AG3 - second assignement

October 20, 2022

1. Consider the morphism $f : \mathbb{A}^2 \to \mathbb{A}^2$, $f(x, y) = (x, xy)$. Determine the image $f(\mathbb{A}^2)$ and specify if it is open, dense, or closed.

2. Consider the morphism $f : \mathbb{A}^3 \to \mathbb{A}^3$, $f(x, y, z) = (x, xy, xyz)$. Determine the image $f(\mathbb{A}^3)$ and specify if it is open, dense, or closed.

3. Let $X \subseteq \mathbb{A}^n$ a Zariski closed consisting of two points. Prove that $A(X) \cong K \oplus K$.

4. Let $f : X \to Y$ be a morphism e let $\Gamma_f = \{(x, y) \in X \times Y \mid y = f(x)\}$ be its graph. Prove that Γ_f is isomorphic to X.

5. Let $X \subseteq \mathbb{A}^n$ and $Y \subseteq \mathbb{A}^m$ be two Zariski closed, and let p_Y be the *second projection*:

$$p_Y : X \times Y \to Y, \qquad p_Y(x, y) = y.$$

   Prove that if $Z \subseteq X$ is closed and $f : X \to Y$ a morphism, then

$$f(Z) = p_Y((Z \times Y) \cap \Gamma_f.$$