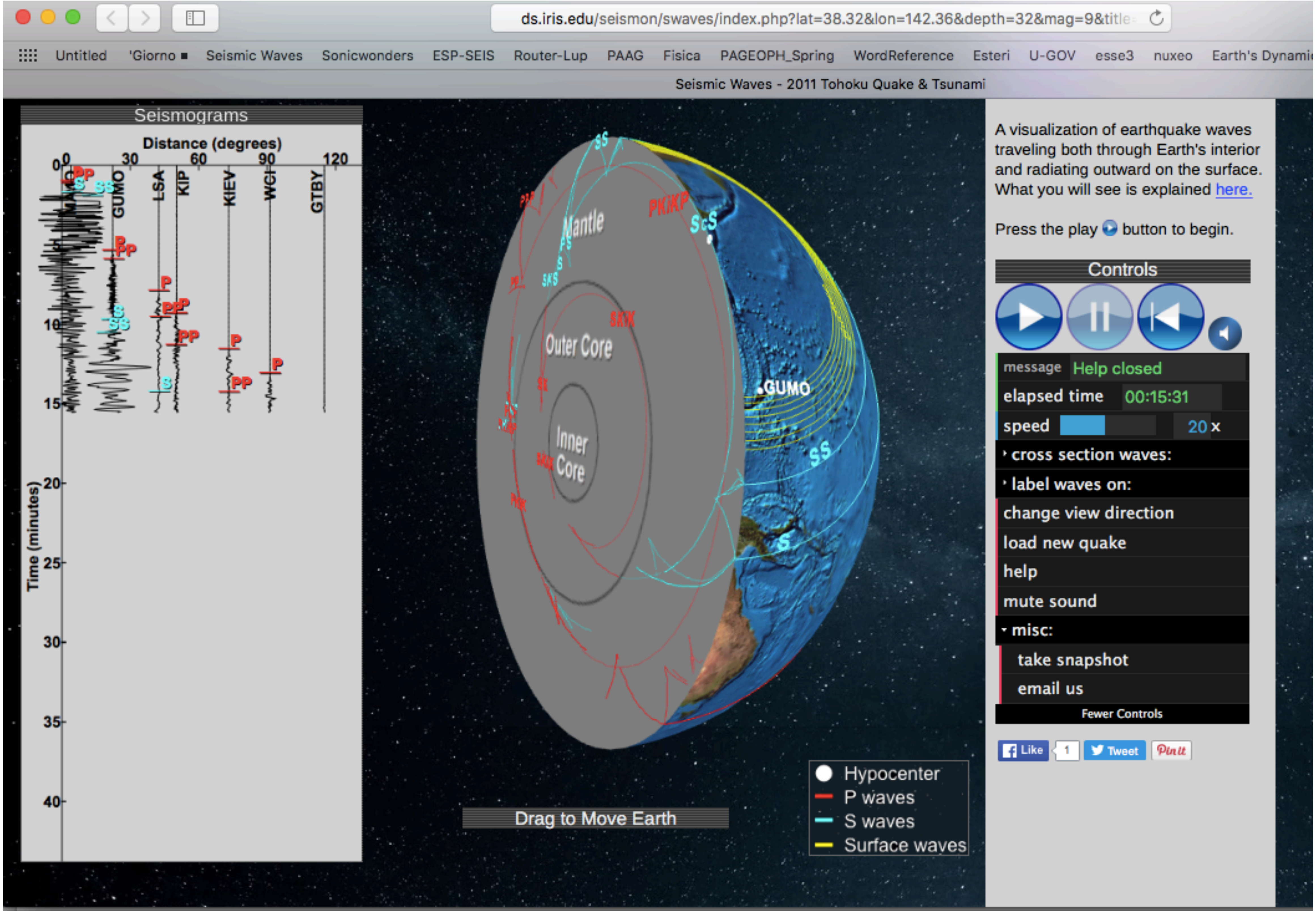
Interference of multiply
reflected **SH** waves
at surfaces (e.g. free surface)
and velocity depends on β



They are **cylindrical** waves
and decay as $(r)^{-1/2}$


Ground Displacement produced in Kodiak Island Alaska
by September 30, 1999 Mw = 7.5 Earthquake in Oaxaca, Mexico
Vertical
Radial
Transverse
Microns
500
0
-500
P
PP
S
SS
SSS
ScS
Rayleigh
Love
500
1000
1500
2000
Time (Seconds after earthquake)

Data example - 2



http://ds.iris.edu/seismon/swaves/index.php

Strong motion seismology

- Strong ground motion is an event in which an earthquake cause the ground to shake at least strongly enough for people to feel the motion or to damage or destroy man-made structures.

- The goal of strong motion seismology is to be able to understand and predict seismic motions sufficiently well that the predictions can be used for engineering applications

- The field of strong-motion seismology could initially be identified with a type of instrument, designed to remain on-scale and record the ground motion with fidelity under the conditions of the strongest ground motions experienced in earthquakes.

Anderson J.G Physical Processes That Control Strong Ground Motion. In: Gerald Schubert (editor-in-chief) Treatise on Geophysics, 2nd edition, Vol 4. Oxford: Elsevier; 2015. p. 505-557.

Strong motion seismology

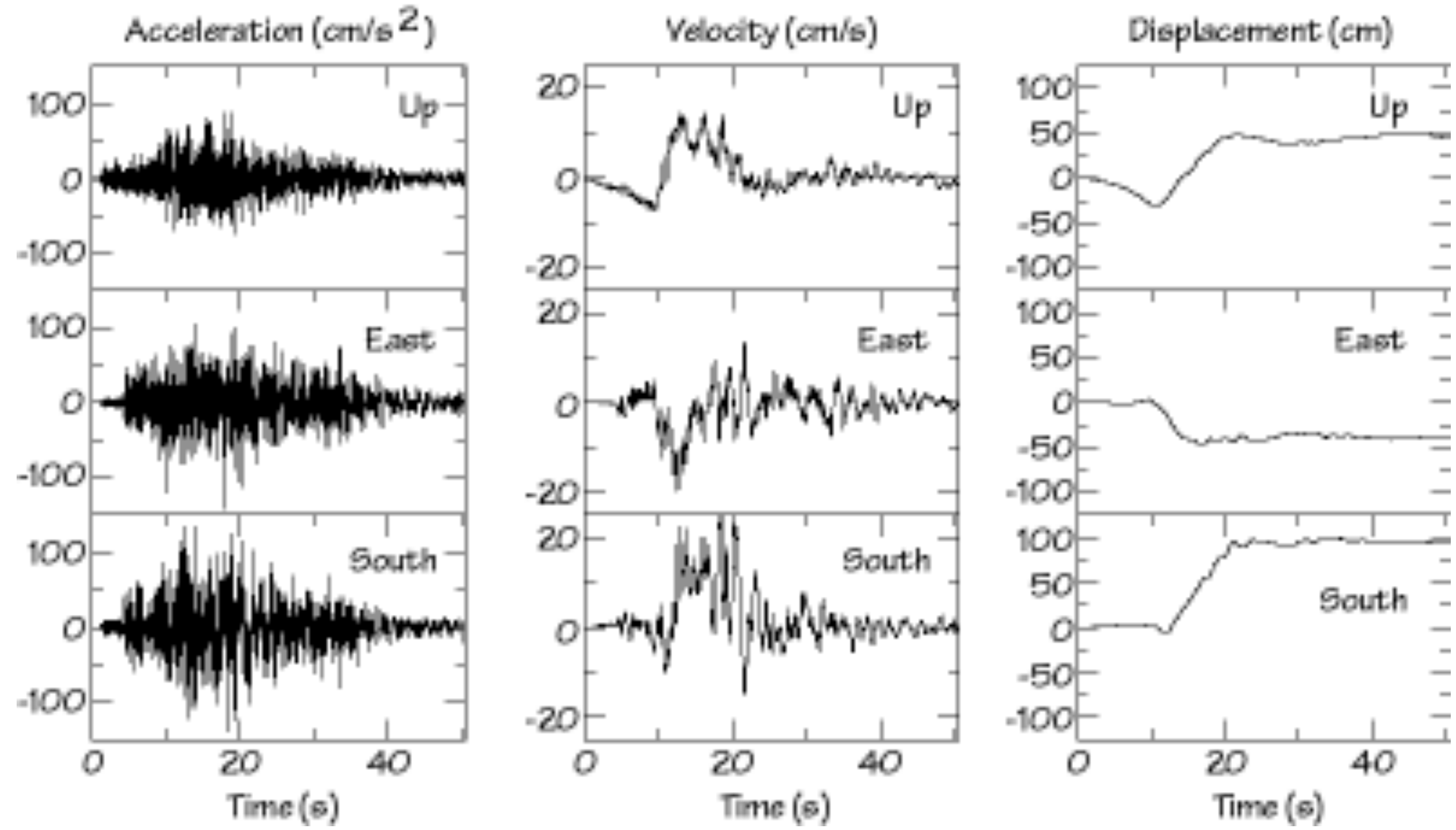
- Early instruments were typically designed so that ground motions up to the acceleration of gravity (1g) would be on-scale.

- The lower limit of ground motion considered by the early strong motion seismology studies was roughly defined by the thickness of the light beam read until the edge of a recorded film. The minimum acceleration resolved is somewhat less than 0.01g, that approximately coincided with minimum ground motions that humans are able to feel.

- Since much smaller ground motions can be recorded on modern instruments, the distinction between strong-motion seismology and traditional seismology is blurred.

Example of Recordings

Ground acceleration, velocity and displacement, recorded at a strong-motion seismometer that was located directly above the part of a fault that ruptured during the 1985 Mw = 8.1, Michaocan, Mexico earthquake.



The left panel is a plot of the three components of acceleration: strong, high-frequency shaking lasted almost a minute and the peak acceleration was about 150 cm/s^2 (or about 0.15g). The middle panel shows the velocity of ground movement: the peak velocity for this site during that earthquake was about 20-25 cm/sec. Integrating the velocity, we can compute the displacement, which is shown in the right-most panel: the permanent offsets near the seismometer were up, west, and south, for a total distance of about 125 centimeters.